Technical Report 40

THE STRUCTURE OF THE SMALL ANNUAL MODEL

David E. Rose

Jack G. Selody

The views expressed in this report are those of the authors; no responsibility for them should be attributed to the Bank of Canada.

ACKNOWLEDGEMENTS

The SAM project began in 1980 under the direction of Paul Masson. Although he did not participate directly in the preparation of this volume, having left the project for other duties in 1982, many of his ideas remain important in the current model. His contribution is particularly important in our modelling of the household sector's consumption and labour supply decisions, as well as in our treatment of the balance of payments and the exchange rate.

Several other people have contributed directly to our work. John Armstrong deserves special mention. As our references will attest, he has provided important input to the estimation of the model. Moreover, he contributed to the analysis of the supply side of the model and should be considered a coauthor of Chapter 4. Bruce Montador did important work on the original data base and on the theory of household decisions reported in Chapter 3. Kevin Lynch and Tom Bennett also participated in the initial stages of the project.

We have enjoyed dedicated research assistance from a talented group of people. In chronological order they are: Pierre Cloutier, Frieda Wong, Louis Fournier, John Parker, Jean St-Gelais, Andrew Karolyi, and Nada Golob. John Parker and Jean St-Gelais survived with grace the worst of the 'Sturm und Drang'. To all of them we owe a debt of gratitude.

Many people have given us the benefit of their support and critical comments. Jean-Pierre Aubry, Pierre Duguay and Brian O'Reilly deserve special mention, but we have also been helped by so many of our colleagues, both within and without the Bank, that listing them all is not possible. We thank them and request their understanding of our decision not to cite individually the more than thirty people who have commented on drafts or discussed particular issues with us. We do wish to specifically acknowledge Charles Freedman's valuable comments on the final draft. We also wish to thank our former department chief, Bill White, for his comments on several of the chapters and for his patience through the long periods of development and experimentation with the model. We are grateful to him and to the Bank of Canada for giving us the opportunity to confront the challenge of building SAM.

In the production of a volume of this magnitude, much more than the labour of the authors is required. We have been extremely fortunate to have had the good-humoured and efficient services of Sylvie Génier and Nicole Larocque on the word processor. The visible part they have played in the preparation of this volume represents only a fraction of their efforts. We also gratefully salute the insightful editorial contribution of Jill Moxley. She has significantly improved the clarity of our exposition.

Finally, we thank Susan and Heather for their understanding and support.

D.E.R., J.G.S.
May, 1985

CONTENTS

Abstract i

Résumé ii

Chapter 1 AN INTRODUCTION TO SAM 1

1.1 Background and Motivation for the Model 1
1.2 An Overview of SAM 11
1.3 Long-Run Equilibrating Mechanisms in SAM 18
1.4 Disequilibrium Dynamics in SAM 30

Chapter 2 SECTORAL ACCOUNTING AND GOVERNMENT FINANCE 35

2.1 Introduction 35
2.2 The Sectoral Accounts of SAM 35
2.3 Some Background on Notation and Function Forms 43
2.4 The Government Sector 47

Chapter 3 HOUSEHOLD CHOICES: CONSUMPTION, LABOUR SUPPLY (THE PARTICIPATION RATE), AND REAL MONEY BALANCES 69

3.1 Introduction 69
3.2 The Basic Model of Consumption and Labour Supply 72
3.3 Real-Balance Preferences and the Equilibrium Price Level 75
3.4 Modifications to the Basic Model 79
3.5 Human Wealth: VHPVN 84
3.6 Estimation Procedure and Results 93
3.7 Properties of the Estimated Equations 105

Chapter 4 THE SUPPLY SIDE OF SAM: OUTPUT, FACTOR USE, INVENTORIES, AND CAPACITY UTILIZATION 113

4.1 Introduction 113
4.2 SAM's Production Technology and Output Equation 114
4.3 Technical Progress: DNPRL, PRODL 120
4.4 The Steady-State Model of Aggregate Supply 121
4.5 Factor-Adjustment Processes 150
4.6 Mechanisms for Short-Run Buffering: Inventories and Capacity Utilization 161
4.7 The Estimation Procedure 165
4.8 Estimation Results 167
4.9 The Technology and Static Properties of the Supply System 178
4.10 Sector Properties: Response to Demand Shocks 183

Chapter 5 LINKS TO THE WORLD IN SAM: IMPORTS, EXPORTS, DIRECT INVESTMENT, AND THE SUPPLY OF NET FOREIGN ASSETS 193

5.1 Introduction: Balance of Payments Identities 193
5.2 The Trade Equations 199
5.3 Estimates of the Trade Equations 205
5.4 Foreign Investment and the Stock of Foreign-Owned Domestic Capital: KWT 210
5.5 Profits Accruing to Foreigners: FOPRO 211
5.6 Relative Trade Prices 212
5.7 Sector Properties 216

Chapter 6 ASSET MARKETS AND THE DETERMINATION OF RATES OF RETURN, ASSET PRICES, AND THE EXCHANGE RATE 219

6.1 Introduction 219
6.2 An Overview of the Asset System 224
6.3 Expected Holding-Period Yields, Asset Prices, and Inflation 237
6.4 Estimation of the Demand System 249
6.5 Properties of the Asset System 260

Chapter 7 MARKET DYNAMICS: THE DETERMINATION OF PRICES AND WAGES 271

7.1 Introduction 271
7.2 An Overview of Price and Wage Determination 272
7.3 The Price and Wage Equations: Structure and Estimates 280
7.4 Identities and Market-Link Equations 290

Appendix A Mnemonic Definitions 295

Appendix B Model Equations 307

Appendix C Cross References 329

References 343

ABSTRACT

This volume contains a detailed description of the structure and sectoral properties of the Bank of Canada's Small Annual Model, SAM. The SAM model, constructed in the Research Department of the Bank, is designed for medium- to long-term simulation. It is small by econometric model standards; the version described in this report has 25 stochastic equations and 103 equations in total.

SAM is a model of the aggregate Canadian economy with complete wealth accounting, in which markets work and generate eventual convergence to a competitive steady state. At the heart of the model is a description of an equilibrium path that exploits restrictions from formal theory, particularly with respect to the long-run determinants of aggregate supply. The complete wealth accounting is an important part of the model because of the attention one must pay to stock equilibrium in long-term analysis. In SAM all flow/stock linkages are explicit and integrated into the budget constraints and other equations that influence the model's long-run properties.

Behaviour in SAM is forward looking. For example, expectations formation is specified to be quasi-rational; that is, it uses information about the steady state to which the system is tending. Some of the behavioural theory is explicitly forward looking -- formal intertemporal optimization; but even where the theory is static the model is given forward-looking properties through the specification of dynamic processes of adjustment towards the steady-state path.

SAM should be most useful for the analysis of the medium- to long-run implications of aggregate foreign disturbances and domestic monetary and fiscal policy initiatives. The model has been designed to be flexible and the documentation attempts to show how it can be used in simulation with various policy prescriptions and assumptions about model structure.

RÉSUMÉ

Dans la présente étude, nous présentons une analyse détaillée de la structure du modèle SAM (Small Annual Model) de la Banque du Canada et des propriétés de ses différents secteurs. Ce modèle économétrique a été construit au départment des Recherches de la Banque pour simuler l'économie canadienne en moyenne et longue période. De taille relativement petite, il contient au total cent trois équations, dont vingt-cinq sont stochastiques.

SAM est un modèle de l'ensemble de l'économie canadienne, où sont prises en considération toutes les relations ayant trait à la richesse et où les mécanismes de marché fonctionnent pour orienter l'économie, par le biais de la concurrence, vers une situation d'équilibre. SAM est fondé sur une description d'un sentier d'équilibre qui tire parti des contraintes formulées dans la théorie économique, en particulier en ce qui concerne les facteurs qui déterminent l'offre globale sur longue période. La comptabilisation des relations stocks-flux associées à la richesse y occupe une place importante à cause de l'attention qu'exige l'équilibre des stocks dans les études portant sur le long terme. Toutes les relations entre les flux et les stocks sont formulées explicitement et sont intégrées aux contraintes budgétaires et aux autres équations qui influencent les propriétés à long terme du modèle.

Dans le modèle SAM, le comportement des agents économiques est tourné vers l'avenir. Par exemple, les anticipations sont formées de façon quasi-rationnelle, c'est-à-dire qu'elles tiennent compte des informations sur la situation d'équilibre vers laquelle tend le système économique. Certaines théories du comportement sont explicitement tournées vers l'avenir en ce sens qu'elles font intervenir le concept d'optimisation intertemporelle. Toutefois, même là où la théorie sous-jacente présente des aspects statiques, on attribue encore aux comportements des agents

économiques des propriétés prospectives en spécifiant des processus dynamiques d'ajustement vers le sentier d'équilibre.

SAM devrait être d'une très grande utilité dans l'analyse des conséquences à moyen et à long terme des chocs globaux d'origine étrangère ou des changements de la politique monétaire et budgétaire suivie au pays. L'étude tente de démontrer comment le modèle, de conception intentionnellement souple, peut servir à des simulations faisant intervenir divers instruments de politique et différentes hypothèses sur la structure de celui-ci.

Chapter 1

AN INTRODUCTION TO SAM

Everything should be made as simple as possible -
but not simpler!
Albert Einstein

1.1 Background and Motivation for the Model

This volume contains the first full documentation of SAM, the Bank of Canada's Small Annual Model. The primary motivation in constructing SAM was to provide a simple but complete model of a small, open economy for medium- to long-term policy simulation. It was decided that to serve this end the model should be as firmly grounded in theory as was practicable, consistent with it being an estimated model. It was to focus on essential relationships for medium- to long-term analysis and not emphasize detail important only in the short run. A substantial background discussion of the authors' views on the state of macroeconomics, goals in building the model, and intended methodology of model construction was published previously (Masson et al. 1980). That report stands as a useful companion piece to this volume, providing more of our roots in the literature than is possible here. In this report we describe how our plans were put into practice in a model that integrates informal dynamic theories of the business cycle with more formal choice-theoretic descriptions of long-run equilibria.

The model presented is a complete and functioning system, incorporating many specific choices for such things as the targets and decision-rules of the monetary and fiscal authorities. We have made considerable efforts, however, to go beyond simply documenting one particular set of equations that constitutes a complete and closed model. The reader will find many instances of discussions of alternative specifications, especially with respect to policy variables. Although SAM has a solid core structure, embodying our interpretation of what theory and accounting identities can tell us, the model has been designed to be as flexible as possible for users in simulation.

This volume consists of seven chapters and three appendices. In this introductory chapter we provide some background to the model, our goals in constructing it, and an overview of its content. We also provide an overview of how the model works, both in terms of its long-run equilibrating mechanisms and its short-run dynamics. There follow six chapters that describe the model in detail. Chapter 2 provides the model's accounting framework and our specification of government activity. Chapter 3 describes our theory of household consumption and labour supply decisions, and the nature and importance of households' long-run, real-balance preferences. In Chapter 4 we describe the supply side of the model. This includes the production technology, the factor demands of firms, and the use of inventory stocks and capacity utilization to buffer shocks. Chapter 5 describes the trade equations of SAM and certain other links to the world. This discussion is completed in Chapter 6, where we describe asset demands and supplies, and the determination of interest rates and asset prices. Finally, in Chapter 7, we describe the details of two important market-adjustment processes in the model, namely, wage and price dynamics.

Appendix B contains a complete listing of the version of SAM described in this volume, including the default simulation rules we have provided for things like policy rules and expectations formation. There is also a double listing (Appendix A) of all variables with their definitions. We suggest that readers remove one of these lists to keep at hand when studying the detailed structure. Appendix C contains complete cross references showing, for each variable and parameter, a list of the equations in which that variable or parameter appears.

Equations reported in the text are labelled using the model's identification system if they are, in fact, model equations appearing in the Appendix B listing. Equations presented for expositional purposes only are labelled using a standard numerical convention.

1.1.1 SAM and its predecessors

The Bank of Canada has a long tradition of construction and use of econometric models. The RDX1 model appeared in 1969 (Helliwell et al.

1969), and the first version of its much larger successor, RDX2, comprising several hundred equations, was published in 1971 (Helliwell et al. 1971). In the years that followed, RDX2 was re-estimated several times and used extensively for policy analysis until its retirement in 1979. The RDX tradition continues at the Bank. A new version of the model, estimated with seasonally adjusted data, is currently used for the quarterly projection exercise: it is called RDXF (Robertson and McDougall, 1982 a and b). In contrast, SAM is an annual model with about two dozen behavioural equations, and less than a hundred equations in all. The large number of equations in RDXF is necessary to give the sectoral detail important for short-run projections. However, this detail can become a hindrance for longer-term analysis. The longer the simulation horizon the less important are likely to be the special factors useful in short-term forecasting, and the more relevant are likely to be the restrictions implied by economic theory. It is much easier to impose restrictions from theory when the model has relatively few equations. The complexity of a large model often makes it difficult to specify exactly what the constraints should be. Even when this is not a problem, the limitations of estimation technology make it impossible to estimate simultaneously a large number of equations.

Although the RDX family of models has proven very useful, there have been problems. Economists have become increasingly uneasy in recent years as to whether models that do not fully exploit the restrictions from theory can provide reliable medium- to long-term answers to policy questions. There has also long been a feeling that the linkages between monetary instruments and both real variables and prices are stronger than those captured in the large models. The evidence of reduced-form analysis and experience seemed to conflict with large-model properties. It was felt, therefore, that there was a place for a smaller model, solidly based in theory, for use in medium- to long-term simulation analysis. It was hoped that such a model could help resolve some of the difficulties that had been identified with using larger models for such purposes, and that the small model could provide a useful check on the medium-term projection properties of the large models.

By choice, the model was to distinguish itself from its predecessors in the nature of its use of historical data to determine structure. Poor fit was to be a concern only where it was judged to arise from inadequate structure as opposed to special factors. As we made clear in Technical Report 22, we were ready to impose our priors on both structure and parameterization. As the project developed, however, we found that there were limitations to such an approach. Even where theoretical arguments are reasonably clear there are usually important quantitative questions that can affect policy judgments. For example, it is often important to know how large an effect is, not simply whether it exists. The use of estimation techniques can eliminate some of the uncertainty and provide important information on relevant regions for tests of the sensitivity of model properties to the particular parameters used. Moreover, we found that for medium- to long-term results to be accepted with any confidence, we had to demonstrate that the shorter-term properties of the model were not at odds with historical experience. As a result, while retaining our primary focus on long-run properties and the exploitation of theory, we have put more effort than originally anticipated into historical validation in estimation. The reader will see that there is a solid core of the model where parameters have come from estimation. We have made extensive use of simultaneous estimation techniques in the process.

1.1.2 Goals for SAM

Exact correspondence between model accounts and markets

In many models the primary focus is explaining movements in particular published data; for example, a particular interest rate or component of the national accounts. It is natural for a forecasting model to take this perspective, since in such exercises there is a big advantage in removing debate over the facts to be explained, in facilitating communication among modellers and users of model output, and in providing a common ground for testing competing hypotheses. It is also natural that short-term forecasting models concentrate on explaining flows.

For SAM we have adopted a different perspective on both points. Our goal was to specify a simple, but complete, set of sectoral accounts and to provide a model of the elements of these accounts using a framework of markets associated with each notionally distinct stock and flow. In the data construction, official published data are combined and transformed to conform with the economic abstraction, and the modelling job is defined to be the representation of these transformed data. For example, government debt is aggregated into one entry on the private sector balance sheet, which we call bonds, and appropriately weighted combinations of 'coupon' rates and yields are constructed. It is these concepts that we try to explain, and not, for example, any particular published market rate. All models contain examples of this type of harmonization of data, model concept and endogenous variable list. What is different about SAM is the scrupulous enforcement of this consistency and completeness. Every endogenous variable in the model has a clear interpretation in terms of the model's abstract markets and is measured accordingly. No explanation is offered for variables outside these markets (e.g., at lower levels of aggregation), and all prices, rates and quantities notionally determined in these markets, with the exception of energy supply, are endogenously determined by model equations. Naturally, there remain important exogenous variables that could be explained in a more general model of the world or more detailed model of the domestic economy.

There are several advantages to SAM's exact correspondence between sectoral accounts and hypothesized markets. First, it is in the modelling of the elements of market processes that economic theory most clearly applies. Moreover, the correspondence of markets and accounts permits a specification in which all elements in balance sheets and income statements (or budget constraints or financing requirements) are explicitly considered in the choice theory and nothing is left residual (possibly with unsatisfactory properties) in the sense of the 'pitfalls' of financial modelling.[1] For example, if a notional market is specified in which a particular measured interest rate is determined, then unless the financial instrument to which this rate applies is explicitly represented in the model's balance sheet accounts, it is difficult to represent exactly the endogenous links between asset stocks, interest rates and the income flow accounts. The lack of an explicit link does not change the fact that there is an identity linking interest payment flows to the stocks. If the link is not explicit in the model it shows up notionally somewhere else. For example, there could be an unrepresented change in the structure of interest rates and/or an offsetting change in some other flow below the level of aggregation of the explicit model.

---

[1] See, for example, W.C. Brainard and J. Tobin (1968).

In SAM all stock-flow linkages are explicit in the endogenous structure of the model. We consider this quite important for a model designed for medium- to long-term simulation of worlds in which sectoral deficits can be an important and long-lasting phenomenon. Indeed, as a general point one can say that stocks are given a prominent place in SAM. An important part of long-run equilibrium is that stocks be willingly held. In SAM, these stock equilibrium conditions are given equal status with flow equilibrium conditions. Indeed, because of the intrinsic dynamics that arise from the links between stocks and flows, it is fair to say that attaining stock equilibrium is the fundamental requirement of a steady state. Such stock equilibrium conditions apply to real stocks, such as producing capital and inventories, as well as to financial stocks. Of particular interest in a small open economy are the determinants of a nation's net debt position vis-à-vis the rest of the world.

The use of restrictions from theory

A second specific goal in building SAM was the exploitation of restrictions from behavioural theory. Behavioural models are always subject to debate. Moreover, the nature of specific restrictions from theory depends very much on the details of the particular abstraction. For example, particular utility functions may imply particular restrictions on demand functions. But theory, per se, cannot specify whether it is 'correct' to impose such restrictions. Some behavioural restrictions are general enough that they virtually transcend uncertainty about functional form and other such details; these are rare. More often, what specific restrictions can be imposed must be an empirical matter. But the absence of detailed theoretical results cannot be taken as licence to ignore the issue. Decisions made by any one agent are subject to interdependencies, almost regardless of the detail of the specific model. For example, except for very special cases, factor demands of firms will be subject to cross-equation restrictions arising from the common technological constraint on the behavioural choices. Similarly, only under special assumptions can household consumption, labour supply, and asset composition decisions be separated. Given a particular choice for a decision framework, a model will not be consistent if it does not incorporate whatever specific restrictions apply. Indeed, failure to impose such restrictions may result in an inconsistency between plans and constraints in the long run. Given the long-run perspective of SAM, it was considered essential that all such inconsistencies be avoided, and all restrictions inherent in the chosen decision models be imposed. Where it is feasible to do so, such as in cases where restrictions are purely parametric, we generally report tests of the restrictions in estimation. Where the restrictions involve such things as particular functional forms, however, we have generally not tested them, especially when such a procedure would have required non-nested-hypotheses techniques.

In choosing what 'theory' to exploit we have taken an eclectic approach. We describe our reasons for particular choices in the detailed chapters that follow. We have been most ambitious in modelling the household sector's decisions, where we specify a full intertemporal optimization problem from which we derive consumption and labour supply functions. In this case there are some truly dynamic elements in the formal theory. For firms, on the other hand, we use a static optimization framework, fully integrating factor demand decisions and exploiting restrictions from the technology, but not formally building in an intertemporal dimension to the decisions. Even though the theory is static, however, there are strong forward-looking elements introduced through variables used as inputs into the decisions. For example, sales expectations and factor price expectations influence factor demands; and all such expectations are forward looking.

Part of the essential core theory of SAM represents a specification of the properties of a steady-state growth path and the integration of the requirements of such a path into the behavioural specification. In some cases this involves explicit use of the conditions in specifying some aspect of behaviour. For example, knowledge of key exogenous elements, such as population and productivity growth, that determine steady-state real output growth, is given to agents in formulating forward-looking plans and expectations. In other cases the steady-state consistency properties are imposed on aggregate decisions as market-generated restrictions. For example, firms' long-run targets are formulated to be consistent with full employment of resources. SAM does not ask whether equilibria exist or whether market processes are sufficient to ensure the eventual attainment of those equilibria. Rather, the equilibrium properties are as strongly specified as is possible from theory, and dynamics are specified as carefully chosen, but formally ad hoc, descriptions of adjustment towards equilibrium on the presumption that markets work. This is not to say that market failures, by any reasonable definition, do not occur, or that adjustment is rapid.

Elaboration of aggregate supply, energy as a factor of production

A third specific goal in the construction of SAM was to elaborate the supply side of the model, relative to previous RDX models, by introducing energy as a factor of production, by allowing a more flexible form for aggregate technology, and by giving more attention to the role of supply-side restrictions on output in the long run. We have done all these things, although our modelling of energy is limited to the domestic demand for energy as a factor of production. Investment, employment, and output decisions of the energy sector are not directly modelled. Energy is treated as available without limit in the world market, at a world price not influenced by Canada. We specify the domestic energy price to be the world price adjusted for domestic price policies and multiplied by the endogenous price of foreign exchange. For this discussion the key point is that in SAM energy is not treated as being subject to a resource constraint or even an upward-sloping supply curve. At the world price

Canada can obtain essentially unlimited quantities and so potential output of the non-energy good is not directly constrained by energy supply. Rather, the scarce factor is labour. More than in any other macroeconometric model of which we are aware, the available supply of labour input constrains output in the long run in SAM. In this regard it is important to distinguish between the level and the growth rate of output. The steady-state real growth rate is determined by exogenous technical progress and population/labour-force growth. The long-run level of output, however, is endogenous and fully respects the requirements of a full-employment, zero-excess-profit, competitive aggregate supply.

Explicit treatment of expectations

A fourth goal of SAM was to make more explicit the way in which expectations enter a macro model. This was considered important not only because expectations are an integral part of forward-looking behaviour, but also because of the Lucas critique of econometric models and simulation practice. Lucas emphasized that among the behavioural rules it is expectations formation that is most likely to change as a consequence of changes in the policy environment. If the Lucas critique is to be taken seriously, the structure of behaviour, and especially the structure of expectations formation, must be explicit, and not bound up in historically estimated parameters of distributed lags that combine many dynamic influences. A major benefit from such explicit modelling of the role of expectations is that modification sensitive to the particular problem is possible. This can facilitate both the investigation of direct questions such as the influence of alternative behavioural specifications of expectations formation, and the direct response to the Lucas point in the context of specific simulations. We wish the Lucas critique to be taken seriously by users, and in the specific analysis will remind the reader regularly that no model of expectations formation, no matter how carefully constructed as a basic simulation rule, should be treated as immutable.

All expectations formation in SAM is forward looking, usually exploiting the equilibrium properties of the model. These expectations can be thought of as quasi-rational. In steady state they are fully consistent in the sense of Muth. However, no attempt is made to impose full rationality in the sense of consistency with the dynamic properties of the model in states of disequilibrium. Rather, we specify expectations that take into account the long-run solutions to which the model is tending, but with convenient, simple assumptions about adjustment processes. The result is a quasi-rationality that goes a long way towards incorporating the fundamental insights of rational expectations arguments.

A simple but complete integration of government and private decision-making

A fifth goal of SAM was to make explicit the constraints facing policymakers and how policy variables affect private decisions. Generally speaking, the constraints on policymakers take two forms: those arising from budget constraints they face, and those arising from other constraints in the economy or in the behaviour of other agents over which policymakers have little control. A complete model must, of course, recognize that all government revenues are payments by some agent in the system and that all government expenditures are receipts for other agents in the system. In addition, however, a complete analysis of the impact of government decisions must incorporate the private sector behavioural response to government actions. Otherwise, there is a risk that some of the more important consequences of policy initiatives will be overlooked, making the analysis suspect.

We wanted SAM to be as highly aggregated (i.e., simple) as possible, while still retaining enough detail for meaningful policy analysis. What this should mean in practice is open to debate. Our operating rule was to retain disaggregation only when we could see it adding something fundamentally distinctive to the policy linkages in the model. Thus, for example, we aggregate all levels of government into one sector. This prohibits us from using the basic model to study questions where factors such as intergovernmental transfers are of primary importance, but in our view such issues are not the fundamental policy issues. Rather, the fundamental issues involve how government decisions influence private sector behaviour. For this, the minimal model needs only one government.

On the other hand, but for the same reasons, we retain considerable detail within the government sector, identifying several different types of expenditures, taxes and transfers, because of the fundamentally different ways these activities impinge on private sector behaviour.

1.2 An Overview of SAM

The previous section has, we hope, given the reader a general feel for SAM. We now wish to provide a more systematic overview of the model's structure. In this overview we do not discuss specific equations. We do, however, describe the agents, markets, and main endogenous variables of the model, along with the general nature of the behavioural theories employed.

1.2.1 The accounting structure of SAM

In SAM there are four groups of economic agents: firms, households, governments, and foreigners. There are three domestically produced goods: primary energy, a non-energy good, and a public good. In addition there is a foreign-produced good that is an imperfect substitute for the domestic good, both in Canada and the rest of the world, and a world supply of energy that is perfectly substitutable for domestically produced energy. Domestic households hold their wealth in four forms: non-interest-bearing fiat money, interest-bearing government bonds, net claims on foreigners and 'equity' claims to the domestic capital stock.[2]

Foreigners, in addition to purchasing the domestic non-energy good, hold domestic government securities (denominated in both foreign currency and domestic currency) and direct claims on the domestic capital stock. They also supply other financial instruments, the net foreign assets of households. Domestic households consume the domestic non-energy good and the foreign good, demand financial instruments and supply labour. Firms demand labour, energy, and goods (for investment) and supply goods and equities. Governments demand goods and labour, and supply a public good and financial instruments.

---

[2] In a data sense, our financial claims usually combine a variety of instruments. For example, 'equities' are notional instruments that combine all forms of claims on firms. In particular, corporate bonds are included in the historical measures.

There are six complete markets in SAM, including those for the four financial instruments held by domestic households and markets for labour and the domestic non-energy good. By 'complete' markets we mean markets with endogenous demand, supply and price determination. There are also a few partial markets in SAM (e.g., energy, imports).

We model the demand for energy but we provide no explanation of the supply side of the energy market. Although we retain the accounting concepts of energy sector investment (and capital stock), labour usage, inventories, and net exports, except for the imposition of the identity forcing energy output to be consumed domestically, exported or put into inventories, these variables are left exogenous. The framework is there, however, for users to provide their own energy-sector equations.

We model the demand for imports, but consistent with our small-open-economy perspective, the price of imports in foreign currency is assumed exogenous to Canada. Any quantity of the import good can be had at the going price.

There is no distinct market for capital goods. The single, non-energy output can be consumed or invested; there is no scope for a separate model of a capital-goods price. The user cost of capital is endogenous, but it is determined as part of the mechanism whereby aggregate demand and supply are equated, and not in a distinct capital-goods market.

There is no true market for the public good. As in the national accounts, output of the public sector is measured as government wage expenditures. This good enters SAM only at the stage of reconciling the modelled private sector output with the usual gross national product concept. There is no consideration of how consumption of the public good influences private sector behaviour, but the income generated from the activity is fully integrated into private sector accounts, as are the consequences of financing the wage expenditures.

Household labour is used in the production of energy and the public good, as well as in the production of the non-energy, private sector good. Labour is taken to be homogeneous, however, and there is only one overall market for labour and one endogenous wage.

The six complete markets, together with three identities linking asset prices to capital income and capital gains, determine eight prices and real rates of interest, and the value of financial wealth. The system is fully simultaneous and we cannot 'assign' the endogenous variables to particular equations or even to particular markets. The normalization rules used in the model code are, in principle, arbitrary.[3] The eight prices and rates determined are: the real wage, the real exchange rate, the market price of outstanding government bonds, the market price of equities, the real cost of (and return to) capital, the real interest rate on bonds, the domestic price level, and either the real domestic return on foreign assets or the expected future exchange rate (one implies the other, given the exchange rate and the exogenous foreign interest rate). Throughout this volume 'exchange rate' refers to the Canadian dollar price of foreign exchange.

1.2.2 The markets of SAM

The product market

In the market for the domestic non-energy good gross output (supply) is made a function of capital, labour, energy, and an endogenous level of capacity utilization. The growth of potential output is determined by exogenous population growth (translated to labour supply growth through an endogenous participation choice by households) and exogenous Harrod-neutral technical progress. Firms' long-run factor demands are derived from a formal framework of static cost minimization, with constraints of zero excess profits and full employment of labour imposed on the aggregate solution. Short-run factor demands, not derived from a formal optimization problem, are influenced by the desired long-run position, but also by the state of the cycle (in particular, aggregate demand).

---

[3] For model simulation, however, normalization is not arbitrary if one wishes to use a Gauss-Seidel algorithm, such as that traditionally used at the Bank of Canada. We have chosen our particular normalization with this in mind. Users of a Newton algorithm, for example, would be free to renormalize virtually at will.

Aggregate demand for the domestic non-energy good comes from domestic households, governments, foreigners, and firms in both domestic sectors. Government demand and energy sector investment demand are essentially exogenous. Export sales are made a function of relative prices and the level of foreign demand, with domestic excess supply having a significant short-run effect. Non-energy sector investment demand comes from the general system of factor demands. In addition to a tendency to adjust to the long-run, optimal capital stock, investment is influenced in the short run by the state of excess demand (an accelerator mechanism) and some financial variables. Household consumption demand comes from a formal, intertemporal, optimization problem, with several disequilibrium features added. In particular, according to our estimates, the level of excess real balances (defined empirically in the model in terms of base money) notably influences consumption demand. This effect arises because households use money balances to buffer shocks and willingly move off their long-run, money-demand function to stabilize other choices such as flow consumption. The model of household consumption demand explains the overall level of consumption. To provide the net demand for domestic goods we deduct imports. The quantity of imports is specified to depend on the scale of domestic activity, relative prices, and a measure of domestic capacity utilization (to capture cyclical import substitution). It is not necessary to think of imports as being 'consumed' as opposed to 'invested'; in SAM there is no explicit treatment of the split.

In the market process, any excess of flow supply over flow demand goes into inventories. There are strong feedback mechanisms from the state of inventory stocks (relative to desired stocks) to other variables in the system, including prices, factor demands, and capacity utilization. Although the price level is determined in the long run by the whole system, it is reasonable to describe SAM's short-run price dynamics as representing response to two sorts of excess demand. First, there are measures of excess demand in the product market itself. Second, there is a measure of the state of excess demand in the money market, calculated by comparing the current price level with that consistent with long-run, real-balance preferences. This latter effect captures both the notion that markets tend to adjust towards equilibrium, and a more specific point -- that in the model an excess supply of money signals a latent excess demand for goods.

A demand shock entering the system will be met, in the first instance, by inventory buffering and supply response through variation in capacity utilization. Also important is buffering through movements in the balance of trade (both imports and exports). The larger and more persistent the shock, the greater will be the effect on factor usage. For purely nominal shocks, the ultimate effects reflect virtual monetary neutrality in SAM. Real demand shocks will have permanent effects on output, but only to the extent that the induced changes in real wages and other variables call forth higher levels of labour supply. As the SAM labour supply function permits only limited scope for such response, one can say that the long-run solution is essentially supply determined, with most of the necessary response consisting of real-exchange-rate and real-interest-rate responses sufficient to reconcile demand with potential output.

Financial markets

In the financial asset markets, demands are specified as a desired allocation of financial wealth across the four instruments, the proportions depending on relative expected rates of return. Asset supplies are specified using the identities describing sectoral financing requirements. Firms issue equities to finance capital formation; governments issue bonds to finance expenditures not covered by tax revenues or new money provided by the central bank. The supply of equities to the domestic market is influenced by the direct investment decisions of foreigners. Similarly, governments can sell bonds to foreigners, and it is the governments' financing requirement, net of such sales, that determines supply to the domestic market. The supply of net foreign assets comes from the balance-of-payments identity and reflects the cumulation of capital flows not otherwise accounted for (i.e., in net government transactions or net direct investment). Sectoral flows of funds are completely accounted for in the above system.

The asset supply and demand equations, combined with identities that express asset prices as present values of expected future capital income or capital gain, 'determine' a system of asset prices and yields. Included is the price of foreign exchange. Recall, however, that the asset system, being part of a fully simultaneous model, cannot determine anything independently of the other equilibrium conditions. In SAM, financial instruments are not perfect substitutes and so asset supplies are important in determining rates of return.

It is not correct to view asset supplies as being passive in the above process simply because one must recognize the sectoral financing identities as part of the system. On the contrary, the same optimization process that determines investment behaviour, for example, provides a behavioural content to equity supply. Similarly, there is absolutely no reason why the supply of bonds must be residual in the long run, with expenditures and taxes exogenous and governments simply accepting the deficit consequences. Indeed, although this view of the process can easily be simulated using SAM, we often use a simulation rule for government finance wherein tax rates are adjusted in the long run such that any structural deficit is eliminated. For this volume we specify a less extreme rule which requires only that real interest on the debt be paid for through personal taxes.

The labour market

Household labour supply from the formal, intertemporal model (with some added disequilibrium effects) must be reconciled with the combined demands of government and the energy sector (both exogenous, essentially) and the endogenous non-energy sector. The labour market, unlike the asset markets, does not clear in every period. Involuntary unemployment can exist, in the short run, for reasons not formally specified in the model.

In the long run, the real wage is set consistent with full employment and the other general equilibrium conditions of the system. It grows with Harrod-neutral technical progress. In the short run, a market process operates that allows the usual unemployment and inflation expectations effects, as well as a tendency to move towards a computed general equilibrium solution.

In the long run, the labour market will clear at a 'natural' rate of unemployment that is exogenous to the model. We do not wish to imply that there are no potentially interesting links between macroeconomic variables and the long-run equilibrium level of unemployment.[4] It seems, however, to be generally accepted in the literature that the first layer of 'explanation' of the long-run rate must come from micro-theoretic issues (for example, search costs), particular institutions (the rules of the welfare system) and other things (such as demographics). We think of these 'micro' influences as providing an exogenous component of the natural rate, a value essentially independent of the macro variables on which we focus. But there is another dimension to the natural rate that can be endogenous in SAM. Although we have chosen to characterize the standard version of SAM as providing an endogenous participation rate and hence labour supply, conditional on a fixed natural rate of unemployment, the same theoretical structure, with some minor modifications, can provide an endogenous natural rate and labour supply, conditional on an exogenous participation decision. In SAM household preferences provide one restriction on the labour supply nexus and either the natural rate of unemployment or the participation rate must be determined outside the model.

In the long run, in SAM, there is no trade-off between inflation and unemployment. Any level of inflation is consistent with the natural rate of unemployment once expectations have adapted and adjustment is complete. In the short run, however, to attain a zero inflation rate from an inflationary starting point would likely require higher unemployment rates during the transition than would prevail in the long run. The nature of the short-run trade-off facing policy makers is described by the model. The exogenous rate of unemployment and the absence of trade-offs between inflation and unemployment are features of only long-run equilibrium states of the world.

---

[4] It might be that in some models the long-run level of unemployment has to be endogenous to close the system. This is not the case in SAM. Regardless of the experiment, assuming the model converges at all it will settle down at whatever exogenous unemployment rate is specified. Of course, the particular solution for the endogenous variables will be very much influenced by the choice.

1.3 Long-Run Equilibrating Mechanisms in SAM

SAM is a model that converges to a full-equilibrium steady state.[5] In this section we focus on what we consider to be the fundamental equilibrating mechanisms in the model-- those processes that ensure that all long-term equilibrium conditions can and will be satisfied. Although we set aside all short-run dynamic considerations here, we cannot avoid using the language of dynamic adjustment, since we are describing mechanisms for convergence to steady state and for long-term adjustment to permanent shocks. The nature of SAM's shorter-run dynamics is reviewed in section 1.4.

In this section two particular points are stressed: (a) that the model has sufficient endogenous variables free to move to satisfy the conditions of long-run equilibrium, and (b) that the model contains appropriate signals of the nature of disequilibrium and processes whereby market solutions or behaviour are modified in response to these signals such that the disequilibrium is eventually eliminated. The discussion is not intended to provide formal proof of 'existence' or 'stability'. Rather, the discussion is general and aimed at providing the reader with an overview of how the long-run solution emerges.

1.3.1 Conditions necessary and variables to be considered

To attain a steady state in SAM the following specific conditions must be satisfied: (a) The full-employment potential output must be demanded and must be produced with optimal factor proportions and normal rates of capacity utilization. (b) Savings flows must be consistent with preferences, in terms of both desired wealth accumulation and desired levels of wealth, and in terms of the composition of wealth. This means that (i) the labour market must clear at the equilibrium or natural rate of unemployment, (ii) the functional distribution of income must satisfy the zero-excess-profits condition, (iii) the rate of capital formation must be consistent with net desired real savings supplied by domestic households, governments, and foreigners, (iv) inventory stocks must be at desired levels, and (v) each asset stock, including money and net claims on foreigners, must be willingly held and must not cause real consumption or capital accumulation to deviate from the stable growth path.

---

[5] In this introductory chapter we will not deal with parametric restrictions necessary for formal stability. The reader can assume that such restrictions are satisfied by SAM's parameters. Details are provided in the chapters that follow.

The key variables that adjust to assure that a steady state will be attained are the real exchange rate, the real return on capital, the real wage, and the nominal price level.

1.3.2 The attainment of nominal steady state

Let us begin with the purely nominal dimension of the discussion. The important restriction that must be satisfied for nominal steady state is that real-balance preferences be respected, i.e., that the stock of real balances be willingly held. We can say very little that is universally valid about the process or the fundamental causation here because, especially in a small open economy, all the key factors represent policy choices. For example, under a policy of fixed exchange rates, the properties of the domestic nominal steady-state path are determined in the rest of the world. If, however, the nominal level and growth rate of the money stock are set exogenously by the central bank, then real-balance preferences imply a long-run restriction on both the inflation rate and the price level, and the exchange rate must adapt. A slightly less stringent procedure would have the central bank maintain its money-growth target, but not concern itself with the levels of nominal variables. When such a perspective is adopted in a model, it is generally true that the price level is determined by other factors, and the level of the nominal stock of money adjusts to satisfy real-balance preferences. SAM can be used with any of these policy assumptions and various other specifications of policy targets.

Without a specific set of policy rules we cannot determine the exact nature of the nominal dimension of the steady-state path. For this volume, however, we have chosen to describe the version in which both the level and the growth rate of money are set exogenously. As such, we can be quite specific. Real-balance preferences depend, in SAM, on nominal interest rates and real wealth. Assuming that we are considering a steady state with constant money growth and stable interest rates, steady-state inflation is directly determined by the money-growth choice, and the price level must adjust to satisfy the level of real-balance preferences. All other nominal variables, such as the money wage, will adapt to the equilibrium price level so as to maintain their equilibrium values in real terms. A relative purchasing power parity condition will hold, in the long run, such that the nominal exchange rate moves to reflect any permanent international inflation rate differentials.

It is important that the reader not be misled into thinking that the price level is solely determined by the money stock, even when the level of money is exogenous. In SAM, the quantity of real balances demanded, and hence the level of prices, is influenced by the level of real wealth and by the level of nominal interest rates (with a real component and an inflation component). The levels of real wealth and real interest rates are endogenous to the model. If these variables change under a shock, the price level will be affected unless the monetary authority neutralizes the shock by changing the level of nominal balances. This does not happen with exogenous nominal balances; so the price level does depend on more than just the money supply. It is fair to say, however, that (given steady-state values for interest rates and wealth and an exogenous money stock) we can 'count' the price level as being determined by real-balance preferences. Throughout this volume, such 'assignment' of variables to particular equations or sectors is referred to as 'proximate determination'.

1.3.3 The attainment of real steady state

Macro constraints and the planning problem of firms

We turn now to the attainment of a real steady state in SAM. A key part of the process involves the planning problem of firms and the imposition of steady-state consistency on these plans. Given a real capital cost, a real energy cost, and the production technology, we can derive the unique real wage consistent with profit-maximizing firm behaviour and the macro consistency constraints — full employment of offered labour and zero excess profits. We can also compute the long-run output and quantities of factors demanded consistent with the macro constraints and given factor prices. Among the properties of this system is one we require for the discussion to follow: the higher the real capital cost, the lower the equilibrium real wage.

It is important that we be very clear that it is not the firms' planning, per se, that provides the macro results. In the formal micro theory, firms are not assumed to be super-rational, or to automatically plan for full employment. Nor do they exploit information about the labour-supply function. The basic theory can be characterized as a standard perfect (or atomistic) competition model. We view the imposition of macro consistency constraints on the solution as the macro theory part of the exercise, the implementation of the general idea that markets work. The result is that firm equations are structured as if those firms consider the requirements for a full employment solution in their planning. Formally, however, there is a micro theory in which this is not so, and there are also macro consistency requirements to be met.[6]

The use of the labour supply function in the determination of the long-run values for output and factor usage to which the system is tending, shows how a key element of supply determination enters SAM. In long-run equilibrium, the level of output is proximately determined by the labour offered at the equilibrium wage.

It is also worth noting that the observation of short-run excess unemployment in the labour market is not necessarily evidence that the real wage is too high. There are a set of conditions that must hold if the system is to generate full employment with demand sufficient to exhaust potential output. It is possible, for example, to construct a state of general disequilibrium in the model in which deficient aggregate demand is associated with too low a labour income or human wealth, in turn associated with too low a real wage, contrary to what a partial labour market analysis would suggest. A benefit of the imposition of macro restrictions on the aggregate firm planning problem in SAM is that we can compute a long-run equilibrium real wage and distinguish among the different possible conjunctures.

---

[6] See Chapter 4 for elaboration of these basic arguments.

The procedure of imposing certain macro consistency constraints on the aggregate firm planning problem is, as far as we are aware, original to SAM. The 'theory' of how market restrictions and individual decision-making interact is not well developed. Because we have imposed full-employment and zero-excess-profit conditions on the long-run values to which the system is tending, we know that these properties are assured in steady state and we do not have to find some explicit mechanism in a market adjustment process to assure that they hold.

Although we have imposed zero excess profits and full employment on the long-run properties of the supply block of SAM, these are not the only macro consistency requirements. Moreover, in computing the implications of the two conditions we do not use a complete, full-model calculation. In particular, the solution takes as given the current 'estimate' of the equilibrium real cost of capital, and certain other endogenous determinants of labour supply. Therefore, particular values computed exploiting full-employment and zero-excess-profit conditions, such as the long-run, desired capital stock, are not exactly correct until these other 'estimates' are themselves correct. Another perspective on the same point is that our imposition of full employment and zero excess profits is not sufficient to guarantee the other macro consistency requirements. In particular, we have not yet explained how aggregate demand and aggregate supply are equated in SAM.

Establishing domestic equilibrium: the role of real interest rates and the real exchange rate

Three 'price' variables are important in the generation of a full, real, steady state in SAM: the real exchange rate, the domestic real interest rate, and the domestic price level. The role of the price level is to generate financial wealth effects and real exchange rate changes. These are important and must be considered in a full description of the process, but since we have already 'counted' the price level as being proximately determined by real-balance preferences, we will leave it aside for the moment and concentrate on real interest rates and the real exchange rate.[7]

The main mechanism by which the real exchange rate can influence aggregate demand in SAM is the standard one, the current account. The trade equations in SAM have the property that a rise in the real exchange rate (a real depreciation) will increase net exports and aggregate demand. There are other effects through asset stocks and real interest flows, but they can be reasonably ignored for this discussion.

Real interest rates have two direct influences on aggregate demand, and one important indirect influence. The direct link through investment demand is standard. We refer to the direct effect of the real cost of capital on the desired capital stock (including the desired stock of inventories) and from that to the level of investment demand. The other direct link is through consumption. A lower real interest rate (and associated lower rate of discount for future income flows) will result in increased market value for certain financial instruments, and a higher present value for a given stream of expected future labour income. Moreover, recall that a property of the general equilibrium solution is that lower real costs of capital are associated with higher values for the real wage. Thus, the whole level of the expected labour income path shifts up, reinforcing the effect of the lower discount rate. For these reasons, both financial and human wealth increase when interest rates fall. According to SAM's intertemporal optimization model, this will lead households to demand more current consumption and save less.

---

[7] To ensure that there is no confusion, let us be clear that a real exchange rate change can be generated by either the price level changing without a corresponding change in the nominal exchange rate, or by a change in the nominal exchange rate with a given price level. For simplicity it is on the latter that we focus in this discussion.

The indirect effect of interest rates on demand is through the real exchange rate. Household preferences influence both the level of financial wealth (there is a stock consistent with current planned lifetime consumption) and the allocation of wealth across the financial instruments. The allocation equations restrict how much of each asset households will hold given a set of relative rates of return and a total portfolio value. In SAM, the real return to net claims on foreigners is very closely tied to the U.S. real interest rate, adjusted for expected exchange rate changes. Other domestic rates -- the real rate on government bonds and the real return on equity claims to the private sector profit stream -- are linked by high, but not perfect, substitutability to the return on net foreign assets and hence foreign real rates. Two things happen when there is downward movement in domestic real rates. First, there is asset revaluation. If the bond rate falls, for example, the value of outstanding bond claims rises. Second, since the return on net foreign assets tends to be very closely tied to the exogenous foreign return, this return rises (relative to domestic returns) when domestic rates fall. Both these effects tend to increase the demand for net foreign assets and this, ceteris paribus, pushes up the price of foreign exchange. In other words, the asset system works such that a fall in domestic real rates will have as an associated effect a depreciation in the real exchange rate.

Let us now ask what market forces might cause movement in real interest rates or the real exchange rate. As an example, take the case of a fundamental disequilibrium with excess supply in the domestic product market. With such an excess supply, there is, simultaneously, too high a flow of real savings from all sources. An interesting, if incomplete, way of looking at this is to note that to the extent that the excess supply results in undesired inventory accumulation, capital formation is being financed by excess savings. Hence, one can imagine the process as one of declining interest rates owing to excess real saving at current levels of demand.[8] If the excess savings are from foreign sources, there would also be upward pressure on the price of foreign exchange. Depending on one's view of the world, this could be directly due to asset effects or to the operation of the nominal interest parity condition (downward pressure on interest rates owing to the excess supply shows up as a higher current exchange rate given expectations for the future exchange rate), or to a rational expectation that the exchange rate must rise to increase domestic demand.

In SAM we compute, whenever we can, equilibrium solutions for variables and use these explicit results in conditioning adjustment processes. In principle, we might be able to extend this approach to the real interest rate and real exchange rate, but we have not been able to find a practicable way to do so. As a workable substitute we postulate an adjustment process in the usual way; that is, we postulate that when there is an excess supply of goods the equilibrium real rate of interest must be lower than our previous best guess.[9] This is imperfect because an excess supply of goods could be consistent with too low a real interest rate, given some combinations of possible severe disequilibria elsewhere in the macro system. In practice, however, we have not found such formal instability a problem. The above idea is implemented in SAM through the real cost of capital. The real equilibrium user costs of producing capital and inventories fall when there is excess supply in the product market. This, in turn, influences equilibrium real profit rates and other domestic real rates through the asset system. Ultimately, the real discount rate used by households is linked to the average real return on the entire portfolio of assets. But, because short-run asset dynamics can produce substantial variation in ex post real returns, only a small weight is given to current market outcomes in discounting the future. More weight is given to a long-term value that moves, with the real cost of capital, in response to the level of excess demand in the product market.

---

[8] Note that we are considering real determinants of real interest rates here. The money-demand function must be respected, however, and throughout this discussion we take it as given that the price level adjusts to reflect any changes in real (and nominal) interest rates and in wealth. See section 1.3.2.

[9] In the case of perfect international asset substitutability this implementation of a real adjustment process would not make sense. In such cases a corresponding process would have to be specified for the real exchange rate.

As explained above, as interest rates change the exchange rate is simultaneously influenced through asset substitution and valuation effects. As domestic rates fall in response to an excess supply of goods, the exchange rate will eventually depreciate, ceteris paribus. There is also a minor related mechanism working through the domestic price level. As domestic real rates fall, the equilibrium price level declines, assuming a fixed money stock. To the extent that this changes real financial wealth, it can also influence the equilibrium real exchange rate.

Given that real depreciation of the exchange rate will tend to generate a stabilizing response to domestic excess supply, and given that we have argued that such a response will indeed follow from the adjustment processes in SAM, we also have the basis of a 'rational expectations' argument for including an excess supply measure in the expected exchange rate equation, but this is not necessary. Moreover, it is important to establish that just because the structural adjustment equation is written in terms of a real interest rate response to excess product demand it does not follow that interest rates are somehow determined by an internal-balance condition and the real exchange rate by the external-balance condition. The system is fully simultaneous. Which 'gap' is used in a particular structural equation does not determine the reduced-form properties of the model. In fact, given the relatively high degree of asset substitutability in SAM as well as the powerful leverage of real interest rates on aggregate demand, it is entirely possible that for most real shocks the exchange rate will move by relatively larger proportional amounts in the adjustment. We do not yet know the balanced answer to this question about SAM's properties.

The question of the substitutability of financial assets

There is a view put forward by some economists, including some at the Bank of Canada, that Canadian domestic assets are so closely substitutable for similar foreign assets as to be virtually perfect substitutes for such assets. This position is based on considerable efforts to obtain relevant empirical evidence.[10] The implication is that domestic real rates cannot, in the long run, deviate from world-determined equilibrium values. In such cases, the long-run burden of adjustment of domestic demand to supply must fall on the real exchange rate. A slightly less extreme view is that the proposition holds for only a certain spectrum of instruments, including government debt, but not for other claims, such as equity claims to the profit stream. As long as all assets are not perfect substitutes internationally, and as long as those that are not are also not perfect substitutes in domestic portfolios, the mechanism of relative real interest rate effects becomes possible in some form.

It is clear from the empirical work that has been done, that Canadian and U.S. real rates have stayed closely tied together. The evidence from other countries is much less clear. Moreover, there are other explanations of the Canadian evidence. For example, the same facts could have been generated in a world of less than perfect substitutability if the domestic monetary and/or fiscal authorities were consciously or otherwise pursuing policies that stabilized the rate differential. The simplest argument of this type is that money or bond supply could have been regulated consistent with such an outcome.[11]

One implication of perfect substitutability is that the Canadian government could sell unlimited quantities of domestic debt in world markets, without leading purchasers to demand higher real rates of return. We do not feel that a country can escape an increasing marginal cost of deficit finance in this way. In SAM we specify and estimate asset demands that do not have properties of perfect substitutability imposed.

---

10. See, for example, Boothe et al. (1985).

11. We do not wish to imply that evidence on rate differentials is the only thing that has been considered. Nor do we wish to suggest that our 'explanation' provides an adequate reconciliation of the historical facts. We want to suggest only that the empirical issue is a difficult one, econometrically, and that the evidence for perfect substitutability, though strong on its own terms, is not definitive.

The results imply fairly high substitutability for domestic assets and foreign assets, but leave an important place for real interest rate adjustment in the overall process of establishing full equilibrium.[12]

1.3.4 The meaning of 'steady state'

Normally, economists tend to think of 'steady state' as implying that there is no force at work that disturbs the solution (except for equilibrium growth). In principle, this can encompass solutions in which not all markets clear; indeed much of twentieth century macroeconomics is devoted to exploring such possibilities. SAM is not a model where market failure can occur in the long run. Throughout this report all references to a steady state should be taken to imply the extra condition that all markets clear. Similarly, the concept of 'rationality' in SAM extends to the requirement that, in a steady state, expectations cannot be systematically inconsistent with actual outcomes. From the discussion of expectations above, the reader will readily see that such consistency is assured if the model solution does in fact converge to the presumed steady state. Hence, our demonstration in section 1.3.3 that the model has mechanisms that provide for movement to the steady-state path also establishes the long-run consistency of expectations.

Relative prices in steady state

It is easy to construct examples where the above conditions hold -- i.e., all markets are clearing at potential, and all expectations are consistent with outcomes -- but where some relative price is continuously changing. Although this may violate some definitions of steady state, for SAM we take an eclectic view. We always try to explain the conditions under which properties like relative price stability on the equilibrium path are assured, but we do not always impose such restrictions on the model.

---

[12] For users wishing to impose perfect substitutability on government bonds, for example, the only requirement is the replacement of the bond-demand function with an equation imposing real interest parity, at least in the long run.

One example of an important relative price that might not be constant in a steady state is the real exchange rate. In SAM, the equations describing trade and other aspects of the balance of payments, as well as the equations describing asset preferences, are made consistent with a constant real exchange rate in steady state for some, but not all, scenarios. If the long-run domestic and foreign real growth rates differ, for example, it is difficult to construct an analysis in which the real exchange rate can be constant. Moreover, we see no reason to discard ever-changing real exchange rates as a real-world possibility. That is, we do not consider it just a mathematical curiosity, but a serious possibility in the process of international resource allocation under such circumstances as differential growth in potential.

Where the analysis is simplified and not seriously limited by considering only steady states with fixed relative prices, we do impose such restrictions. For example, in the historical data there are discernible trends in the shares of the various financial instruments in the household portfolio. If these trends are interpreted as causally related through the demand functions to the growth in wealth over time, then on a constant-growth path where all asset supplies are growing at the same constant rate, the relative real returns on the assets would have to change continuously in the market-clearing process. Real rates could not settle down on any fixed structure unless asset supplies were growing at the appropriate differential rates, and it is unlikely that this could be consistent with other requirements of steady state. For our basic model we impose a simpler view and leave complexities such as non-homogeneities to be introduced as required. As such, we interpret the historical trends in asset shares that are not attributable to sample movements in relative returns as exogenous and historically specific. This is made explicit in the equations so that users can include or suppress such exogenous trends, at their pleasure, in simulations over future periods.

1.4 Disequilibrium Dynamics in SAM

1.4.1 Adjustment towards steady state

In the previous section we established in some detail the factors and processes in SAM that work to assure the existence of a steady state. That discussion is directly relevant in any discussion of disequilibrium dynamics, because when there is a permanent shock to the system it is precisely the mechanisms discussed in section 1.3 that must operate as the system seeks a new equilibrium path. Putting aside all other sorts of disequilibrium influence for the moment, we can characterize SAM's disequilibrium dynamics as a system of partial adjustment towards equilibrium. We view this as the representation of basic equilibrating forces in markets, and not some super-rationality of particular agents. Wherever possible, we implement this idea by using computed measures of equilibrium values and specifying that one of the factors influencing adjustment is a tendency to move towards those values. For example, we use long-run, real-balance preferences to compute an equilibrium price level consistent with the exogenous money stock and current best estimates of the steady-state levels of variables determining desired real balances. This equilibrium price level is used as one of the determinants of price dynamics; we specify a market tendency to move the actual price level towards this value. Similarly, factor demands reflect partial adjustment towards long-run equilibrium (as well as a variety of other influences).

It is worth repeating that the computed 'equilibrium' values are never exactly correct, except in steady state. In each case, not all essential information is incorporated into the calculation. The results are not true full-model solutions, always approximations. Technically, we do not know a way to solve SAM analytically for full equilibrium.[13] But we can solve chunks, conditional on assumptions about certain endogenous variables. This is what we do. Moreover, the system is stable and the approximate solutions will converge on the correct solutions as a simulation progresses.

---

[13] It is always possible to find the solution by simulating the core model, but we do not consider a second layer of simulation in each period a 'practicable' procedure.

We interpret our approximations as capturing a piece of a real-world information problem. It seems to us that it strains credulity to claim that plans are always formulated in the light of full information about the steady state. Moreover, we feel that such information or planning problems are an important source of cyclical behaviour. Our procedures allow us to introduce this idea into SAM, albeit in a somewhat ad hoc manner.

In summary, an important part of disequilibrium dynamics in SAM is a general tendency to adjust towards full equilibrium. This can be offset in the short run by other factors, but ultimately dominates. Even in the absence of complicating factors, however, adjustment to equilibrium need not be rapid, owing to the real costs of adjustment and other inertias in the system.

1.4.2 Other sources of dynamic properties

In addition to the fundamental equilibrating processes, SAM has another layer of disequilibrium effects that enrich the short-run properties of the model.

Deliberate buffering behaviour

In the real world, the economy is subject to a continuous stream of shocks, both permanent and transitory, and no one would expect to observe states of full equilibrium. This very fact leads us to conclude that it is important to build into a model explicit behavioural response to shocks. That is, in an uncertain world, it is certain that there will be surprises. Rational agents will develop institutions and response mechanisms for such surprises. In SAM, we provide two explicit mechanisms that firms can use to buffer shocks and alleviate the costs of cycles in factor usage and output. These are the use of inventories as a buffer stock and the use of explicit variation in capacity utilization to limit the final flow excess supply of goods. We also specify that households use financial assets, especially money balances, to buffer shocks to their income flows or balance sheets so as to limit disruptions to flow consumption.[14] Finally, we specify that both exports and imports move to limit fluctuations in aggregate domestic demand.

Initially, when shocks enter the system, such buffering mechanisms are stabilizing. They tend to blunt the impact of shocks on real flows and limit the necessity for prices to overshoot final values in the short-run temporary equilibrium solutions of markets. As time passes after a shock, however, the need to re-establish equilibrium in the buffering mechanism, say the stock of inventories, can itself become a source of prolonged disequilibrium.

Expectations errors

Above we dealt with information problems, wherein plans are not formulated on exactly correct evaluations of steady state. There is a parallel set of related effects that arise through expectations errors. In SAM, expectations are forward looking. They are generally formulated as average expectations about the variable over some horizon relevant to the decision. These expectations errors are notionally distinct from errors about the steady state. They involve difficulties in anticipating the dynamic path as opposed to the end point.

Although our specification of how expectations are formulated has some unique features, the ways they influence behaviour and market solutions are, in the main, standard. For example, expectations of inflation influence wage and price dynamics, interest rates, and the exchange rate. Inflation expectations are very important in dynamic processes in SAM. They provide a major source of the propagation and prolongation of cycles through their influence on wages and prices.

Confusions, misconceptions, and miscellaneous interdependencies

Generally speaking we avoid basing model properties on irrational behaviour, permanent illusions and so on. However, the absence of formal theory about optimal dynamic behaviour, both in SAM and more generally, leaves us little clear guidance as to what is 'irrational' in the short run. We sometimes add variables to dynamic equations, in a form such that they disappear as the system converges to full equilibrium, because of a particular interest in a short-run property or because others have found the effect empirically useful. For example, we add to labour demand a short-run sensitivity to the extent of wage disequilibrium. If labour is relatively cheap in the short run, our results suggest that more labour will be demanded than would be the case if only the long-run signals were considered. This is not an expectations error or a planning error, though one might consider it a case of deliberate buffering. There are a few such examples in SAM of effects that do not fall clearly into one of the categories above.

---

[14] We also specify a special disequilibrium role for current wage income that works in the opposite direction, amplifying rather than damping cycles.

In particular simulations we sometimes add confusions and misconceptions about shocks as an explicit part of the experiment. These then become temporarily part of the dynamics and can dominate the process of adjustment. SAM's dynamics are very much predicated on the processing of information. Because of the model's emphasis on forward-looking behaviour, an explicit introduction of errors can have significant consequences.

Chapter 2

SECTORAL ACCOUNTING AND GOVERNMENT FINANCE

2.1 Introduction

This chapter has three parts. We begin with a detailed overview of the sectoral accounts of the model (section 2.2). This expands the discussion in section 1.2 and provides an overview of the variables considered in the chapters that follow, albeit in simplified notation and with certain details suppressed. We then present (section 2.3) some miscellaneous material necessary for a full understanding of the detailed discussions of model equations, including a review of the operator notation used in the Bank of Canada's mainframe simulation software. The chapter concludes (section 2.4) with a detailed examination of the government sector in SAM, with emphasis on the possibilities for and implications of various long-term financing rules for government.

2.2 The Sectoral Accounts of SAM

In this section we present a stylized version of the accounts of SAM. Some of the detail is supressed to simplify the exposition. Among other things, we ignore the energy sector, government wage expenditures, all government financial transactions with foreigners, and all aspects of asset valuation. The material discussed here is reconsidered in full detail in the relevant sections of the report. Our goal for this section is to provide a simple, but reasonably complete, overview of the accounting framework of the model.

SAM has four economic agents: firms, households, governments, and foreigners. The accounts of the model can be conveniently summarized by specifying the financing constraints or flow budget constraints on intersectoral transactions. The notation for this discussion is provided in Table 2.1.

Table 2.1

NOTATION FOR THE SIMPLIFIED ACCOUNTS OF SAM

Mnemonic	Definition
c	real consumption expenditures (price, p)
i	gross real investment expenditures (price, p)
g	real government expenditures on goods (price, p)
x	real exports (price, p)
m	real imports (price, pm)
dinv	real inventory accumulation (price, p)
py• y	nominal domestic output (price, py, volume, y)
w• l	wage income (wage, w, employment, l)
r• k	gross capital income (gross profit rate, r, capital stock, k)
cca	capital consumption allowances
gint	interest payments on government debt
trf	net nominal transfers to foreigners
trh	net government transfers to households
txi	indirect tax revenues
txc	direct corporate (profits) tax revenues
txh	direct household (income) tax revenues
dkw	real net direct foreign investment (stock, kw, price, p)
fid	net interest and dividend payments to foreigners
rg	nominal rate of return on government debt
rf	nominal domestic-currency rate of return on foreign assets of households
rn	net profit rate (r net of depreciation, taxes)
dvg	household accumulation of government-sector liabilities (stock, vg)
dvc	household accumulation of firm-sector liabilities (stock, vc)
dvf	net household accumulation of foreign-sector liabilities (stock, vf)

Firms

For firms we begin with the financing constraint associated with capital formation. The income-flow aspects of transactions with firms are considered later. Gross investment in producing capital and inventories, net of capital consumption allowances and direct investments of foreigners, must be financed by selling financial instruments to domestic households:

$$dvc = p\cdot(i+dinv-dkw) - cca. \tag{2.1}$$

Note that we do not account explicitly for retained earnings. For our wealth accounting we consider all asset acquisition by governments and firms as asset acquisition by households. Here, for example, we assume that all profits are actually paid out. Retained earnings in the data are treated as if paid out and reinvested; thus all capital formation results in asset acquisition, either by foreigners directly (dkw, real) or by domestic households (dvc, nominal). Similarly, any intersectoral asset transactions between governments and firms are treated as transactions with the ultimate wealth owners -- households. Finally, we assume that capital consumption allowances are identical to economic depreciation, so that household wealth grows with net investment. In effect, any differences between economic depreciation and national accounts depreciation are treated as part of the payments to households by firms.

Foreigners

Transactions with foreigners can be summarized in the balance-of-payments identity, that any surplus of export revenues and net direct investment flows over import purchases plus payments for capital services (interest and dividends) plus net transfers to foreigners must generate net acquisition of claims on foreigners:

$$dvf = p\cdot x + p\cdot dkw - pm\cdot m - fid - trf. \tag{2.2}$$

An important point arises here regarding the measurement of fid. Consistent with our general principles of wealth accounting we include retained earnings accruing to foreigners in fid. They are treated as paid out and reinvested so that our national wealth accounting will be correct. Numerically, this adjustment is extremely important; for example, the 1983 official current account surplus becomes a substantial deficit, from the national wealth perspective, when the effect of unrepatriated profits is considered.

A related point arises when we introduce the identity linking fid to the stocks and rates of return in the model. For each asset stock SAM explains the interest rate relevant in determining the income flow for that asset.[1] Thus, the model must respect an exact hypothetical relationship:

$$fid = rn \cdot kw - rf \cdot vf, \qquad (2.3)$$

where rn is the net profit rate (after depreciation and taxes) and rf is the rate of return on foreign assets. Independent measures are available for all components of identity (2.3), and these measures do not respect the notional identity. We use the identity to derive a model measure of fid and put the difference between this measure and the official fid data (adjusted, as noted above, for unrepatriated profits) into transfers to foreigners, trf. The historical balance-of-payments data are respected, but the composition is altered to respect identity (2.3).[2]

---

[1] We ignore here the distinction between average coupon rates and rates of return. In the model this distinction is carefully maintained.

[2] Strictly speaking, we do not have independent measures of the interest rates. We construct them ourselves, deliberately not imposing identity (2.3). If we use (2.3) to define rf, for example, we get very erratic results. Since rf is a variable explained in the asset system, imposing (2.3) on the data is a questionable procedure. Our compromise procedure respects the flow data without distorting the rate measures.

Government

The transactions of domestic governments can be summarized in the financing requirement or government budget constraint:

$$dvg = p \cdot g + gint + trf + trh - txc - txi - txh. \qquad (2.4)$$

In this simplified framework, any excess of spending on goods and services, p•g; interest payments, gint; transfers to foreigners, trf;[3] and transfers to households, trh, over total revenues from profits taxes, txc; indirect taxes, txi; and personal income taxes, txh, must result in asset acquisition by the household sector, dvg. SAM is more complex than this stylized equation in that it also includes government employment and financial transactions with foreigners, and because several of the general categories are disaggregated. But the model does have the feature of identity (2.4) that government asset acquisition is treated as household asset acquisition in the wealth accounting. Such government asset acquisition in the data (after consolidation of intra-government stocks and flows), net of the miscellaneous revenues associated with such stocks, is treated as a transfer to households.[4]

Just as we must respect the identity linking fid and claims held by foreigners (equation 2.3), we must respect an identity linking gint to the stock of claims on government, vg:

$$gint = rg \cdot vg. \qquad (2.5)$$

---

[3] For simplicity we treat all transfers to foreigners as government transfers. See p. 50 for details.

[4] An important distinction between household and national wealth can arise when there is a structural deficit. To illustrate, consider the case where transfers to households are financed by bond sales to households. Even when households realize that future taxes must be levied to pay for the bond interest and therefore do not count the bonds as net additions to sector wealth, they do still count the transfers. This is correct from the household perspective, but not from the national perspective. National wealth, the stock that generates the total real output flow, is not increased by the transfers. To the extent that there is a persistent structural deficit, SAM's household wealth overstates national wealth because of this double counting.

Once again we have data on all components of equation (2.5) and the identity does not hold in terms of these measures. Moreover, we cannot use the identity to determine rg for the same reasons that we cannot use (2.3) to obtain rf. We resolve the difficulty in the same way. Identity (2.5) is used to define a model version of gint. The national accounts measure is respected by treating the difference as another form of transfer to households, in trh.

Several other specific identities are imposed on components of government spending and revenues. These are described in section 2.4.

It is perhaps useful to identify specifically the measures of government deficits in SAM, especially the link between the national accounts deficit and the financing requirement that determines liability issue, dvg in equation (2.4). The national accounts deficit differs from the financing requirement owing to government asset acquisition and net 'off-budget' transfers by governments. As noted above we merge government asset acquisition with transfers. We similarly incorporate the off-budget items as transfers and record the financing requirement consistent with data on debt issue (after appropriately consolidating to remove intra-government transactions, transactions involving government-guaranteed debt, and transactions with the Bank of Canada, and to reflect our decision to include the public sector pension funds with government). We maintain the various national accounts measures (e.g., the deficit and transfers) in the data base and compute the historical balancing entries to permit linkage of these values to model concepts on the assumption that all changes in simulation come from the endogenous core of the model.

Households

Household income consists of wage payments, capital income, and net transfers from other sectors. In the simplified accounts of this section, there is only one employer (the private sector) and we can write wage income as w•l. Gross capital income, r•k, plus government interest payments, gint, net of capital income accruing to foreigners, fid, capital consumption, cca, and corporate profits taxes, txc (i.e., r•k +gint-fid-cca-txc) defines capital income accruing to households. All transfers between other sectors and households are consolidated in trh. The household stock/flow constraint can be summarized in the requirement that any excess of this income over nominal consumption, p•c, and personal direct taxes, txh, must result in asset acquisition. In equations (2.1), (2.2), and (2.4) we define the forms this acquisition can take. The constraint is then:

$$\begin{aligned} dvc + dvf + dvg = {} & w \cdot l + r \cdot k + gint - fid - cca \\ & - txc + trh - p \cdot c - txh. \end{aligned} \tag{2.6}$$

It is perhaps wise to point out at this juncture that dvc, dvf, and dvg represent nominal savings. Actual asset-value changes are influenced by asset-price changes as well -- these are fully dealt with in SAM. We ignore them here for simplicity.

Aggregate flow accounts

If p•c is moved to the left-hand side of equation (2.6) then the equation embodies the identity that disposable income is either consumed or goes into wealth accumulation. We can generate the model's domestic income identity by substituting equations (2.1), (2.2), and (2.4) into equation (2.6) to eliminate the dv terms. After cancellation and reorganization we obtain:

$$\begin{aligned} p \cdot c + p \cdot g + p \cdot i + p \cdot dinv + p \cdot x - pm \cdot m & \\ = w \cdot l + r \cdot k + txi, & \end{aligned} \tag{2.7}$$

or

$$py \cdot y = w \cdot l + r \cdot k + txi, \tag{2.8}$$

where y is real output and py is the aggregate deflator. In this simplified framework py differs from p only because of pm, but in the full model several other relative price distinctions are retained and when government employment is included there is an additional distinction between domestic and private sector output and the corresponding deflators.

Equations (2.7) and (2.8) represent the identity stating that simplified domestic income equals domestic output. This can be viewed as a constraint on the aggregate economy. All output generates income, and (recalling equation 2.6) all income generates consumption, taxes or asset acquisition. There are three parts to the embodiment of these identities in model equations. Equation (2.8) appears as a residual-gross-profits equation. Inventory changes are determined as y (from an output-supply equation) less the other real components of expenditures, so that real output is equal to c+i+g+x-m+dinv. The overall deflator, py, is set to reconcile the nominal units (i.e., $py=[p\cdot(c+i+g+dinv+x) - pm\cdot m]/y$).

Equations (2.1), (2.2), (2.4), (2.6), and (2.7) (or 2.8) represent a singular system. Any of the five equations can be derived from the other four. In SAM, equation (2.6) is not explicit (although the definition of household income does appear in the personal-tax equation). Some other models, including the Bank's RDXF model, retain equation (2.6) and omit equation (2.1). In SAM the three sectoral financing equations that lead to changes in household-sector assets are explicit, as is the aggregate-income identity.

Energy

We define the energy sector narrowly to include primary energy production. Its output is treated as entirely consumed as a factor of production by domestic firms or exported. Details are provided in Chapter 5. From an accounting perspective, an important feature of SAM is that we define the output of the non-energy sector as inclusive of the value of energy inputs and not just the value added by capital and labour. Correspondingly, the extra output from the energy sector itself that we add to non-energy output, UGPC, to get total private sector output, UGPB, is limited to net exports of energy plus energy inventory accumulation. See equation EQ53UFI in Appendix B.

Reconciliation with the national accounts

To reconcile SAM's private sector output with standard national accounts concepts such as gross national product, we must add the government sector's wage expenditures and adjust for net capital payments to foreigners net of withholding taxes. Since SAM's measure of the capital payments differs from the national accounts measure a small balancing item is necessary for exact reconciliation. We routinely provide for such reconciliation with standard measures in a block of equations that transforms model output but does not influence the simultaneous solution. For purposes of illustration we report three such equations (EQX01, EQX07, and EQX08 in Appendix B) which provide for reconciliation with Gross National Expenditure in real and nominal terms and for the GNE deflator.

2.3 Some Background on Notation and Function Forms

In this section we provide some preliminary information on the notation used in this report, and on the form we use to represent, in discrete time, processes that arise from continuous-time models.

2.3.1 The mnemonics of SAM variables and equations

Variables

Complete lists of the endogenous and the exogenous variables in SAM are provided in Appendix A. Wherever possible, the mnemonics have been chosen to conform with the economic significance of the variable or standard labelling conventions of the literature. For example, any variable beginning with P is a price. Similarly, any variable beginning with R is a rate or proportion. All interest rates are measured as fractions not percentages. All wealth measures begin with V, all real-output measures with U, all capital-stock measures with K, and all labour measures with L. All mnemonics beginning DN indicate a rate-of-change variable, the symbol coming from the change, D, in the natural log, N, of the variable (the continuous-time rate of change). Each variable is defined the first time it appears in the text. In addition, a duplicate listing of the mnemonic definitions has been provided that can be removed and kept at hand.

Equations

Each equation in SAM is represented by a seven-character identifier, such as EQ41AGS. The first two characters are always EQ and identify the mnemonic as an equation name. The next two characters provide an integer between 0 and 99 and identify each particular equation in the model. In general, equations with low numbers solve for variables with strong and direct links to the exogenous variables. Equations with higher numbers solve for variables where simultaneous feedbacks are stronger.

The fifth character in the identifier refers to the market to which the equation belongs. Equations that are not clearly associated with any particular market are identified as either referring to an income account or are grouped with the market on which the variable described by the equation has its largest impact. The market identifiers are:

Market	Character
Output	U
Asset	A
Capital	K
Labour	L
Energy	E
Income	I

The sixth character in the identifier refers to the agent who is most likely involved in the action described by the equation. Equations not associated with any particular agent are grouped as a description of the operation of a market. The agent identifiers are:

Agent	Character
Government	G
Households	H
Foreigners (Aliens)	A
Firms	F
Market	M

The seventh and final character in the identifier refers to the type of equation being considered. Equations can often fit into more than one of these groups and hence the labelling is somewhat arbitrary. The type identifiers are:

Type	Character
Demand	D
Supply	S
Price	P
Identity	I
Government policy rule	R
Expectation	E
Long-run condition	L

2.3.2 TSP operator notation

SAM is maintained and simulated on the Bank of Canada's Univac mainframe, using a version of TSP (Time Series Processor) developed at the Bank. The equations of the model are presented in this report in their TSP form. Some special operator notation is employed. Each operator begins with J. This is followed by a two-character identifier that establishes what operator is involved and an expression in parentheses to which the operator applies. The following particular operators are used in the equations of SAM:

1. JiL means a lag or delay operator that shifts the variable i periods. For example, J1L(X) means a one-period lag of variable X. For SAM the frequency is annual, so J1L means a one-year lag.

2. JiA means an i-period average. The average contains the contemporaneous value and i-1 lags. Thus, J2A(X) means a two-period average of this year's X and last year's X.

3. JiD is an i-period difference. Thus, J1D(X) is this period's X less last period's X.

4. JiP is the percentage change over i periods. Thus, J1P(X) is 100*J1D(X)/J1L(X).

2.3.3 Continuous and discrete time, approximations

We generally think of the basic theory behind SAM as representing continuous decision processes. For estimation and simulation, however, such processes must be represented in discrete time. Consider, for example, a production function. In continous time a technology represents the rate of output coming from flows of services of the factors. Although SAM's technology is more complicated, for this discussion we assume that the service flows are proportional to the relevant stocks, where appropriate, and write

$$y = f(k, e, l), \tag{2.9}$$

where y is the rate of output from the stock of capital, rate of energy use, and employment: k, e, and l, respectively. If we integrate the above relation over an arbitrary interval (for us it is one year) we obtain the flow of output on the left side, and unless f is linear (a rather unsatisfactory condition for a production technology) a non-tractable mess on the right side, in the sense that there is no exact identification possible of a relation between integrals of factors and integrals of output that preserves f or even provides a clear modification to f. All is not lost, however. Armstrong (1985b) shows that the approximation error induced by passing the integral through f to write

$$Y = \int y = f(\int k, \int e, \int l) \tag{2.10}$$

is very small, at least for the SAM technology if not in general.

This leaves us with the problem of interpreting and measuring the integrals. Y itself is observable; it is simply the annual flow. The capital stock data are end-of-period data. We approximate the annual integral of the stock as the average of end-of-period values:

$$\int_{t-1}^{t} k(s)ds = (k_t + k_{t-1})/2 = J2A(k). \tag{2.11}$$

Wherever a stock measure appears in the model it is recorded at year-end value and appears in 'flow' equations as a J2A. Similarly, stock equations normally have a form in which the change in (or growth of) the end-of-period stock is related to flows during the period.

To conclude our technology example, we measure energy as the annual use (integral) and labour as average annual employment. Of course, in the complete model complications such as productivity growth must be considered as well.

Where a mathematical product must be integrated, for example when a value (equal to a price times a quantity) is integrated, we write:

$$V = \int v = \quad p \cdot q \approx \int p \cdot \int q = P \cdot Q. \tag{2.12}$$

We must recognize that the integral of a product is not generally the product of the integrals. In the case of valuation equations like (2.12) the error that results from ignoring the problem would be small. But we take a slightly different approach. If we were to measure each integral separately, the integrated identity would not hold. To make it hold, we measure only two of the components, usually the value and the quantity, and define the third residually.

2.4 The Government Sector

2.4.1 Introduction

When economists consider the behaviour of private sector decision makers they usually rely on certain basic taste and technology parameters and structures being fixed and, at least in principle, estimable. When government decisions are under consideration, however, there can be no presumption of stable behavioural rules that are capable of explaining all episodes of history. Government decision rules by their very nature are subject to alteration in response to a socio-political process that generates discontinuous changes in perceptions as to the appropriate role of government and the set of particular targets to be pursued. Furthermore, one of the main reasons for building a policy-simulation model is to explore the effects of different government behavioural rules on the rest of the economic system. For both reasons it does not make sense for us to try to provide a theory of government in the same way we provide a theory of private sector decisions. Indeed, we are content to let many aspects of government behaviour be exogenous historically and set by simulation rules or explicit forecasts in dynamic solutions of the model.

There are, however, certain aspects of government sector behaviour that must be explicit in a complete macroeconomic model. The most important of these arises from the fact that government expenditures must be financed. The way in which governments choose to finance their activities affects the long-run properties of a model because the different possible mechanisms (money issue, debt issue, taxes) impinge in different ways on private sector decisions. Indeed, whether a steady state exists can depend on the long-run rules of government finance.

To ensure that a macro model is logically complete it is not sufficient to impose the government financing requirement in the accounting sense. For example, if the expenditures are financed through a tax that nobody pays -- one not fully integrated into private budget constraints and behaviour -- then the model is not complete. The government appears to be able to get something for nothing, which cannot be a correct specification of the long-run constraints on the government and the economy. Neither is it satisfactory to have expenditures financed on a permanent basis by extra bond sales, either at home or abroad, without a formal consideration of the consequences of the growing bond stock and associated interest flows. In particular, for correct long-run properties it is essential that the identity linking interest payments to the stock of debt be explicit in the model. It is possible, for example, to combine interest payments with other transfers and to specify some rule for overall transfers that is independent of the bond stock. This means that 'other' transfers move to offset any endogenous changes in interest payments on the debt. Although this is not logically impossible, it is a severe restriction that is at odds with historical experience and fundamentally changes the analysis of long-run properties of an economy under different government policies.

Several other identities affect government behaviour. They are considered further in section 2.4.3 where SAM's government sector equations are presented. In each case, we take the view that for a model that focuses on the medium to long term these identities should be explicit in the model and that any offsetting behaviour should be similarly explicit.

2.4.2 The government accounts of SAM

We aggregate the three levels of government into one sector. Public sector corporations are treated as part of the private sector, but public sector pension funds are considered part of government. Wherever possible we consolidate the accounts appropriately, but it is not always feasible to do so, given data limitations.

To aggregate federal and provincial decisions has some costs. In particular, some issues of long-term capital flows are obscured, since provincial governments are relatively important foreign borrowers either directly or through public utilities. Similarly, we cannot study directly questions concerning the effects of intergovernmental transfers. SAM is a small model, however, and it seems to us that most of the interesting macro questions can be posed in the framework of a model with a consolidated government sector.

Although we aggregate all levels of government we maintain considerable detail concerning types of expenditure and sources of financing. The basic financing identities for the consolidated government sector are:

$$\text{EQ42AGS}\quad \text{J1D(LGT)} - \text{PFX*J1D(FGT)} + \text{J1D(HT)} = \text{GFR}$$

$$\text{EQ41IGI}\quad \text{GFR} = \text{GEXPW} + \text{GEXPNW} + \text{GUIB} + \text{GTIN} + \text{TRANSF} + \text{TRANSP} - \text{TAXIP} - \text{TAXIC} - \text{ROY} - \text{TCC} - \text{TAXP}.$$

The model's financing requirement, GFR, is composed of the sum of expenditures and transfers net of all forms of tax revenue. Recall (see section 2.2) that GFR differs from the financing requirement on a national accounts basis, essentially because government asset acquisition is treated as a transfer to households. The formal 'budget constraint' is imposed in EQ42AGS in that any positive financing requirement must result in bond sales in Canadian currency, J1D(LGT), bond sales in foreign currency, -J1D(FGT), or new issues of base money, J1D(HT).[5]

In SAM, eleven elements of the financing requirement are identified. GEXPW and GEXPNW are nominal wage and non-wage expenditures (on goods and services), respectively. Government investment expenditures are treated as part of GEXPNW. GUIB represents unemployment insurance benefits. TRANSF represents transfers to foreigners. It is measured from the balance-of-payments data and includes private transfers. As a result, government transfers to persons, TRANSP, are adjusted to preserve the household wealth accounts (see section 2.2 and Chapter 5). The remaining transfer, GTIN, represents interest payments on the public debt.

Five forms of taxation are recognized. TAXIC and TAXIP are, respectively, indirect taxes net of tariff revenues and subsidies to corporations, and tariff revenues. In SAM's notional accounts, tariffs are paid directly by households. ROY represents total indirect taxation of the energy sector firms. It includes true royalties and all other forms of tax on primary energy except for profits taxes. Profits-tax revenues from both sectors are represented by TCC. Finally, we have personal direct taxes, TAXP.

---

[5] Both LGT and FGT are divided into two components in the model. For LGT we recognize a split by residence, separating purchases of Canadian dollar debt by foreigners. For FGT we recognize a split into interest-bearing and non-interest-bearing components. See Chapters 5 and 6 for details.

Selody and Lynch (1983) report that the most disparate attributes of government activities by level of government are the 'trends'. Since we treat trends as exogenous to the model we lose little in this regard by aggregating the three levels of government. Indeed, the aggregated series are generally more stable than the components, reflecting reallocation of activity without major changes in the overall levels. In considering aggregation of detrended categories, Selody and Lynch report two main conclusions. First, there is a danger in aggregating too far by type of activity. In particular there is a considerable loss of information if one aggregates all types of decision to look only at 'the deficit'. There is little loss in combining components of broad categories (e.g., types of non-wage expenditures), but broad categories (e.g., expenditures and transfers) can only be aggregated at the cost of a somewhat greater information loss. Thus, by and large, our aggregation choices are consistent with the findings of the more detailed study of Selody and Lynch.

Our choices have also been made in the light of two theoretical considerations. First, where government variables enter other economic identities they must be appropriately accounted for. For example, we cannot aggregate non-wage expenditures and wage expenditures or any transfer because non-wage expenditures must be identified separately as a component of the demand for private output. Second, where a particular aspect of government activity has a unique influence on private sector behaviour, there is a case for retaining that aspect of government as a separate entity in the model. For example, unemployment insurance benefits could be considered part of a broader concept of transfers. We do not do so because the theory of household behaviour suggests that the relative wage and unemployment insurance benefit rate will influence the participation decision and hence potential output. Similarly, each tax that we identify has a distinct impact on private sector decisions.

2.4.3 Reaction functions

Economists have devoted considerable resources to the study of government and central bank behaviour in an attempt to specify how these agents react to economic signals. If stable behavioural rules do exist for the authorities, then a model that does not incorporate these rules will miss something of the dynamics of the economy.

The standard version of SAM is structured such that governments choose exogenously the scale of their expenditures and transfers. Whatever view one has as to how the process works, governments choose to do things: to provide certain services, to provide a certain degree of redistribution through transfers, and so on. Questions as to how these activities are financed are all derivative. Although one sometimes hears that the level of activity is restricted by the ability to tax, we do not think that this truly represents a target level of taxation, rather it is another way of saying that expenditure levels are set not absolutely but relative to the scale of activity or potential output. For SAM we represent long-term policy by choices of such ratios of activities to potential output. These choices are exogenous to the model and represent the potential levers of fiscal policy.

The assumption that it is the ratio of government activity to general activity that is exogenous has important implications. The nominal level of such spending then rises automatically with the general price level and with the level of real activity, creating extremely important endogeneities for simulation exercises. Note that our specification of government behaviour implies that productivity growth in the private sector is incorporated in government variables. For example, the level of transfers per capita grows with private sector productivity and not just with the price level.

For the standard version of SAM we limit our explicit reaction functions to these ratio equations. It is interesting, however, to think about a distinction between target ratios that are attained in the long run and actual behaviour that can respond to the cycle. In earlier work (Rose and Selody, 1983) we reported estimates of short-run reaction functions for several categories of government activity. To get estimates of cyclical response we first had to specify a trend model to serve as the long-term target. Such trends, sometimes quite complex, were estimated jointly with disequilibrium response to output and labour market gaps. We had no success in finding evidence of government stabilization of the output market, but some success in finding evidence of short-run stabilization of the labour market (i.e., increased activity when unemployment was high, relative to equilibrium unemployment). The coefficients were not stable, however, over various historical periods. Moreover, the specification of trend models was a tenuous exercise because of the often erratic changes in the tendencies of actual values. Therefore, although we sometimes use our estimated results in experiments where some average historical cyclical government behaviour is desired, for the standard model we assume that historical ratios were the target ratios. For simulation over future periods a rule must be specified for each category of government behaviour not established through an identity.

2.4.4 Financing government activities

Expenditures and transfers must be financed. Government can do this by selling bonds to the monetary authority (creating high-powered money), selling bonds to the domestic public (creating future interest liabilities), selling bonds to foreigners (creating interest liabilities and exchange rate exposure if such bonds and payments are denominated in foreign currency), or by imposing taxes.

In SAM we posit a relatively independent monetary authority that chooses a rate of growth of high-powered money based on some implicit target inflation rate. There is no problem of principle in reformulating the monetary rule in terms of other targets, but our standard model posits a simple money rule. Essentially, new money is given to government to spend, but because the monetary authority has a target for monetary growth it does not respond passively to finance government deficits. So government is not residually financed by money creation.

We have said that the main exogenous expenditures and transfers are set according to targets. Certain other government variables are set by identities; for example, the interest on the government debt. Indirect tax rates, tariff rates, and business profits- tax rates are set as exogenous policy decisions and the revenues determined by the appropriate identities. Only the first of these provides a potential residual source of finance for government. In practice the bases for tariffs and profits taxes are insufficient for their use as a residual tax. We feel that it is clearer to have only one residual tax, so we have specified indirect tax rates to be exogenous and focused on personal direct tax rates as the potentially endogenous residual variable. We say 'potentially' because the financing of government is one of the choices that must be made explicitly as part of the design of particular simulations. Given an independent monetary authority, the system must be closed by fixing personal tax rates and letting bond sales provide residual financing, or by specifying some target bond stock (to validate an interest rate target, for example) and letting personal taxes adjust to provide residual financing, or by some combination of the two. The choice matters in SAM under some circumstances. For example, if the authorities decline to use taxation to finance interest payments on the debt, then any structural deficit will result in unstable increases in debt issues to finance interest payments and no stable path is possible.

For the standard model we adopt an intermediate long-term financing rule. Essentially, we force personal direct taxation to adjust to provide for real interest payments on the debt. The explicit equations are described in section 2.4.5, below. Given the rest of the standard formulation, this is sufficient to ensure a stable long-term solution, in particular a stable debt/output ratio. This will not be true under all reasonable formulations, however. For guaranteed stability it is necessary to have tax rates set residually to ensure that the deficit in the government sector is just sufficient to sustain a stable debt/output ratio.

A model of government in a macro model must be more than simply logically consistent. A model that in simulation simply diverges, not attaining an equilibrium path, albeit in a logically consistent manner, may be interesting but is not very useful for the analysis of other matters. In our view it is sensible to specify a basic model in which convergence to a steady state is assured. The model can then be changed as appropriate for the analysis of particular questions, even those where convergence to steady state is no longer possible. Although the requirement that the model must converge to a steady state in its standard form involves more than government sector behaviour, the financing of government and the consequent stock-of-wealth and interest-flow repercussions demand particular attention.

2.4.5 Government sector equations

We now turn to the details of SAM's government sector equations. We begin with the expenditures and transfers and then move to the taxes set by explicit rules or policy decisions. We conclude with a discussion of the residual financing of government.

Government wage expenditures: GEXPW

Given national accounts data on government sector wage expenditures (our GEXPW) and a measure of government sector employment, LG, we can define an implicit average wage for the government sector, WG, such that:

EQ33IGI $\quad GEXPW = WG*LG = W*WREL*LG.$

If we do so we find that WG does not look very much like the private sector market wage W; WG is higher and has slightly different trend properties.[6] The model purports to explain a wage level, but not a relative wage between the sectors. For historical purposes we treat the difference as exogenous, defining WREL such that EQ33IGI holds. In simulation WREL is exogenous and the government wage moves in proportion to the endogenous private sector wage.

---

[6] Our measure of LG is constructed by adding together explicit measures of various types of government employment. Private sector employment is then defined as total employment (from the labour force survey) less our measure of government employment. There is evidence that we have understated government employment, because of the omission of some post-secondary teachers whose pay is included in GEXPW. This will be corrected in subsequent versions of the data base. To the extent that we understate LG, measures of WG and WREL will be too large and W will be too small (because W is the rest of labour income divided by the rest of employment).

The default simulation rule for government employment is that government takes a certain portion, LGST, of the available equilibrium labour supply, LSS.[7] The proportion can be considered a policy choice, linked to the target level of per capita provision of the services of public goods. This idea is made operational as follows:

EQ32LGD $LG = LGST*LSS$.

In the standard model we simply define LGST from EQ32LGD, so that government employment is assumed to be on target, historically. Note, however, that LG is not exogenous in simulation; LGST is fixed and any endogenous changes in LSS are passed into LG. For simulation over future periods a simulation rule must be specified to determine government policy on the provision of the public good; i.e., a choice for LGST.

Recall (section 2.4.3) that we have done some work to estimate disequilibrium reaction functions for certain government decisions. Government employment is one of these. To do so we specify a trend model for LGST and add the unemployment gap as an explanatory variable. The data are too erratic (growth in the government employment share to 1972, followed by rapid decline) to enable us to specify a target or trend share with any confidence. As such we have little confidence in the identified disequilibrium response.[8]

Government non-wage expenditures: GEXPNW

Non-wage expenditures by governments are important because they influence aggregate demand directly. Moreover, the ratio of such expenditures to nominal output has been somewhat erratic. This ratio

---

7. See Chapter 3 for a more precise definition of LSS. Essentially, it is the value generated by the model's labour supply function, evaluated under conditions of full equilibrium in the model (i.e., all markets).

8. See Rose et al. (1983) for estimates of this and other disequilibrium reaction functions. For government employment we found some evidence of stabilizing behaviour, but the coefficient was not significant and residual autocorrelation was severe.

follows a pattern similar to that of government employment to total employment: rapid relative growth during the 60s, slow relative growth in the 70s. This pattern is not inconsistent with the view that goods are combined with labour to produce public sector output according to some technology. The basic trends are similar.

In the standard model, the ratio of non-wage expenditures to nominal private sector output is specified to be an exogenous policy-determined value, GNWT:

EQ31UGD GEXPNW = GNWT*P*UGPBSS.

The nominal output value used as the scale is potential real output, UGPBSS, evaluated at current prices, P. We find this to be the most neutral form because it makes current real expenditures, GEXPNW/P, independent of the price cycles and responsive only to endogenous changes in potential.

Our default procedure is to define GNWT from EQ31UGD, so that actual government expenditures are specified to be on target historically. In simulation, however, nominal expenditures move with the price level and with potential output. Note, in particular, that productivity growth in potential output will result in equivalent growth in the level of real government spending.

Interest on the public debt: GTIN

As described in section 2.2, the model equation for GTIN is an identity linking stocks of debt outstanding to interest payments using an endogenous measure of the interest rate. In section 2.2, however, the equation described ignores the distinction between domestic- and foreign-currency debt and the distinction between yields and average 'coupon' rates of interest. The model equation is:

EQ29IGI GTIN = RAC*J2A(LGT) - RACUSG*PFX*J2A(FGBT),

where RAC and RACUSG are respectively the average coupon rates on government debt in domestic and foreign currencies, and where J2A(LGT) and -PFX*J2A(FGBT) are the respective average stocks. The negative sign on FGBT appears because we measure the foreign-currency debt as an asset rather than a liability. The average coupon rates are endogenous to the model and are determined by updating equations that appropriately weight old debt and old average interest rates and new issues at the current interest rate. See Chapter 6 for precise definitions of these concepts as well as the equations for RAC and RACUSG.

Recall that we treat EQ29IGI as a model identity to preserve the exact relationship between interest payments, stocks of debt, and interest rates. We define a model version of GTIN from the identity. This differs from official measures of interest payments, even after we consolidate as best we can intra-government payments and transactions with the pension funds. We treat the difference as an 'other' transfer to households. Thus, we respect the payments data and the notional identity, but we change the interpretation of transfers.

We believe that the use of some such procedure is vital for medium- to long-run analysis. Other modellers have faced the same problem by putting estimated coefficients in an equation like EQ29IGI. Although this may allow the equation to fit the historical data, the coefficients pervert the simulation properties of a model by breaking the logical link between stocks, interest rates, and interest payments. These links must be imposed precisely, especially when questions about the long-run consequences of various forms of government behaviour or the implications of sectoral deficits are considered.

Unemployment insurance benefits: GUIB

We define an unemployment benefit rate, UIB, such that

GUIB = UIB*RNU*LS, (2.13)

where GUIB are actual payments, RNU is the unemployment rate, and LS is the labour force. We compute GUIB in the model but remove it from the simultaneous block by substituting equation (2.13) wherever GUIB appears.

The UIB rate is a policy variable in the model. In the 1960s the rate was relatively constant and did not respond automatically to inflation or real productivity growth. In 1972, major reforms were introduced. The UIB rate was increased dramatically and linked more closely to actual price changes. Since 1972, several rounds of minor reform and at least one more major change in the rules of eligibility, coverage, and so on, have been introduced. Although it is interesting to speculate on how such a process of reform evolves, for this version of SAM we are content to let the UIB rate be an exogenous policy variable. We specify that the exogenous policy choice is the ratio of the UIB rate to the equilibrium wage:

EQ30IGR UIB = RUIB*WS.

The equation defines RUIB historically. For simulation over future periods it must be set along with the other policy variables. It is important that the UIB rate is linked to the wage in simulation. The relative wage/UIB rate is important to households, especially for labour supply (participation) decisions (see Chapter 3).

Transfers: TRANSF, TRANSP, TAXIP

TRANSF is net transfers to foreigners, based on the current account data, but adjusted to reflect the imposition of some model identities on other variables that influence the current account. For example, we impose an identity similar to the GTIN identity on interest and dividend payments to foreigners. Any difference between the official data and the model data for such payments is treated as a transfer. In reality TRANSF is not solely a government sector variable. Some transfers reflect private transactions. However, we felt it unnecessary to retain the disaggregation. We prefer to focus on the possible policy lever, government transfers or foreign aid, and to record all the transfers as a government activity. To keep the household and government accounts correct, we adjust government transfers to households by an amount equivalent to household transfers to foreigners. These transfers then go notionally from households to governments and then to foreigners.

TRANSP is a residual category in the data construction. As noted, all adjustments of convenience in other government sector transactions result in implicit changes to transfers. At the behavioural level we make one further simplification. Tariff revenues are linked to imports in an identity:

$$\text{EQ34IGI} \quad TAXIP = (RINDT*(1+RTAR)+RTAR)*PMNEID*MNEID,$$

where RTAR is the tariff rate and it is assumed that indirect taxes are then levied on the gross of tariff price. These indirect taxes are presumed paid by households. But variations in tariff revenues are assumed offset by other changes in transfers[9] so that:

$$\text{EQ58IGR} \quad TRANSP - TAXIP = TST*PC*UGPBSS.$$

TST is the exogenous policy variable. As usual we have two versions, one in which TST is computed from EQ58IGR so that actual net transfers are presumed to be always on target, and the second in which a model is specified for TST and a disequilibrium reaction function estimated. In the standard model we take the former approach. Note that in simulation any changes in tariff revenues are offset by changes in TRANSP such that net transfers remain an exogenous fraction of nominal output. Note further that we use PC, the consumption deflator, to value potential output. This is because it is PC that appears in the household valuation of transfers, so EQ58IGR provides the most neutral reaction function. See Chapter 3 for details on these calculations.

---

[9] This unusual procedure does not reflect a judgement that such an offset actually occurs or has some particular logic. The equation is written in this form to preserve the integrity of our wealth accounting, which was completed on the assumption that tariffs would appear in the firm accounts and not in the household accounts. As such, our measure of household wealth did not consider tariffs as an applicable tax. In future versions of the model the data generation will be made consistent with the accounts in this regard and the offset in EQ58IGR will become arbitrary.

Indirect taxes: TAXIC, RINDT

Tariffs and taxes on energy are identified separately; all other forms of indirect taxation are combined in TAXIC. The rate of indirect tax (net of subsidy), RINDT, is set such that the following identity holds:

EQ35IGI   TAXIC = RINDT*(P/(1+RINDT))*UGPC.

The price P is a market measure, inclusive of indirect taxes. Dividing by 1+RINDT puts the valuation of output on a factor-cost basis. Revenues are defined to be the indirect tax rate times output, UGPC, valued at factor cost. RINDT is treated as an exogenous policy variable.

Recall that the domestic income identity is imposed, such that the market value of output is just exhausted by payments to the factors plus indirect taxes. The version of equation 2.8 that holds in the non-energy sector is:

$$P \cdot UGPC = W \cdot L + R \cdot K + PEN \cdot EN + TAXIC, \qquad (2.14)$$

where L, K, and EN are the quantities of labour, capital, and energy employed, at prices W, R, and PEN, respectively. By substituting EQ35IGI in 2.14 and rearranging we obtain:

$$(P/(1+RINDT))\cdot UGPC = W\cdot L + R\cdot K + PEN\cdot EN, \tag{2.15}$$

or

$$UGPC = [W\cdot(1+RINDT)/P]\cdot L + [R\cdot(1+RINDT)/P]\cdot K + [PEN\cdot(1+RINDT)/P]\cdot EN. \tag{2.16}$$

Equation 2.15 shows that EQ35IGI and the income distribution are consistent in the sense that P/(1+RINDT) is the price index at factor cost that yields the value of output just exhausted by nominal payments to factors. Equation 2.16 provides a set of real factor prices, using the factor-cost deflator, that just exhaust real output in terms of notional real payments to factors -- but these are inclusive of the factor's 'share' of the real indirect tax. We present this equation here because it is the form used in the firm's optimization problem presented in Chapter 4.

Royalties: ROY, ROYH

All forms of indirect taxation of primary energy are subsumed into a single notional royalty tax. Total revenues from such taxes are called ROY. The fact that we recognize two domestic sectors, energy and the rest, means that in principle there are two distinct real profit rates possible, and that in general we would require two forms of equities. The existence of economic rents in the energy sector and the taxation of those rents has received much attention in recent years. Clearly, governments have made an attempt to capture at least part of the energy rents for general use. We impose a simplifying assumption about government taxation of the rents in the energy sector — that governments set royalty rates to remove all excess rents. By excess rents we mean profits over and above those available in the non-energy sector. This is not to say that there can be no allowances for risk premia or special incentives for exploration and development. Most of the gains of simplification require only that the two rates of return move together in simulation. Nevertheless, we compute the hypothetical royalty rate that exactly equates the rates of return in the two sectors, and employ a simulation rule that moves actual royalties to the hypothetical level in the future. For the historical period we respect the data and ignore the problems this causes for the interpretation of the single homogeneous market for claims to the capital of both sectors.

The hypothetical royalty-tax revenue that equates the rates of return in the two sectors is given by:

```
EQ37IGL   ROYH = PEN*(ENC+J1D(INVENT)) + PENW*PFX*(XEN-MEN)
                 - W*LEN - R*(J2A(KENT) + (PEN/P)*J2A(INVENT)),
```

where PEN is the price of energy and is used to value domestic sales, ENC, and inventory accumulation, J1D(INVENT); PENW*PFX is the domestic currency price of net energy exports, XEN-MEN; W*LEN is the wage bill of the energy sector; J2A(KENT) is the annual average stock of capital in the energy sector and (PEN/P)*J2A(INVENT) is the stock of energy inventories in the units of KENT; and R is the nominal gross profit rate in the non-energy sector. The equation shown above is slightly simplified. The full version contains an additional term to allow for differential depreciation in the two sectors. We equate the returns net of depreciation. The full version also uses an explicit link to the profit rate in the non-energy sector that necessitates a somewhat complex expression for R. See Appendix B for the complete equation and Chapter 4 for an explanation of the return to capital in the non-energy sector.

The actual ROY data are respected in the equation:

```
EQ38IGR   ROY = (ROYH+ROYEX)*(TIME<1974)
                + ROYH*RTR*(TIME>1973).
```

Before the first OPEC price increase an additive adjustment is specified. After the OPEC price increase a proportional adjustment seems more appropriate. For simulation over future periods we move RTR to unity from its end-of-sample value of roughly 0.5.

Corporate profits taxes: TCC, RBTAX

The institutions of corporate taxation are very complex. Our current specification ignores the complications and substitutes a simple link to profits. The definition of private sector profits, YB, is simply total nominal output (energy included) less wage costs, capital consumption allowances[10] and indirect taxes (including royalties):[11]

$$\text{EQ92IFI}\qquad YB = P*UGPB - W*(LC+LEN) - CCAB - ROY - TAXIC.$$

Corporate profits taxes are then given by:[12]

$$\text{EQ61IGR}\qquad TCC = \underset{.391\ (40.1)}{AG34}*RBTAX*YB,$$

Sample, 1960-83 RSQ = 0.955 DW = 1.1

where RBTAX is the marginal profits-tax rate, constructed as a weighted average of the rates for small and large corporations. The parameter AG34 is necessary because effective average tax rates are far below the marginal rates. Estimated by ordinary least squares, AG34 indicates that the effective tax rate is just 39% of the marginal rate.

Equation EQ61IGR is not meant to be a serious model of corporate taxation. Many particular rules exist to break any simple link between taxes and actual profits. We consider the influence on the user cost of capital of investment tax credits, interest deductibility and variation in the tax rules for depreciation, but we have not incorporated any consequent effects on corporate tax revenues. Implicit in EQ61IGR is the assumption that other changes offset any revenue consequences of changing a fiscal incentive. In other words, a marginal incentive can be introduced without changing the average tax rate. This can be changed as deemed appropriate in particular experiments, but this version of SAM is not configured to deal automatically with the revenue implications of fiscal incentives.

---

[10] Recall (section 2.2) that capital consumption allowances are defined on a proportional basis and not according to the rules of depreciation for taxation purposes or according to the conventions in the national accounts. The equation for CCAB is EQ91IGI in Appendix B. CCAB is specified to be a proportion (depreciation rate, DELK) of the replacement cost of capital.

[11] Profits, YB, are equivalent to r.k in the overview discussion in section 2.2 and to R.K on page 61 (except that YB is for the entire private sector) but in both cases net of capital consumption allowances. Profits are the residual in the income distribution identity, EQ92IFI, but the model contains equilibrating mechanisms that force the value of YB to adjust to a 'normal' level. See Chapter 4.

[12] In all estimations reported in the form of EQ30IGR (i.e., coefficients reported with the equation) the figures in parentheses are t-ratios.

Personal direct taxes: TAXP, RATAX, RATAXN, RMTAX

As described in section 2.4.4, personal direct taxation is given a partial role in the residual financing of government. In particular, the average tax rate, RATAX, is specified to adjust such that real interest payments on the government debt are paid for through personal direct taxes.

We begin with a basic identity, defining RATAX to be the rate that links tax revenues to personal income:

EQ40IGI TAXP = RATAX*YPERS

```
EQ84IHI   YPERS = W*(LC+LEN+WREL*LG) + TRANSP - TAXIP
                + RNU*LS*UIB + RAC*J2A(LGDT)
                + RACUS*PFX*J2A(FHT) + YBD - FOPRO.
```

Personal income consists of labour income from the three domestic sectors, W*(LC+LEN+WREL*LG), net transfers from government, TRANSP-TAXIP, unemployment insurance benefits, RNU*LS*UIB, and capital income in the form of interest on domestically held government debt, RAC*J2A(LGDT), interest on net foreign assets, RACUS*PFX*J2A(FHT), and the net profits from domestically owned private sector capital, YBD-FOPRO. YBD is after-tax profits and FOPRO is the share that goes to foreigners.

To implement the endogenous link to interest payments we separate RATAX into two components:

EQ39IGR RATAX = RTAXB + RRGTIN.

The variable RRGTIN is defined to be the tax rate that yields revenues sufficient to cover real interest payments on the public debt. We use the household consumption deflator, PC, to measure inflation because it is the real value of debt to households that matters. Households make all real decisions based on the price of their consumption bundle. The base tax rate, RTAXB, is exogenous or tied to some policy goal, depending on the experiment, but RRGTIN is specified to move endogenously such that changes in real interest payments result in corresponding personal taxes:

EQ44IGR RRGTIN = (GTIN-(J2A(LGT)-PFX*J2A(FGBT))*.01*
J1P(PC))/YPERS.

Recall (p. 57) that GTIN includes interest payments on debt denominated in foreign currency. Hence, when the exchange rate moves, any resulting changes in Canadian-dollar interest payments on that foreign debt are passed on to households via RRGTIN and personal direct taxes.

The tax rates, both average and marginal, affect household wealth calculations (see Chapter 3). Moreover, to allow average tax rates to change endogenously while holding marginal tax rates fixed would introduce very important tax effects. For example, if average taxes rise with fixed marginal rates the degree of progressivity in the tax system changes and the supply of labour is systematically affected. We specify a link between marginal and average rates:

EQ36IGI RMTAX = 1 - AG40*(1-RATAXN),

where RATAXN (EQ07IHE) is a smoothed version of RATAX. We estimate AG40 from EQ36IGI with a correction for first-order autocorrelation, using marginal tax rate data derived from the detailed taxation statistics. The value we obtain is 0.8634. The coefficient is very well determined and not much affected by the autocorrelation correction. The correlation between marginal and average tax rates is high enough that we can reasonably think of the degree of progressivity in the tax structure as being independent of the level of taxes.

Chapter 3

HOUSEHOLD CHOICES: CONSUMPTION, LABOUR SUPPLY (THE PARTICIPATION RATE), AND REAL MONEY BALANCES

3.1 Introduction

One objective in constructing SAM was to build a model as firmly grounded in theory as was practicable, given that it was to be estimated. Another objective was to focus on real/financial linkages and to respond to a concern that traditional Keynesian econometric models were not adequately capturing the processes whereby money affects real variables. Our attempt to meet these objectives led us to pay special attention to the modelling of consumption and labour supply.

The theory specified for the household is an intertemporal utility-maximization model: each period the household determines simultaneously the pattern of lifetime consumption and saving and the pattern of lifetime participation in the labour force. The simultaneous determination of consumption and the participation rate builds in a key macroeconomic link between demand and supply. To consume, the household sector must either generate income through labour (and in so doing provide the input necessary for production) or sacrifice future consumption.

The use of an integrated model of the consumption and labour supply decisions of households represents an important extension to the theoretical base found in most macroeconometric models. We have not tried to extend the theory further to encompass a joint determination of the savings decision and the asset allocation decision. The allocation decision is discussed in Chapter 6. We do estimate the real-balance-preference function jointly with consumption and labour supply in order to respect a cross-equation restriction, but the equation for real balances is not derived formally from the preferences of households.

Intertemporal models of household choice are not new. The consumption/labour supply decision is relatively well understood and has received a great deal of empirical attention. In estimation, consumption functions tend to be relatively stable and well determined, perhaps reflecting the fact that aggregation problems and special disequilibrium influences are less important for consumption than for other macro variables. Moreover, optimizing models produce highly non-linear structural links between consumption, labour supply and their determinants, such as interest rates. Thus, there is every reason to expect that theory can provide important gains of efficiency in estimation -- because of the restrictions on functional form and parsimony in parameterization provided by formal theory. Moreover, the non-linearity of formally derived models may be important in counterfactual simulations of large shocks or situations different from historical experience.

We view the formal household optimization model as providing the structure for consumption and labour supply functions under steady-state conditions. Our formal theory does contain some dynamic elements. For example, within the optimization framework, we allow for some response to disequilibrium in the labour market as reflected in the deviation of the unemployment rate from its long-term value (and hence the effective wage from its long-term value). But, although the standard intertemporal maximization model provides a reasonable description of how consumption and labour supply would respond to permanent changes in their determinants, its general ability to describe responses to transitory fluctuations is less clear. For example, if we ask what would happen if there was a change in the expected path of real wages owing to a change in the rate of technical progress or what would happen if real interest rates were to change permanently, the standard intertemporal model seems an ideal vehicle for the analysis. However, the response to an asset valuation effect or a temporary (before prices fully adjust) change in financial wealth or a temporary interest rate fluctuation following a monetary shock, for example, may be quite different. Although, in principle, such matters could be handled in a formal optimization framework, the required model would go far beyond the standard one. Given the state of theory, we feel it is better for an applied model to treat certain key disequilibrium processes as adjuncts to rather than part of the formal maintained theory of the model. With a more careful specification of the equilibrium behaviour, it is reasonable to expect that the disequilibrium effects will be more easily and accurately identified empirically. For consumption and labour supply we modify the results of the formal model by adding two types of influence from terms that are zero in full equilibrium but which have value otherwise: a money disequilibrium term and a wage disequilibrium term.

Consider first the notion of money disequilibrium. The authorities can determine nominal money balances, but it is private sector preferences that determine the long-run level of real balances. If the private sector finds itself with excess real balances, this monetary disequilibrium can be eliminated through spending on goods or assets. In the process the price level may be bid up (reducing the excess balances directly) or the desired level of real balances may rise because of higher activity levels. At full employment, the price-adjustment mechanism is primary. It has long been argued by applied macroeconomists that standard econometric models tend to underestimate the short-run influence of money on real variables. Some economists, for example P.D. Jonson and his colleagues,[1] have argued that this is largely due to the omission of a money-disequilibrium transmission mechanism. We have used this idea in SAM, treating money as a buffer stock in that households allow real balances to deviate from long-term desired levels in response to shocks and cycles. The resulting gap is posited to influence behaviour, in particular consumption demand, in a transitory manner. We have found this idea empirically useful.

The second 'disequilibrium' influence considered involves the short-run distribution of income. What should happen in the short run if, with a given output and total income, the share of wages falls? Our basic theory tells us that if the higher profit stream is reflected correctly in financial wealth, then, ignoring tax effects, the pure distribution change would not influence consumption or labour supply through wealth, although labour supply would be reduced owing to the pure substitution effect (lower real wage). We were curious, however, to see whether the explanation of the historical data was improved by giving returns to labour a special role. We added a term that measures the difference between human wealth using the current wage and human wealth using the equilibrium wage. In the wealth dimension, this measures the equivalent of a transitory labour income effect. Although this variable provides some empirical success, we do not consider the mechanism an essential feature of SAM.

---

[1] See, for example, Conference in Applied Economic Research, Reserve Bank of Australia, December 1977.

The rest of this chapter is organized as follows. Section 3.2 is an overview of the intertemporal optimization model that provides the core equations for consumption and labour supply. In section 3.3, we introduce the notion of long-term, real-balance preferences and the implications for the price level. The details of our extensions to the basic model to enhance real financial linkages and dynamic properties are described in section 3.4. Section 3.5 describes SAM's concept of human wealth, a variable that is critical to all household decisions. Readers wishing only a basic understanding of the model can skip section 3.5 and turn to the discussion of estimation procedures and results in section 3.6. The chapter concludes with a review of the properties of the estimated system.

3.2 The Basic Model of Consumption and Labour Supply

Consumption and labour supply are treated as resulting from the same intertemporal utility-maximization decision. Within any time period, utility is assumed to be a log-linear function of consumption C(t) and leisure Lm-L(t), where Lm is the maximum labour supply and L(t) is actual labour supply, both measured as participation rates. We posit that identical households, consisting of a head and NK dependants, value present utility more highly than future utility and apply a rate of time preference ρ (a parameter) to the latter. Their time horizon is assumed infinite. Furthermore, utility from leisure is presumed to increase as the size of the family, (1+NK), increases.[2] The function that households maximize is thus

---

[2] This feature enables us to explain in estimation the trend in the participation rate over the sample. In the formulation we adopt, each household treats its NK as fixed over the planning horizon; i.e., the aggregate household sector does not foresee movements in the aggregate dependency ratio.

$$U = \int_0^\infty e^{-\rho t}[\alpha \log(C(t)) + (1+NK)^{\tau}\beta \log(Lm-L(t))]dt. \qquad (3.1)$$

The maximization is subject to a budget constraint: the excess of labour income, government transfers to individuals and interest income on financial wealth over consumption and taxes equals the change in financial wealth. Account is also taken of the possibility of unemployment and of collecting unemployment insurance benefits. All households are assumed identical, hence transforming from individual decisions to aggregate results involves multiplying by the adult population, NPOP. If LS is the aggregate labour supply (in person-years) and CON is aggregate constant-dollar consumption expenditures, the resulting[3] aggregate equations (ignoring tax considerations) can be written as:

$$\begin{aligned} LS/NPOP = {} & Lm-\rho[(\beta/\alpha)(VFR+VHPVN)/NPOP]/[(1+(\beta/\alpha)(1+NK)^{\tau})* \\ & \{WS*(1-RNU) + UIB*RNU\}/PS] \end{aligned} \qquad (3.2)$$

$$CON/NPOP = \rho\{(VFR+VHPVN)/NPOP\}/[1+(\beta/\alpha)(1+NK)^{\tau}]. \qquad (3.3)$$

The real value of financial wealth, VFR, is the sum of the net financial assets owned by Canadian households, divided by the consumption deflator, PC. In terms of SAM aggregates, it includes the real value of: base money, government debt held by domestic residents, claims to producing capital (including housing), and net foreign assets. The variable VHPVN represents human wealth, the after-tax present value of expected wage and transfer income. Market interest rates play a role through the discounting -- details are provided in section 3.5. WS, PS and UIB are the equilibrium nominal wage, consumption price, and the unemployment insurance payment per unemployed worker, respectively, and RNU is the unemployment rate. The concept of an equilibrium real wage is fully explained in Chapter 4 after the introduction of the demand side of the labour market. The concept of an equilibrium price level and its use in the calculation of the equilibrium nominal wage, WS, are described later in this chapter.

---

[3] See Masson, Montador and Selody (1981) and Masson and Montador (1982) for derivations of the optimal path of consumption and labour supply for each household.

Equation (3.3) shows that, given NK, consumption is proportional to total real wealth, both human and financial. Only total wealth matters, not its composition. The optimization that yields equation (3.3) also generates a plan for future consumption. Planned consumption grows at a rate equal to the difference between the real interest rate (used to discount future income in the wealth calculation) and the time-preference rate. In a steady state these plans will be realized. In the simplest, closed-economy case real wealth will grow at the real growth rate and this will also be the difference between the real interest rate and the time-preference rate.[4] Taxes and open-economy considerations complicate matters but need not change the basic result that growing per capita consumption from equation (3.3) is consistent with intertemporal optimization.

Labour supply is negatively and linearly related to real wealth, deflated by the current trend real return to labour (which takes account of the possibility of unemployment). Note that the unemployment rate acts as a probability here, so that the real wage used in the participation decision reflects an unemployment-weighted average of UIB (the unemployment benefit rate) and the equilibrium wage. An increase in the real return to labour produces a substitution towards labour, ceteris paribus. It also increases human wealth, but the combined effect will lead to an increase in LS.[5] Note that if unemployment rises the composite real return to labour falls. Moreover, this effect outweighs the opposite influence that comes from the simultaneous decline in human wealth. As a result SAM has a 'discouraged-worker' effect; LS moves inversely with the unemployment rate. Finally, a rise in the unemployment insurance benefit rate will increase the supply of labour, ceteris paribus, because it will increase the return to being in the labour force. It is often argued that the 'equilibrium' unemployment rate, RNAT, is dependent on UIB. If the level of unemployment went up simultaneously with a rise in UIB, then the net effect on effective labour supply is ambiguous. A link between RNAT and UIB is not explicit in SAM. To consider such issues a user would have to introduce special simulation rules.

---

[4] The movement of the market real interest rate to reflect conditions necessary for a steady-state growth path is a key part of SAM's long-run equilibrating mechanism. See Chapter 1 for an overview and Chapter 4 for details.

[5] This and some other statements about model properties that follow depend in part on particular parametric results from estimation.

In addition to the equation for the participation rate/labour supply, SAM contains an equation for a longer-term concept of labour supply. To get a notion of the steady-state supply of labour we remove the disequilibrium influences from the basic model. Essentially, this involves evaluating the LS function assuming full employment. As part of this evaluation we define an equilibrium version of human wealth, VNSS, that is used to replace VHPVN in the labour supply function. The calculation of the steady-state supply of labour, LSS, is important because it provides a key input into the calculation of potential output (see Chapter 4). The equations for VNSS and LSS are EQ16IHL and EQ19LHL, respectively, in the model listing. To fully understand these equations, however, the reader must first consider the short-run equivalents discussed later in this chapter.

3.3 Real-Balance Preferences and the Equilibrium Price Level

3.3.1 Links between real balances and other household choices

Some readers may wonder why real-balance preferences are discussed in the midst of a consideration of consumption demand and labour supply, especially since the formal utility analysis is not applied to real balances. Our operating principle in grouping equations was that things estimated together must be reported together. Although there are no cross-equation restrictions on the real-balance-preference function that comes from optimization, there are two other forms of cross-equation restriction. One comes from the fact that we make human wealth part of the scale variable that influences the desired level of real balances. Human wealth is a model-based concept and depends on a parameter that must be estimated.[6] The second form of cross-equation restriction comes from our disequilibrium money stock term to be described below. The extent of disequilibrium is measured relative to the model's concept of desired real balances. Hence, parameters from the real-balance-preference equation appear indirectly in both the consumption and labour supply equations. For both of the above reasons, the system is econometrically simultaneous.

3.3.2 Real-balance preferences, the long-run demand for money

The existence of a stable money-demand function has been vigorously debated over the years. For monetarists, the existence of such a stable function, although an empirical issue, is critical because it forms the basis of all long-run analysis of the relationship between money and prices. There is another school, perhaps reaching its zenith in terms of practical influence in the Radcliffe Committee report in the United Kingdom, that maintains that velocity is virtually infinitely variable and that no useful policy conclusions can be based on the existence of any such stable function. Our view, one that we believe would be accepted as reasonable by the vast majority of economists, is that for long-term analysis the existence of a stable real-balance-preference function is not a contentious issue.

The existence of such a function, per se, says nothing about what determines the nominal levels of variables in a model or in the economy. If the money supply is determined by an exogenous process, however, then there are long-run implications for the exchange rate, the price level, nominal wages and indeed all nominal values. Conversely, if the exchange rate is pegged, or some exogenous process of wage determination is at work, then money becomes endogenous and determined in the long run by the same real-balance preferences. In such cases it is the exogenous wage process that pins down nominal values, determining the price level and the nominal money stock in a causal sense. There is still a correlation between money and prices, but the causation runs from prices to money, not from money to prices.

---

[6] The behaviour generated from a utility function such as equation (3.1) has a maximum participation rate, Lm. This is treated as a parameter to be estimated. It appears in the labour supply function (equation (3.2), above) and in the definition of human wealth, which influences consumption, labour supply and desired real money balances. See section 3.5 for further details.

For the core version of SAM we posit an independent monetary authority that determines the level and growth rate of the stock of nominal balances. We therefore interpret the causation as going from money to other nominal values including the price level, the nominal wage, and the exchange rate. The model can be used under other interpretations of how monetary policy is implemented, and under different paradigms for the fundamental determination of nominal values (e.g., exogenous money wages) as long as the real-balance-preference function is accepted as a structural relationship that must be respected. It is debatable, however, whether we would choose to write the wage and price dynamics of the model in the same way if the wage were viewed as the exogenous nominal variable.

For our data on money we use high-powered or base money. There is no banking sector in the model and all other forms of 'money' disappear in the sectoral aggregation. As an economic concept, however, money in SAM should be given a broader interpretation. The accounting consolidation means that in the model the monetary base represents the entire system of financial instruments with 'money' characteristics. In a model with a complete financial system it would be natural to focus on who literally demands the base and hence concentrate on institutional details such as reserve requirements for banks. In a model like SAM, however, the demand function characterizes the overall preference links between macro variables and desired holdings of the monetary instrument.

Our specification of real-balance preferences is fairly standard. We postulate that the desired level of real balances is directly proportional to total real wealth and inversely proportional to a nominal interest rate (the average return on the portfolio of assets). The function is written as:

$$\log(\mathrm{HR}) = \mathrm{AS00} - \mathrm{AS01} \cdot (\mathrm{RDRH} + \mathrm{DNHE} - \mathrm{DNUBSS}) + \log(\mathrm{VFR} \div \mathrm{VNSS}), \qquad (3.4)$$

where HR is real balances, VFR is real financial wealth, VNSS is real human wealth evaluated under conditions of full employment and where (RDRH+DNHE-DNUBSS) is an 'expected' nominal interest rate. The 'expectation' is long term; it is broken down into a real component, RDRH, which is endogenously determined as part of the model's adjustment to full-equilibrium steady state, and a long-term expected inflation rate given by the money-growth rate less the real growth rate of output.

Two restrictions are imposed in equation (3.4): a unit wealth elasticity, and common coefficient for RDRH (the real interest rate) and expected inflation. These restrictions were tested and not rejected in single-equation estimation of equation (3.4); they are imposed on the systems estimator discussed in section 3.6. The unit wealth elasticity is convenient, but not critical. If we relaxed the restriction, however, we would have to redefine the link between money growth and inflation (used widely in the model, particularly for long-term inflation expectations and nominal interest rates) to account for the resulting 'velocity' trend. Money-growth rules could be easily redefined to be consistent with the same inflation target. As such, since the restriction was not rejected, it seemed reasonable to opt for the simplicity of the unit-elastic form.

3.3.3 The equilibrium price level: PS

In SAM, real-balance preferences provide a long-run restriction on the macro variables. That is, equation (3.4) is binding only in the long run. In the short run, households willingly vary their actual real balances to buffer both real and nominal shocks. We describe the details of the short-run, money-demand function in Chapter 6. In the long run the system generates a solution wherein the short-run function and the real-balance-preference schedule intersect at the data point described by the equilibrium interest rate and wealth. The price level adjusts to guarantee this result.

Because real-balance preferences provide a restriction on the system -- a long-run demand condition that must be respected -- it is important that these preferences be represented in the model. In SAM, the role of the real-balance-preference equation is to provide a long-run link between the stock of money and the price level. We 'solve' the equation for PS, the 'equilibrium' price level, the value of the deflator for nominal balances, PC, that must eventually prevail, given the determinants of desired real balances and the exogenous money stock. The model equation, with actual money balances measured as the average of end-of-period stocks, is

```
EQ17UML  log(PS) = log(J2A(HT)) - AS00 + AS01*(RDRH+DNHE-DNUBSS)
                   - log(VFR+VNSS).
```

In fact, values of PS generated by EQ17UML are approximations to the true equilibrium price because the interest rate and wealth measures are only approximate measures themselves. This is a minor qualification, however, and does not change the fundamental role of PS as a signal of the price level required for full equilibrium. We use this model-generated measure of the equilibrium price level extensively in SAM. Two particularly important examples are its use in the model's price dynamics (Chapter 7) and its use in the measure of monetary disequilibrium (section 3.4). The measure is also used to convert many steady-state real values to a nominal level. For example, we measure WS, the equilibrium nominal wage, by multiplying the equilibrium <u>real</u> wage by PS (see Chapter 4).

3.4 Modifications to the Basic Model

3.4.1 The impact of monetary disequilibrium

The most important change we make to the basic model is to augment the 'usual' real/financial linkages with terms that, in our view, capture the direct effects of excess money balances on real variables. It has often been observed that econometric models tend to have very weak links between money and real variables. This is thought to be implausible -- inconsistent with evidence from reduced-form studies and practical historical experience. Indeed, one of our goals in constructing SAM was to try to provide a model with clearer and stronger real/financial linkages. An important part of our attempt to do so is the use of explicit monetary disequilibrium measures in some of the real-side equations.

It is our view that one reason most econometric models have produced weak links, sometimes despite considerable effort to strengthen the estimated effects, is that the focus of attention has been inappropriate -- too rooted in equilibrium constructs and indirect transmission mechanisms.

The theoretical model of consumption presented above, like any similar model, shows that there are potentially powerful effects on aggregate demand when the discount rate changes-- the entire future income flow is revalued. But there is no presumption in the theory that all fluctuations in market interest rates result in equivalent changes in the discount rate. Perceived permanent changes would be so treated by rational agents, but transitory movements would influence only the discounting of income to be received over the immediate (transitory) future. Most economists accept the idea that real interest rates are primarily determined in the long run by real forces, not monetary forces. The influence of money on real rates is transient. Such transient effects may be very important for household decisions. Indeed, we provide for short-run effects on the discounting of future labour income and transfer income as well as for asset-valuation effects. Although these provide important real/financial linkages, we find an additional role, empirically, for a disequilibrium-money effect.

Since real money balances are part of financial wealth, which appears in the consumption and labour supply functions generated by our theory, it might appear that the possibility of direct links between money and household decisions has already been considered fully. This is true in a sense -- the influence of equilibrium growth in real balances is captured appropriately by these functions. But when there is a monetary shock the change in nominal balances does not represent a permanent change in real wealth. Indeed, in conditions of full employment, most of the initial change in real wealth will eventually be eliminated by price response. There is no reason why the short-run response of households to a transitory fluctuation in wealth, especially one arising from a monetary shock, should be modelled as if it were a permanent change.

For these reasons we use a measure of monetary disequilibrium as an extra influence in several model equations, including the consumption function. This term allows for a stronger link between money and consumption demand than would be captured by standard interest-rate and wealth effects. In the consumption function, the estimated coefficient can be interpreted as the rate of dissipation of the excess real balances through transitory excess consumption. It is worth noting that any identified link from excess money balances to real demand not only increases the real effects of a monetary shock, but also provides an excess demand in the product market that can be exploited in explaining the mechanism whereby prices are driven to the new equilibrium level. However, since the comparative statics of the experiment require only prices to change, there is no logical necessity for an excess product demand to be created before prices move. Whether excess money balances help to explain fluctuations in consumption and whether either excess money balances or excess product demand help to explain price movements are empirical issues. To summarize the results, we find a powerful role for monetary disequilibrium in explaining real demand, and we find both a direct monetary disequilibrium and an excess product demand signal useful in modelling price dynamics.

Strictly speaking, we do not use an exact measure of excess real money balances. Instead, we use a very similar measure based on the extent of price disequilibrium. To show the exact difference between the two approaches let us write desired real balances as:

$$HRD = f(r) \cdot VR = H/PS, \tag{3.5}$$

where HRD is the desired real stock conditional on interest rate r and real wealth VR. Excess real balances are then:

$$\begin{aligned} EHR &= H/P - H/PS = (H/PS)\cdot((PS/P)-1) = (H/PS)\cdot PGAP \\ &= f(r)\cdot VR\cdot PGAP = HRD\cdot PGAP \end{aligned} \qquad (3.6)$$

Thus excess real balances can be written as desired real balances times the proportional price gap, PS/P-1. Our measure excludes the influence of f(r), since we use terms with the structure $\alpha\cdot VR\cdot PGAP$, with α a parameter to be estimated. The difference between the two approaches is that f(r), sensitive to the nominal interest rate, falls as we notionally increase the money growth and inflation rates. Thus our measure states that for a given real wealth, a given PGAP has a given influence (on consumption, for example), whereas the pure excess-balance version states that this link diminishes in magnitude at higher rates of inflation. Our choice does not reflect a strong preference for one form over the other, and we have not tested empirically for differential power to explain history. Our initial choice was made without serious consideration of the question and we do not feel the difference warrants special investigation. Only the details of the disequilibrium process are affected.

We include the money disequilibrium term in both the consumption and labour supply functions. We had a fairly strong prior expectation, however, that this variable would play a more important role in the consumption equation than in the labour supply equation. If one has excess balances it seems more reasonable to spend them than to reduce labour participation (so that with constant expenditures, assets, particularly money balances, would be reduced). This perspective rests on the notion that there are higher adjustment costs in changing labour market participation than in marginally changing expenditures. In any case, we retain the variable for both functions.

3.4.2 The impact of wage disequilibrium

In SAM, labour receives the benefits of technical progress in the form of higher real wages. On a steady-state path, however, the ratio of the wage bill to nominal output is fixed. Thus, although there is a trend in output per capita, there is no trend in the share of labour income in total income. What can change the distribution of income in this sense is a change in another factor price (i.e., the steady-state real capital cost or real energy cost). Given such a change, it is our opinion[7] that, aside from tax-induced effects, SAM would generate offsetting changes in human and financial wealth. Thus, most of the effects on real output of such changes would come from supply-side response, not from demand response via consumption, although there would be some demand response from a pure substitution (towards leisure) effect.

Even if the equilibrium model is relatively neutral in terms of demand-side response to changes in the distribution of income, a different set of issues arises regarding short-run response to wage and profit cycles. In a world of perfect markets one might expect offsetting wealth effects (human vs. financial) regardless of whether the shock was permanent or transitory. However, the idea that wage income might have a liquidity effect on consumption is worth considering. It is far from clear that the household sector can borrow freely against expected future income to smooth cycles. Therefore, we decided to include a disequilibrium term and test for such an effect. To do so we specify and add to the consumption equation a disequilibrium term, α (VHPV-VHPVN), where VHPV is human wealth calculated using the current wage (in place of the equilibrium wage used in the calculation of VHPVN) and α is a coefficient to be estimated. By definition this term will be zero on a steady-state path. See section 3.5 for a description of VHPVN and EQ75IHI for the specific definition of VHPV.

We have focused on the consumption argument above, but it is also interesting to consider the possible influence of wage disequilibrium on labour supply. The question is whether, empirically, we can identify a short-run response of labour supply to wages, or whether such decisions are dominated, even in the short run, by long-term wage expectations. Our results suggest that labour supply is determined by long-term wage expectations. The separate influence of the current wage is small and insignificant in estimation.

---

[7] We say opinion because the asset-valuation system is not tranoparent and we have not as yet tested the proposition in simulation.

3.5 Human Wealth: VHPVN

3.5.1 An approximation procedure

Human wealth, VHPVN, is influenced by the expected present values of three things: unemployment insurance benefits, other net government transfers to persons, and wage income (all adjusted for taxes). Each present value is an integral over an infinite future horizon. It is not possible to solve such integrals analytically except under very special assumptions. It is not possible to do so, for example, when growth rates or discount rates are expected to vary over future periods. We specify approximate solutions to the forward integrals using step-function approximations, where necessary, for those factors that are expected to change over future periods.[8] Briefly, each approximation consists of the sum of five subintegrals, one each for 0-1, 1-3, 3-6, 6-12, and over 12 years into the future. The variables 'step' towards steady state values at rates depending on the particular variable. For example, the discount rate takes relatively large steps; we specify that no disequilibrium influence is expected beyond three years into the future. At the other extreme, wage-growth disequilibria are assumed to linger. For example, our specification implies that for the interval three to six years into the future, only about half of any disequilibrium is expected to have been resolved.

3.5.2 The discount rate

Present-value calculations are based on a real discount rate that varies over calendar time and over projected lead time at each decision point. For estimation, the sample-average, after-tax, real return on the financial portfolio is taken as the expected steady-state real discount rate, RDRH. We had difficulty in successfully introducing in estimation any variation of the discount rate in response to variations in ex post market rates. Preliminary versions of the household equations were estimated with a constant (sample mean) discount rate because results were better without any variation associated with movements in the return on the portfolio. However, the refinements to calculation of VHPVN, reported in Rose et al. (1983), have permitted us to introduce successfully in estimation the real/financial linkage of a variable discount rate. 'Success' is a relative concept, of course. The fit of the model would not be much affected by constraining the rate to a constant (the log-likelihood function would fall by only about 0.7), but it would be worse. Furthermore, it is useful to have the evidence from the data indicating the order of magnitude of such effects. Given the qualitative importance of this link for policy analysis, even a minor improvement in estimation is noteworthy. An important lesson remains, however. There is no support for the proposition that fluctuations in market rates influence consumption and labour supply as they would if agents were myopic and could not see through temporary fluctuations. On the contrary, the only way in which this linkage appears to work is when it is recognized that most ex post variation in real rates is noise, so that fluctuations in market rates influence the discount rate only a little. Only when changes in market real rates are considered permanent will discounting be affected.

---

[8] See Rose, Lecavalier and Montador (1983) for details on our approximation procedure and a derivation of the approximate solutions.

We apply considerable smoothing to variation in market interest rates in defining how the discount rate of households is influenced by the market. Our particular choice was made using two pieces of evidence. First, we looked at the temporal correlation structure of RRFINE, the expected return on the financial portfolio.[9] If, for example, we estimate a model,

$$\text{RRFINE} = \text{A0} + \text{A1} \cdot \text{J1L}(\text{RRFINE}-\text{A0}), \qquad (3.7)$$

---

[9] RRFINE is defined as the weighted average real return on the four financial assets held by households. The weights are the asset shares in the portfolio. See EQ71AMI in Appendix B. Also see Chapter 6 for a description of the individual expected returns.

where A0 is the mean (assumed constant) and A1 the parameter reflecting the extent of first-order correlation in deviations from A0, we get (for the sample 1961-81) an estimate of 0.19 for A1 with a standard error of 0.21. Although there may be some structure there, it is not well determined and gives no support for a hypothesis that RRFINE would be expected to stay away from its mean for an extended period. In fact, the weight applied to the current deviation in computing a forecast for three periods ahead would be only about 0.007. As such, there is no evidence to support giving weight to RRFINE (as opposed to the long-term constant RDRH) for anything but the first two subintervals (0-1 yr., 1-3 yrs.). Indeed, based on this evidence, the influence of RRFINE even on the first two intervals must be quite small.

Our second set of evidence comes from estimation of the household equations -- it strongly reinforces the time-series evidence. Any current interest rate variation permitted to influence the discounting of the future more distant than three years induced too much variation in human wealth and led to a deterioration of model fit. It was also abundantly clear that even on the first interval the effect of RRFINE had to be severely limited. We did not search exhaustively for the best fit, but we did a few runs to see whether any weight on RRFINE could be allowed without a deterioration of the fit and if so, how much such variation could contribute to the fit. We settled on weights that give 10% influence to RRFINE on the first interval and 5% influence on the second interval. As such, very little fluctuation in current market interest rates is permitted to affect the discount rate.

3.5.3 Expectations formation

The present value integrals that define human wealth reflect expectations. Generally, we impose limiting, steady-state restrictions on these expectations. Consider three examples.

First, we stipulate that households expect the unemployment rate to return to the natural rate. The model generates complicated dynamics in the labour market. We make no attempt to specify rational household expectations in the sense of Muth-consistency with the dynamic path.

Rather we specify a rough approximation, that RNU is expected to approach RNAT exponentially, such that 95% of the adjustment is complete within five years:

$$\mathrm{RNUE}(t+s) = (\mathrm{RNU}(t)-\mathrm{RNAT}(t))\exp(0.6s)+\mathrm{RNAT}(t). \quad (3.8)$$

Although the model rarely generates monotonic adjustment paths, the difference between a wealth measure using our simplification and one using an exact ex post path would be small. The parameter, the value 0.6 above, could be varied in cases where very unusual paths were encountered. Generally, however, we shall not consider this detail, as this particular expectation influences only a small part of the human-wealth calculation.

Second, we specify that households expect that in the limit the rate of change of real wages will be the rate of productivity growth. The steady-state version of this growth rate is parametric in SAM. We assume a value of 1.3% per annum but this can be changed for particular experiments. Whatever the value, it is assumed known by households. The details of the specified expected path determine the 'steps' in the approximation to the forward integral. Briefly, we specify that the current growth rate of real wages is expected to prevail for the first year, but after that to return in four steps to the steady-state value, the rate of productivity growth, over a total of 12 years.

The third expectation on which some consistency restriction is placed concerns government transfers. Recall that the government sector equations are specified such that, in steady state, net transfer payments attain a target share of private sector output. In simulation the target values and the adjustment paths can be treated as exogenous policy choices. For the historical period we have estimates of the target paths and their limit values, with which 'rational' expectations of government behaviour can be specified. We have chosen not to impose complete consistency on these expectations because an exact solution of the present-value integral in which they appear is not available. We do, however, approximate the exact solution. The current growth rate of government transfers is specified as the rate of technical progress plus an adjustment. The adjustment is calculated to be consistent with the current rate of transition towards the steady state where transfers per capita reach the target value. The expectations are thus consistent with current government behaviour and steady-state behaviour, but because of the step-function approximation do not reflect exactly 'consistent' expectations.

We have not applied the same degree of consistency to expected future taxes. As detailed in Chapter 2, personal taxes provide the long-run residual source of government finance in the basic version of SAM. Where such a system operates, consistent expectations would require households to consider the course of future taxes implied by the current and future activities of governments. However, the nature of government decision rules is one of the things we expect to vary frequently from experiment to experiment, particularly the form of residual financing of expenditures. Therefore, we decided not to specify any regime-dependent expectations with respect to taxes for the household sector. Rather we link the expected average tax rate to the current measured tax rate. We then assume that households form expectations about relevant after-tax values. For estimation we tie our measure of average expected tax rates closely to current actual values. In simulation one can use any appropriate expectations-generating rule. For example, we could specify an average expected tax rate that was consistent with the actual future path of taxes. But, although it is possible to allow for different expected tax rates in a temporal average sense, it is not possible (with the current specification) to distinguish between different expected paths with the same 'average' effect.[10] With the default tax-rate expectation rule, a switch between bond and tax financing will have a short-run effect on consumption and labour supply. A reduction in bond financing that reduces future tax liabilities, replaced by higher current taxes, will reduce consumption initially. As actual tax collections fall, however, this influence is removed. Whatever procedure is used to define the expected average tax rate, our equation will be consistent in steady state with any assumption about government targets.

---

[10] The forward integrals could be modified and an additional dimension to the step-function approximations introduced if more time-path consistency was thought necessary.

3.5.4 Human wealth: VHPVN

We now consider how the material in sections 3.5.1 to 3.5.3 comes together in the definition of human wealth. Human wealth is composed of purchasing power generated by current and future transfers from government and by current and future labour income. In defining transfer wealth we distinguish between unemployment insurance benefits and other transfers. Specifically, human wealth, VHPVN, can be thought of as the sum of three components:

$$\text{EQ74IHI} \qquad VHPVN \equiv VHPVT + VHPVW + VHPVU,$$

where VHPVT is the present value of expected transfers from government (excluding unemployment benefits), VHPVW is the present value of expected future wage income plus those expected unemployment benefits that would arise on a steady-state path (where the unemployment rate is the equilibrium rate, RNAT), and VHPVU represents an adjustment reflecting the influence of the deviation of unemployment from the natural rate. In SAM the identical households anticipate being unemployed a proportion RNAT of the time in steady state. Hence, the basic wage wealth computation in VHPVW reflects both wage income and transfers in the form of unemployment benefits. The term VHPVU is zero in steady state, and varies in sign depending on the actual unemployment rate. If the actual rate, RNU, exceeds RNAT, then since the benefit rate is less than the wage, human wealth is lower than it otherwise would be and this is introduced through a negative VHPVU. Similarly, if RNU is lower than RNAT, VHPVU is positive.

Human wealth from wages and normal unemployment benefits: VHPVW

The equations for VHPVT, VHPVW and VHPVU are very complex. Consider for example, VHPVW:

```
EQ72IHI VHPVW = NPOP*BASEW*((EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24
        -(AH20*RDRH+(1-AH20)*RRFINE)))-1)/(AH25*GWAGE+(1-AH25)
        *AE24-(AH20*RDRH+(1-AH20)*RRFINE))+(EXP(AH35*(AH25*GWAGE
        +(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE)))*(EXP((AH36-
        AH35)*(AH26*GWAGE+(1-AH26)*AE24-(AH21*RDRH+(1-AH21)*
        RRFINE)))-1))/(AH26*GWAGE+(1-AH26)*AE24-(AH21*RDRH+
        (1-AH21)*RRFINE))+(EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24
        -(AH20*RDRH+(1-AH20)*RRFINE))+(AH36-AH35)*(AH26*GWAGE+
        (1-AH26)*AE24-(AH21*RDRH+(1-AH21*RRFINE)))(EXP((AH37-
        AH36)*(AH27*GWAGE+(1-AH27)*AE24-RDRH))-1))/(AH27*GWAGE+
        (1-AH27)*AE24-RDRH)+(EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-
        (AH20*RDRH+(1-AH20)*RRFINE))+(AH36-AH35)*(AH26*GWAGE+
        (1-AH26)*AE24-(AH21*RDRH+(1-AH21)*RRFINE))+(AH37-AH36)*
        (AH27*GWAGE+(1-AH27)*AE24-RDRH))*(EXP((AH38-AH37)*(AH28*
        GWAGE+(1-AH28)*AE24-RDRH))-1))/(AH28*GWAGE+(1-AH28)*AE24
        -RDRH)-(EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+
        (1-AH20)*RRFINE))+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-
        (AH21*RDRH+(1-AH21)*RRFINE))+(AH37-AH36)*(AH27*GWAGE+
        (1-AH27)*AE24-RDRH)+(AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24-
        RDRH))/(AE24-RDRH))).
```

This monstrous equation represents the step-function approximation to the solution of the forward integral for wage and normal UI benefits. NPOP is simply the working-age population base to convert the 'per unit' calculation to a total. BASEW is a concept used only for exposition here. In the code BASEW is replaced by its definition,

$$\mathrm{BASEW} = \mathrm{AH00}*\{(1-\mathrm{RNAT})*\mathrm{WRAT} + \mathrm{RNAT}*\mathrm{UIBRAT}\}. \tag{3.9}$$

Essentially, BASEW is the steady-state, after-tax, effective value of wage income and leisure per annum. 'Effective' means that the wage and the unemployment benefit rate are weighted by the proportion of time spent employed (1-RNAT) and unemployed (RNAT), respectively. The whole thing is multiplied by a parameter, AH00 (Lm in the simplified exposition in section 3.2), that represents the maximum participation rate.

It is interesting to consider why the base for the calculation of human wealth would use the maximum participation rate. What this means is that time spent at work and time spent at leisure are both valued at the effective wage. But only leisure up to a proportion AH00 of available time generates utility and is therefore valued. Furthermore, the marginal utility of leisure increases without limit and total utility decreases without limit as participation approaches AH00. Hence, even at infinite wages, participation will not rise above AH00. The fact that the wealth measure involves valuing a proportion AH00 of available time at the effective wage is not arbitrary. The optimization provides consumption and labour supply behaviour that is linear in this particular measure of wealth. If any other definition of wealth is adopted, the behavioural functions would have to be modified to produce the same results.

The variables WRAT and UIBRAT are the real after-tax wage and unemployment benefit rates, respectively. In the model code these are replaced by their definitions in terms of model variables.[11] As explained previously, 'after-tax' need not mean simply adjusting for the current tax rate. Expected future tax rates can also be taken into account. In estimation we use a three-period moving average of actual tax rates. The rest of EQ72IHI represents the solution to the forward integral under the step-function approximation. There are five separate terms, one for each of the five intervals in the approximation. GWAGE is the exogenous 'local' trend growth of real wages. It is a smoothed version of the actual historical data. For future simulation it is moved gradually to productivity growth. RDRH is the steady-state discount rate and RRFINE is the average expected real return on the portfolio of financial assets. Parameters AH35 to AH38 are the break points for the subintervals: 1.0, 3.0, 6.0, and 12.0, respectively. Parameter AE24 is the assumed steady-state rate of productivity growth, 0.013 (1.3% per annum). Parameters AH25 to AH28 are the weights applied to GWAGE to get the expected average real wage growth for the five intervals (the rest of the weight is applied to the steady state value, AE24). Parameters AH20 and AH21 are the weights applied to RDRH to get the discount rate for the two intervals (the rest of the weight is applied to RRFINE). For interval 1 (the first year) a 10% weight is given to RRFINE, and for interval 2 (year 1 to year 3) RRFINE receives a 5% weight. Thereafter, the discount rate is simply RDRH.

---

[11] Nominal variables are deflated by PC, a consumption price index, that combines the prices of domestic goods and imports. The adjustment for taxation includes recognition of the change in 1972 when unemployment benefits became taxable.

We will spare the reader a parallel discussion of VHPVU. Its structure is similar to VHPVW except for two points. In each term of the integral approximation an extra +0.6 appears in the discount rate. This represents the speed of adjustment of unemployment back to RNAT. There is also an extra multiplicative term that represents the deviation of annual income from what would have been received at full employment. This makes VHPVU zero in equilibrium. The effect of equilibrium unemployment is captured in VHPVW. See EQ73IHI in Appendix B for the definition of VHPVU.

Human wealth from transfers: VHPVT

To complete the discussion of the components of human wealth we must consider transfer wealth, VHPVT:

EQ13IHI VHPVT = NPOP*BASETR*INTEGRAL.

The term INTEGRAL stands for an integral approximation paralleling that described for VHPVW. GTRAN, the trend growth rate of per capita transfers, replaces GWAGE, and some of the weighting assumptions are slightly different, but the term has the same functional form and interpretation. GTRAN is specified as the trend productivity plus an adjustment consistent with the changes in the government's target level of transfer activity.

BASETR is a model variable serving as the base for the forward integral. It represents the after-tax, per capita real net transfers from all levels of government. It is converted to a total value by multiplying by the working-age population, NPOP. The equation for BASETR is EQ12IHI (Appendix B). Unlike the wage computation, where we explicitly separate the equilibrium and disequilibrium level effects, for transfers we combine the two, letting BASETR be a weighted combination of the current actual transfer rate and the long-term government target rate. There is sensitivity in BASETR and hence in VHPVT to changes in the current per capita transfer rate, so even transitory changes in transfer payments can have an influence on consumption demand.

3.6 Estimation Procedure and Results

3.6.1 Background on the data

Consumption expenditures, CON, are calculated as total personal expenditures on goods and services including durables. We make two adjustments to this series: (i) the portion of the data on gross fixed capital formation in residential construction that is in fact real estate commissions is counted as consumption, (ii) we exclude from the series an estimate of consumer expenditures on hospital care before 1961. This adjustment eliminates the break in consumption that occurs in 1961 as a result of the transfer in the national accounts of hospitals to the public sector. We contemplated a similar adjustment for medicare, but this change seemed quantitatively less important. In retrospect, however, an adjustment should have been made, since it became apparent in estimation that there was a significant dip in the per capita real consumption in the late sixties that was not associated with any of the macro variables. This can be seen clearly on Figure 3-4 in section 3.6.4, below. We attribute the shift to the introduction of medicare and control for it, in estimation, using a constant dummy variable in the consumption equation from 1969.

SAM's labour supply is the labour force as conventionally measured plus military personnel. Recall that for our data on money we use high-powered or base money.

3.6.2 Steady-state values in estimation

The equilibrium wage

Our theoretical model contains several variables that can be defined only by the model itself. One is human wealth. It depends on AH00 (a parameter to be estimated) and on the equilibrium wage. In turn, the equilibrium wage cannot be known without the equations from the firm sector — in particular we need to know the technology. In principle, all household and firm equations could be jointly estimated with the necessary definitions included as identities. But the estimation of the production sector was a formidable problem in its own right. The reader can see in Chapter 4 that for the 'supply side' we estimate a thirty-parameter, five-equation system using full-information, maximum-likelihood methods. We decided not to try to add the household sector equations to this estimation problem.

This left us with a difficulty. We need the equilibrium wage for the household equations, but we need the labour supply function to close the model of firm behaviour before we can generate that wage. One possibility is to iterate, to take a wage measure, estimate the household sector, generate the necessary results for the firm equations, estimate those equations, calculate an equilibrium real wage series, and then return to start the process again. This is not an attractive option given the complexity of the estimation problem for the supply side.

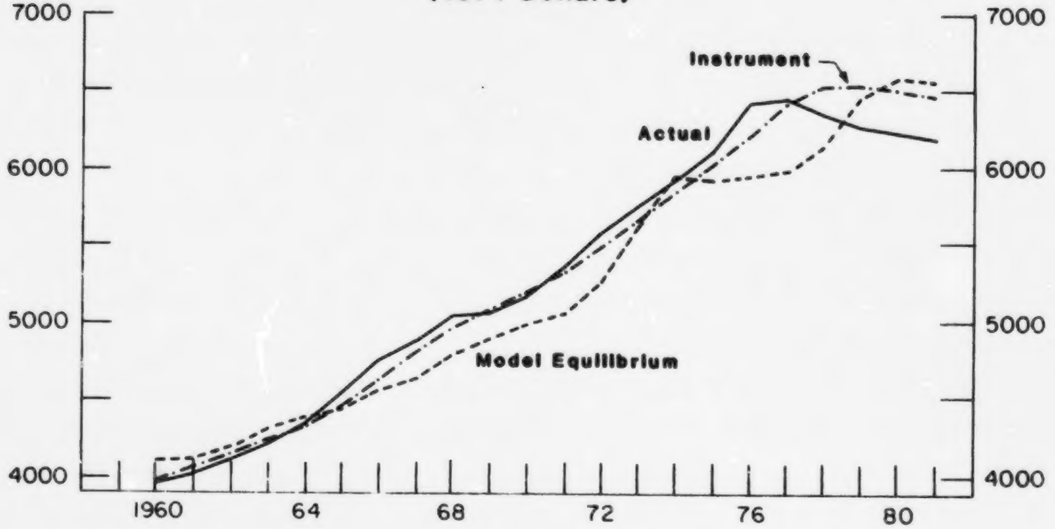
Our initial decision was to use a proxy or instrument for the equilibrium wage in estimating the household sector. This proxy was generated using an equation that generates a trend through the historical data.[12] The actual after-tax real wage, our proxy, and the later-generated, model-based measure are shown in Figure 3-1. There was considerable uncertainty, when we were creating the proxy variable as to what to choose for the end of the seventies and early eighties. For this

---

[12] The generating equation was `PROXY = (J1L(PROXY)*exp(GWAGE)+AH07*J1L(WRAT-PROXY)+AH08*WRAT)/(1+AH08)`, with AH07 and AH08 set to 0.38 and 0.1 respectively, and with an imposed starting value for PROXY equal to the actual WRAT in 1958. WRAT is the measured real after-tax wage, our 'actual' in Figure 3-1.

Figure 3-1

REAL AFTER-TAX WAGE
(1971 Dollars)



estimation our data base did not include the major revisions to the labour income figures in the national accounts issued in June, 1983. We knew, however, that a substantial revision was pending that would eventually lead to a revision of our 'actual' data. Although it was clear that earlier trends in the real after-tax wage had not been sustained, it was also clear that the measured decline in that variable considerably overstated the extent of the change. We also knew that while growth in consumption per capita had slowed, there was no evidence of the kind of dramatic decline that would be predicted by our model based on the real wage profile in the 'actual' data. We decided to use a trend model for our proxy of the equilibrium solution that diverged markedly from the data, cutting off the peak in 1974-75 and holding the value fairly constant over 1976-81. It is important to remember that the equilibrium wage need not, in principle, stay close to the actual data. This is not a model of the wage data. The important point for estimation is how close it is to the model's equilibrium wage.

Figure 3-1 also allows the reader to compare our instrument with the equilibrium version generated later from the entire model. Our speculation that the instrument should lie considerably above the data at the end of the sample is supported by our calculated series. The two are very close over the last few years, but our model calculation shows a much bigger difference between the data and equilibrium for the middle seventies.[13] There is also a long period, 1966-72, when our proxy is too high, according to the model measure.

The differences between the instrument and our model-generated series were large enough, and erratic enough, to lead to a change in estimation strategy. We decided to go one more step and re-estimate the household sector equations using the first-round model equilibrium wage. We do not, however, complete a full second iteration by re-estimating the supply side. This should not be very important since the structural parameters affecting the supply-side problem do not change very much in iteration 2 on the household sector equations. We discuss only our final estimates here.

The equilibrium price

To estimate the parameters of the real-balance-preference function we decided to normalize the equation on the price level. The simulation role of the equation is to provide PS, the equilibrium level of PC. For estimation, however, we need a proxy for PS. In the results reported below we use the actual price PC. Measurement errors for the dependent variable are less damaging than similar errors for explanatory variables. As long as the measurement errors are statistically independent of the explanatory variables no consistency problem arises. We can, moreover, report that using a smoothed PC, a 'trend' through the data, rather than PC itself, does not materially affect our results.

---

[13] See Chapter 4 for a description of the determinants of the equilibrium real wage. The decline after 1973 is due to both the real energy price increase and a rise in the efficiency cost of capital coming from an apparent decline in normal capacity utilization.

3.6.3 The estimation procedure

We have noted that SAM's measure of human wealth depends on a parameter, and that the wealth variable appears in all three equations of the stochastic system. Normal procedure in a case like this would be to estimate the complete system, including the wealth definition, so that the parameter and data would be jointly determined. In this case, however, the standard technique is likely to be inefficient. The measured money disequilibrium term is highly correlated with the estimation errors from the real-balance-preference function. In the limit, as a system explanatory variable becomes perfectly correlated with a system disturbance, the parameter on the variable ceases to be identified. Even when the problem is less severe we can gain efficiency in estimation by imposing a covariance matrix restriction.[14] We impose the restriction on the stochastic specification that the real-balance-preference disturbances are independent of those in the other two equations. To impose the cross-equation covariance restrictions in a single estimator is not possible in our TSP software, although it is in principle.[15]

To implement our specification, therefore, we used the following iterative procedure. Given a starting value for AH00, the parameter in the human wealth definition, we estimate the parameters of the real-balance-preference function as a single-equation problem.[16] Given the results, we can generate a money-disequilibrium measure and estimate the consumption/labour supply system. Here we allow a general covariance matrix and use an iterative Zellner estimator that yields results asymptotically equivalent to maximum likelihood. This estimation produces a new value for AH00. We then return to step one and begin the sequence again. This process must continue until AH00 has stabilized. In practice, convergence was rapid. At most, four global iterations were required. Once a converged estimate of AH00 has been determined, the system is run one last time with AH00 imposed at its final value. In this run, all cross-equation restrictions have been imposed and the results including the estimate of AH00, are asymptotically equivalent to maximum likelihood. We did not repeat the iteration on AH00 for the second-round (using the model-based equilibrium wage) estimation of the household sector. Instead, we held it fixed at its first-round value.

---

[14] See Armstrong (1985a) for the details of this argument and background on our estimator.

[15] The system could have been estimated directly using the GQOPT2 package employed for the supply side and asset demands (see Chapters 4 and 6), but that system was still being developed when this estimator was designed. Indeed, a similar covariance restriction is embodied in our FIML estimator of the supply-side equations.

[16] The covariance restriction implies, precisely, that aside from the parametric restriction, the two systems are econometrically independent.

3.6.4 Estimation results

The system of equations estimated consists of the human wealth definition along with the three stochastic equations:

```
EQ17UML log(PS) = log(J2A(HT))-AS00+AS01*(RDRH+DNHE-DNUBSS)
                  - log(VNSS+V/PC)

EQ76LHS LS/NPOP = AH00-((AH01*AH02*((1+NK)**AH03)*(V/PC
                  +VHPVN)/NPOP)/((1-RMTAX+AH02*(1-RATAXN)*
                  ((1+NK)**AH03))*((WS/PS)*(1-RNU)+(UIB/PS)*
                  RNU*((1-RMTAX)**(QTXRFM-1)))))+AH06*
                  log(PS/PC)*(V/PC+VNSS)/NPOP+AH62*(VHPV-
                  VHPVN)/NPOP

EQ77UHD CON/NPOP = AH01*((V/PC+VHPVN)/NPOP)/
                   (1+AH02*(1-RATAXN)*((1+NK)**AH03)/
                   (1-RMTAX))+AH05*log(PS/PC)*(V/PC+
                   VNSS)/NPOP+AH61*(VHPV-VHPVN)/NPOP
                   +AH04*(QTIME.LE.-2).
```

There is one change to the equations for estimation. As noted above, we had a dilemma with respect to the last few periods of real wage data. We did not believe the extent of the decline in the preliminary figures, and we knew that substantial revisions would be forthcoming in 1983 from Statistics Canada. The main difficulty for the estimation is that VHPV, being strongly influenced by the current wage, falls enormously and creates a large gap. Preliminary experiments showed that this dominated the results. Things that were stable and well determined over other forms of sample variation and minor specification change became much less so when the data for the wealth gap were used for the period 1979-81.[17] Rather than exclude the last three observations, or make arbitrary adjustments to the wage data to anticipate possible Statistics Canada revisions, we decided to simply remove the wealth-gap variable from the estimation problem for the period 1979-81. This is done by multiplying by a dummy variable that becomes zero in 1979. For simulation, however, the data problem is not important. The model generates solutions for the real wage that do not decline so precipitously, and we do not need to remove the gap term. So in the coded model the term appears without adjustment, as reported above. The value for AH00 was constrained in the final run that produced the rest of the estimates. The statistics reported for this parameter are from the final run in the iterative sequence that determined AH00. The parameter estimates are presented in Table 3.1.

All parameters have the expected signs and most are very well determined. The result for AH62 indicates that we find little effect from short-run fluctuations in the market wage on labour supply. According to these results, participation-rate decisions are dominated by longer-term expectations about wages. Similarly, the effect of monetary disequilibrium on the participation rate, AH06, is small and not significant at the usual confidence levels. We were surprised, however, to find that the t-ratio is above 1.0; some weak evidence for an effect comes through. The negative sign indicates that when there are excess balances, labour is withdrawn and more leisure chosen, so that income will be lower and asset holdings will fall towards desired levels. We do not wish to make much of this result. It adds little to historical explanatory power and is not essential to SAM's description of disequilibrium dynamics. We can

---

[17] In estimation we used a VHPV measure that evaluated wealth as if the current, disequilibrium wage would last forever. For large wage disequilibria it will thus greatly overstate the size of the gap. For the rest of the sample this does not matter, but for the final observations it may have been a factor exacerbating the basic problem. We have since redefined VHPV, taking into account the fact that the wage level will return to equilibrium. We have not, however, re-estimated the system. The revisions published by Statistics Canada in 1983 were important, and do raise the measured real wage, but do not completely remove our empirical problem.

Table 3.1

PARAMETER ESTIMATES, HOUSEHOLD SECTOR
(Sample, 1960-81)

Parameter	Point estimate	Standard error	Asymptotic t-ratio
AS00	-6.6406	0.0275	241.7
AS01	0.5603	0.3878	1.4
AH00	0.7945	0.0515	15.4
AH01	0.0148	7.5*E-4	197.5
AH02	0.0546	0.0031	17.6
AH03	3.9137	0.1511	25.9
AH04	149.8	26.1	5.7
AH05	0.0032	9.7*E-4	3.3
AH06	-1.08*E-7	9.0*E-8	1.2
AH61	0.0076	0.0016	4.8
AH62	5.6*E-8	1.5*E-7	0.4

constrain AH06 and AH62 to zero without affecting either the fit of the system (the restrictions are easily accepted by a formal test) or the estimates of the other parameters in any important way. So it is reasonable either to use these results or to suppress them as desired. The rest of the estimated results are unaffected. Note, however, that both the money-disequilibrium effect on consumption (AH05) and the transitory wage-income effect on consumption (AH61) are highly significant. The coefficient estimates are considered in greater detail in the next section where we discuss the properties of the estimated equations.

In Table 3.2 we report some individual equation statistics, and in Figures 3-2 to 3-4 we provide the actual and fitted values from the three equations. The price-level equation produces results that we find quite plausible. The monetary contraction in 1970 generates a relatively strong price-level gap that would have put downward pressure on prices had not the following years witnessed rapid money growth. From a SAM perspective, this money growth *precedes* the rise in inflation partly because of the levels gap. In the period 1972-75 actual price movements are virtually identical to what the model judges to be equilibrium movements. Thereafter, whether because of expectations effects associated with the announcement of money targetting (and the planned gradual reduction in money growth), the influence of the Anti-Inflation Board, or whatever, actual price increases are below those consistent with money growth and the model's long-run real-balance preferences. In 1979, however, the reduction of monetary growth starts to assert itself, and by 1980 we see a gap opening up that foreshadows the strong downward pressure on prices to come.

Table 3.2

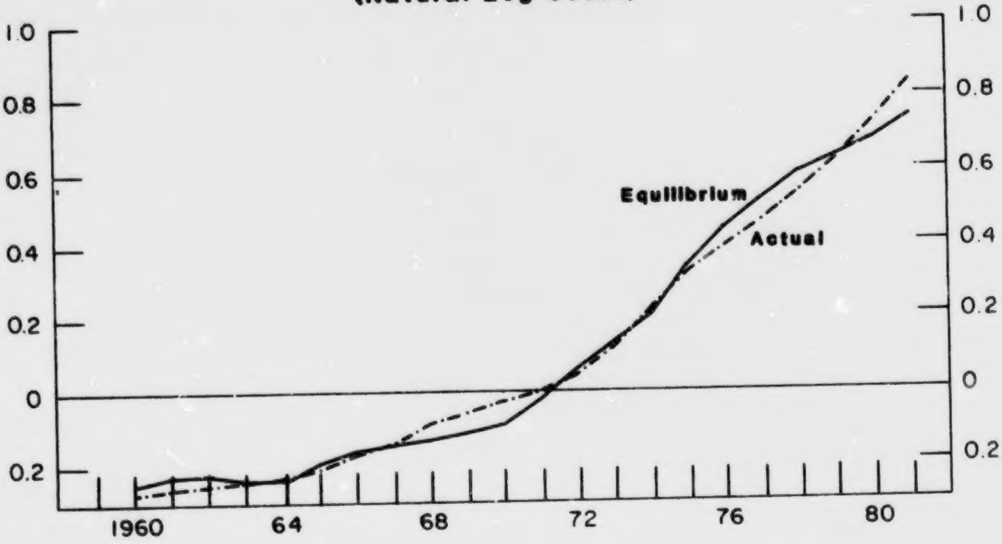
INDIVIDUAL EQUATION STATISTICS
(Sample, 1960–81)

Equation	Dependent variable	RSQ	DW
EQ17UML	log(PC)	0.988	0.55
EQ76LHS	LS/NPOP	0.965	0.78
EQ77UHD	CON/NPOP	0.992	0.59

If we use the estimated base money-demand equation to generate values for PS for 1982-83, we find that the gap opens dramatically. Indeed, the actual price level is about 30% above its equilibrium value. We are at risk in computing such gaps owing to the possibility of level shifts of the behavioural functions. If there has been a reduction in the desired level of real balances (independent of the macro variables), then our model will understate the equilibrium price. Although we do not have enough evidence to make a final empirical judgement on the issue, we know there have been important reductions in required holdings of reserves by banks and there is strong evidence of other technology-based shifts. Our current best estimates indicate that we have understated the positive price-level gap (PS relative to PC) in the mid-seventies and overstated the negative gap at the end of our sample. Our current best estimate is that the equilibrium price is about 11% below the actual outcome in 1983.

Figure 3-2

CONSUMPTION PRICE INDEX
(Natural Log Scale)



This leaves a substantial gap that continues into 1984-85. If this is so, then there is still a substantial nominal disequilibrium to be eliminated that, in the short run, will keep measured inflation performance below what would otherwise be expected.

The low Durbin-Watson statistic for EQ17UML is not surprising. This equation is not meant to generate a measure that will track fluctuations in actual prices. Our model of actual prices is described in Chapter 7. The absence of positive residual correlation would be a danger sign here.

For such a highly constrained and simple system, the equations for per-capita consumption and the participation rate do a remarkably good job. It is difficult to interpret the Durbin-Watson statistics in the context of a systems estimator, but it is clear that both equations have serially correlated fitting errors. Nevertheless, our objective was not to provide an explanation for all the fluctuations in the data. It would no doubt be possible to find special historical features that would enable the model to reduce the extent of residual correlation. However, it is

Figure 3-3

PARTICIPATION RATE

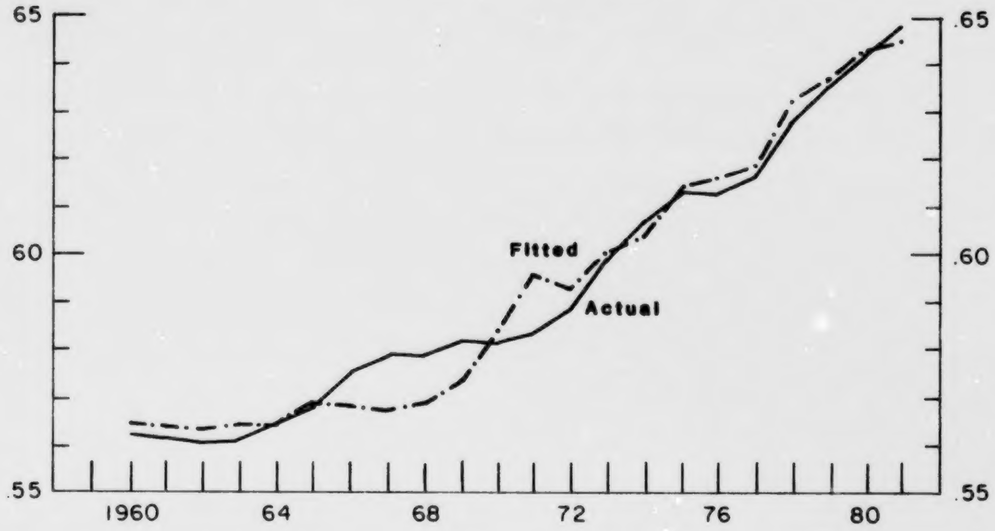
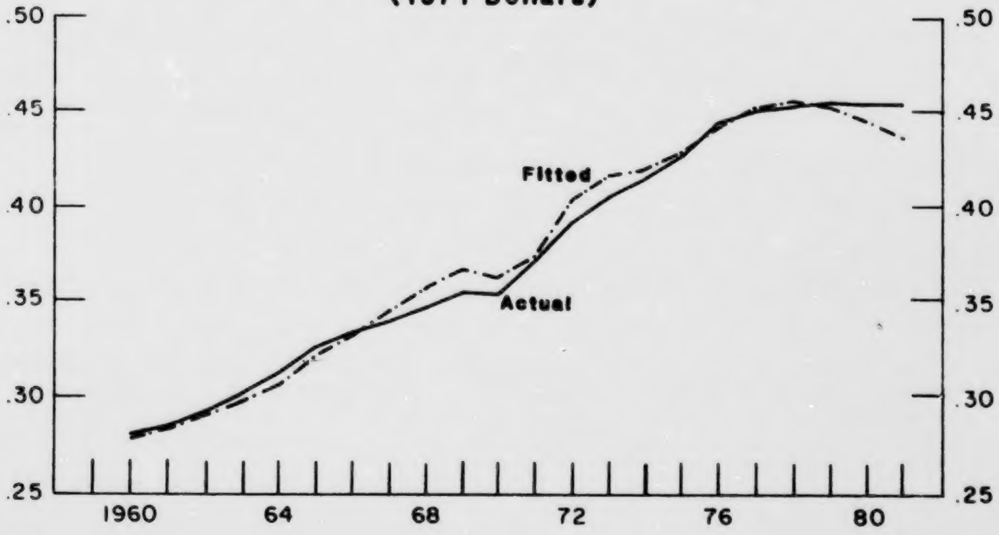


Figure 3-4

CONSUMPTION PER CAPITA
(1971 Dollars)



not clear that this would improve the model as a simulation tool. We are satisfied that the intertemporal optimizing model has given us both a useful framework for the analysis of the effects of policy shocks and a potentially important check on the conclusions from short-term forecasting equations.

The prediction errors for the participation rate over the period 1966-71 are caused by various factors. Two are of special interest. The unemployment insurance benefit rate lagged behind wages for a time in the late sixties, pulling down the model's predicted participation rate. Similarly, part of the large predicted increase is a response to the delayed 'catch-up' of the UIB rate. The other major determinant of the 'overshoot', reversed by tax reforms in 1972, is a marginal tax rate effect. In our data, RMTAX declines sharply in 1970-71, for reasons that are not entirely clear. This boosts the after-tax wage and our predicted participation rate. We suspect a data 'error', since replacing the measured RMTAX with a trend that links linearly its 1965 and 1971 values removes most of the overshooting. However, for the period 1966-68 the actual rise in the participation rate cannot be explained by the model.[18] The reader may recall that in this period the actual wage exceeds the model's equilibrium wage, and it is natural to wonder whether this calls into question our use of equilibrium concepts in the core model rather than actual current values. It is true that over 1966-68 the model would track better using the actual wage, although it still would not 'explain' the rapid upward movement in participation from 1963 to 1967. Moreover, the very good performance over the seventies, including all the episodes where the largest deviations of equilibrium from actual wages are observed, shows that the 'gain' from fitting 1966-68 a bit better would be completely dominated by larger deteriorations of fit in the 1970s. Indeed, the success of our model in the seventies, when the two wage series are so different, provides considerable support for our basic approach.[19]

---

[18] We have recently changed our methodology for measuring RMTAX. The original equation now has much less severe problems fitting the data for the period 1965-71. We have not re-estimated.

3.7 Properties of the Estimated Equations

3.7.1 Sensitivity of the price level to the money-growth rate

Our coefficient on the nominal rate, AS01, indicates that if money growth is increased by 1% per annum and inflation and nominal interest rates fully adjust, then desired real balances fall by 0.6%. With an exogenous money stock, this would imply that for equilibrium the price level would have to rise by 0.6%, over and above the direct effect of the change in the underlying inflation rate on prices. In the context of a reduction of the money-growth rate, this gives rise to the so-called 're-entry problem', wherein the price level must fall more than is implied by the lower inflation rate. In the version of SAM reported here, the money supply is presumed set without any adjustment for level-shift effects. This, however, is simply a default simulation rule. It would be easy to implement an alternative rule in which the authorities adjusted the level of the money stock to offset the effects of growth-rate changes.

The coefficient AS01 is not well determined in two senses. First, its standard error is relatively large. The confidence interval includes values up to 1.34. Second, these results are based on an expected nominal rate using the steady-state expected inflation rate, DNHE-DNUBSS. Replacing this measure of inflation expectations with our shorter-term, cyclically sensitive measure, DNPCE, we obtain a point estimate for AS01 of about 1.1. It is also worth noting that our estimate declined by about 0.4 when we replaced the equilibrium wage proxy with the model-generated values. AS01 is the only parameter that was substantially affected by this change. Our methodology requires us to take the lower, second-round result as our best point estimate.

---

[19] In historical dynamic simulation the model appears to do better than the estimated equation for the 1960s. Indeed, the largest simulation error is less than half the size of the large estimation errors. The results are not quite so good for the 1970s, but no systematic problem emerges. These statements are tentative, as only preliminary historical controls have been computed.

We cannot present strong arguments about the appropriate size of AS01. Evidence from demand-for-money studies for Canada indicate much larger interest elasticities for narrow definitions of transactions balances, such as M1 (currency and demand deposits), and somewhat smaller interest elasticities for broader monetary aggregates. We find our estimated elasticity a plausible characterization of the interest sensitivity of the link between the base and the price level. We can report that the estimates of other household-sector parameters are not seriously affected by a constraint on AS01; users of SAM who prefer higher interest rate sensitivity can at least double the coefficient without concern for the validity of the rest of the estimated model. The higher this sensitivity, however, the greater will be the extent of the 're-entry problem', and the more the equilibrium price will vary with movements in real interest rates.

3.7.2 Properties of the consumption/labour supply system

The rate of time-preference, AH01, has a point estimate of 0.0148, or 1.48%. In the simplest closed-economy growth models, the market after-tax real interest rate must equal the rate of time-preference plus the real rate of growth for a golden-rule, steady-state path. With our assumption of a 2.4% steady-state growth rate, we would look for a steady-state, after-tax real return on the asset portfolio of 3.9%. Using the average tax rate for 1983, this would imply a before-tax return of 5.1% per annum. This is slightly lower than the average real return on the asset portfolio over the estimation sample (1960-81), 5.8%, but real growth has been higher over this period than we assume will be the case in the future. In fact, the historical average real return is too low to be consistent with sustained real growth at the average sample level. If, however, we allow a heavy discounting of the baby-boom bubble for these steady-state calculations, then the average historical return is close to the hypothetical, closed-economy, golden-rule value.[20]

Parameter AHO2 is the model representation of the ratio β / α in section 3.2. It determines the 'substitutability' of leisure and consumption in the households' preferences. In terms of model properties, AHO2 is instrumental in determining the labour-supply or participation-rate response to real wages. If we hold everything else fixed, including human wealth, and focus on the pure substitution effect (between leisure and consumption), we find an elasticity of 0.34. In other words, a 10% increase in the equilibrium real wage will induce a rise in the participation rate of 3.4% or, at 1981 levels, about 400,000 person-years of labour supply or 2.2 percentage points for the participation rate. This considerably overstates the actual response, of course, because there is a simultaneous offsetting wealth effect. If we repeat the experiment and permit wealth to change we find an elasticity of about 0.06. The pure wealth elasticity, roughly -0.28, does not reverse the positive effect, but does offset about 80% of it. For a 10% real wage increase, a net elasticity of 0.06 translates to about 71,000 person-years of labour or 0.4 percentage points for the participation rate.[21]

The model predicts a rise in the participation rate when the unemployment insurance benefit is increased, but the effect is quantitatively minimal. If changes in the equilibrium unemployment rate are ignored, the net labour supply elasticity (including the wealth effect) is about 0.0014. A 10% increase in the UIB rate would increase the participation rate by only 0.014%. Moreover, this result assumes no 'funding' through increases in the contribution rate.

---

[20] We cannot provide an equivalent equilibrium condition for the general open-economy case. If however, a country's net debt to foreigners, as a share of national wealth, remains relatively small, then the condition for a steady-state growth path will be close to the closed-economy equivalent. If a country has a positive net debt position, then domestic real interest rates may have to be higher than in the closed-economy case in order to generate sufficient savings to service the foreign debt in addition to sustaining equilibrium capital formation.

[21] There is some oscillation in the period-by-period values, and a slight downward trend. The 1981 values are lower than the reported sample averages.

The 'discouraged-worker' effect, the response to a rise in the unemployment rate, has an elasticity of -0.012. Thus, if unemployment rises by 10% (say from 10% to 11% of the labour force), 0.12% of the labour force would withdraw. At 1981 values this represents about 14,000 person-years of labour. The discouraged-worker effect is almost entirely a cyclical phenomenon. If we change the level of the *equilibrium* unemployment rate, the long-run labour supply elasticity is much smaller in absolute value, only -0.0016. The reason for the difference is that when $\hat{R}$NAT changes there is a substantial wealth effect that offsets most of the short-run 'discouragement' at the lower effective wage. If a rise in unemployment is judged permanent, workers stay in the labour force because things are not just temporarily worse, they are not going to get better. If the rise in unemployment is judged temporary, however, some workers leave to await the recovery.

Our estimate of the *maximum* participation rate, AH00, is about 12.8 percentage points above the 1981 value. This seems reasonable. It is important to remember that it is the value for the participation rate that would be generated in the limit as the real wage increased without limit or real wealth went to zero. In these terms the result does not appear too large. But, given recent trends in the participation rate, could it be too small? The large increases in female participation, seen in recent years, seem likely to continue, but at a diminishing rate. Given current estimates of what the steady-state participation rate might be for Canada, from a demographic perspective, and given the actual participation rates in countries with a higher and more stable female participation rate, our estimate is large enough to leave considerable room for supply response even under 'high-participation' scenarios.

Our measure of the participation rate rises from 56.2% in 1960 to 64.8% in 1981, while the dependency ratio (number of dependents as a percentage of the adult population) declines from 53.6% to 32.4%, reflecting the movement of the baby-boom generation into the adult population. With the estimates of the parameters quoted above, the movement in the dependency ratio alone would explain a participation rate increase of 8.0 percentage points, almost exactly the rise that occurred.

The remaining variables, the increase in the real value of wealth, which decreases labour supply, and the rise in the value of the real wage, which causes a substitution away from leisure and hence increases it, almost exactly offset each other in their effects on participation. In fact, if everything except the dependency ratio is held fixed at 1960 values, the predicted value for the 1981 participation rate is 65.4%, only 0.5 percentage points above the full-model fitted value. This result does not arise because other variables stayed relatively constant. For example, if other variables are held fixed, the rise in wealth is sufficient to send the participation rate to 20% by 1981.

It is interesting to speculate on whether AH03 captures purely demographic effects, as the model 'story' maintains. It is clear that the dependency ratio is not going to continue its declining trend into the 1980s. The implication is that our model will generate 'predictions' of a stabilizing participation rate. While we feel that the NK variable does capture something real in terms of the dependency ratio, there are undoubtedly other trend factors that are embodied in the estimate, based on influences correlated with NK. For example, the trends in female participation may be only partly related to NK influences. Thus, we anticipate that our model will slightly overpredict the decline in the trend in the participation rate.

Next we consider the impact of taxes. If we change both average and marginal tax rates, using the equilibrium relationship between marginal and average rates presented in Chapter 2, we find a very small effect on labour supply (elasticity -0.009) and a somewhat larger effect on consumption (elasticity -0.136). These are mean values over the sample 1960-81. Both series have trends, the elasticities becoming larger in absolute value over time. The end-of-sample values of the elasticities are -0.017 and -0.188, respectively. If we hold the marginal rate fixed and change only the average rate, the consumption elasticity is -0.102, not too different from the results above. However, the labour supply elasticity is +0.033. With only the average tax rate changing there is a wealth effect. When average taxes rise, households work more to protect their private consumption power, but a change in the marginal tax rate induces a strong substitution effect in the other direction. As we saw above, this latter effect dominates when both rates change in equilibrium configuration.

3.7.3 Partial effects of a monetary shock

In this section we report household response to a 10% decline in the level of the stock of money. For this partial simulation exercise we ignore the government budget constraint. We hold all other real financial wealth constant, but allow the money and price response to influence total real financial wealth. We do not permit any interest rate changes or other financial sector feedback. Government transfers and taxes are held fixed, and we assume that money wages respond in proportion to any price level changes so that real wages and all other determinants of human wealth are unchanged by the shock. Finally, we assume that any influences on aggregate demand and/or aggregate supply are offset by other things so that the only process at work is the elimination of excess real balances through price response to the new equilibrium level. Thus, we include in the partial simulation the real-balance preferences in the form of an equilibrium price equation, the consumption and labour supply equations, and the model's price-dynamics equation (though only the price level disequilibrium has any shock-minus-control effect here).

Like any partial experiment, this one is artificial from a full-model perspective. Its purpose is simply to demonstrate the working of the monetary disequilibrium term in the household sector equations, with price dynamics that ignore the full-model complications of real-cycle effects and wage adjustment.

The results are shown in Figures 3-5 and 3-6. The 10% reduction in the money stock results in a very slightly smaller reduction in the equilibrium price in the first period (9.99%) owing to the small initial decline (0.2%) in real financial wealth. This decline results from the delayed response of prices to the nominal shock. In the first year actual prices are down only 1.5%. Over the next five years the system generates over 90% of the eventual full response. By the tenth year the real equilibrium has been re-established (99.94% adjustment).

Figure 3-5

MONEY, FINANCIAL WEALTH AND PRICES
Shock Minus Control %

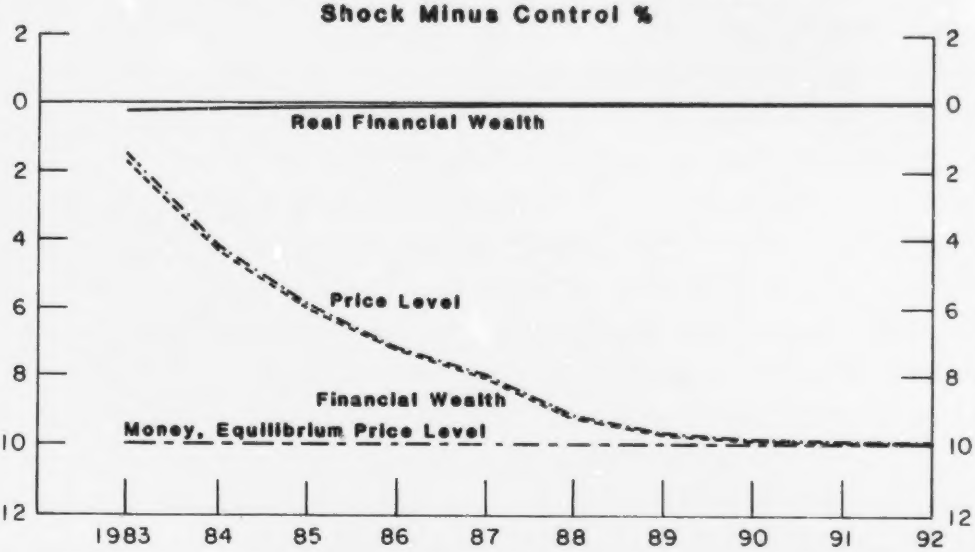
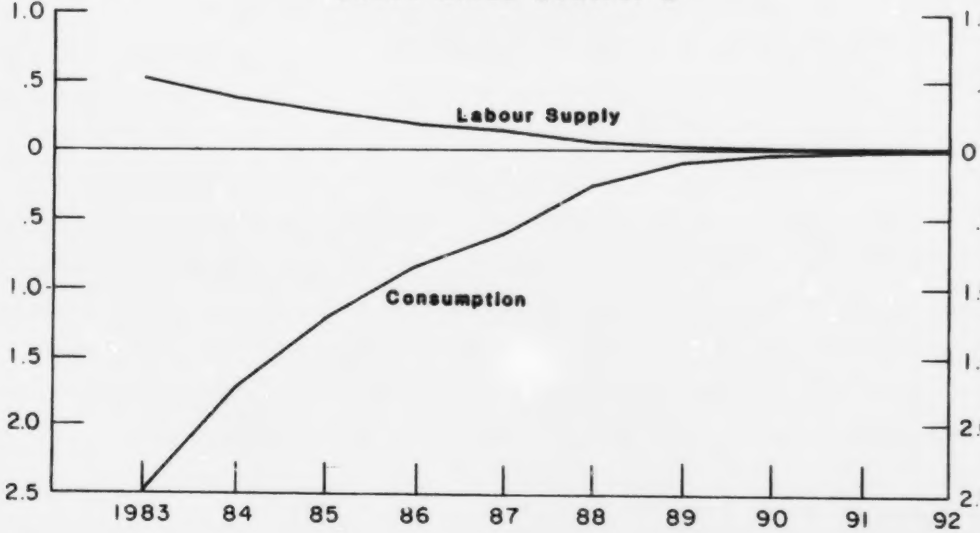


Figure 3-6

CONSUMPTION AND LABOUR SUPPLY
Shock Minus Control %



The decline in real financial wealth is very small because money is a small proportion of total financial wealth. In turn, human wealth is much larger than financial wealth and so the overall impact of the real-wealth change is negligible. For the real variables the responses shown come almost entirely from the money disequilibrium term. For example, of the 2.48% consumption response in the first year, only 0.05 comes from the pure wealth effect and 2.43 from the monetary disequilibrium term. Similar results obtain for the labour supply response. This is important because it illustrates clearly our point in distinguishing disequilibrium from equilibrium wealth effects.

The 10% money-level reduction induces a real contraction of about 2.5% in consumption in the first year, and a labour supply increase of 0.5%. Both these effects will tend to increase unemployment. In this partial setting the real effects of the purely nominal shock largely disappear after six years, but there are some lingering small effects after nine years. To repeat, these are not full-model properties, but they do illustrate the nature of the contribution of the household sector equations to those properties.

Chapter 4

THE SUPPLY SIDE OF SAM: OUTPUT, FACTOR USE, INVENTORIES, AND CAPACITY UTILIZATION

4.1 Introduction

In this chapter we discuss the determinants of the supply of goods other than energy. To do so we must examine the production technology, the factor demands, and the roles of inventories and capacity utilization. We provide both a long-run and a short-run perspective on these variables. The long-run perspective comes from a steady-state model that describes a full-employment, zero-excess-profits nexus of optimal capital and inventory stocks (and investment flows), optimal energy and labour usage, as well as potential output. The short-run perspective comes from a system of equations describing movements in these variables in states of less than full equilibrium. In addition to equations describing actual factor use, this includes the use of inventories as a buffer stock and variation in the degree of capacity utilization as a means of moving off the long-run technology in response to demand cycles. The equations for output, capacity utilization, and the short-run use of the three factors of production are estimated simultaneously, using a full-information, maximum-likelihood technique.

The reader will note a certain imbalance in the nature of the theory we use to explain firm behaviour, when compared with the model of household decisions. We do not specify a formal intertemporal optimization problem for firms.[1] Instead, we opt for the simplicity of a framework of static optimization to describe equilibrium behaviour and rely on ad hoc descriptions of the dynamics. We have nevertheless been careful to incorporate some lessons from formal dynamic models in our specification of the adjustment processes. For example, the literature establishes that only in very special circumstances is it optimal to determine the disequilibrium settings for a variable using only information on the state of that variable itself. This point is especially pertinent for fixed-coefficient models, such as the standard partial-adjustment model. In general, since the disequilibrium states of many variables are related in the economic system, one would expect agents to use a variety of information in determining optimal adjustment of any particular variable. We postulate a system in which adjustment eventually proceeds towards formally derived equilibria. All disequilibrium behaviour is thus forward looking. Moreover, adjustment is specified to depend on various measures of the nature and extent of disequilibrium. Finally, we specify a form (original, as far as we know) for the factor-adjustment processes that permits us to generate different dynamic responses to temporary and permanent shocks. Thus, although our theory does not incorporate restrictions from any formal dynamic analysis, we do present a highly structured and integrated model of output and factor use.

---

[1] To generalize our approach it would not be sufficient to extend the theory of the firm to a dynamic framework. What would be required is an integration of household savings and asset-acquisition decisions (including investment in real capital), building on the model of Abel and Blanchard (1983), for example.

The chapter begins with a description of the production technology and how variation in capacity utilization combines with the technology to give SAM's output equation (section 4.2). This is followed, in section 4.3, with a discussion of our treatment of technical progress. In section 4.4 we describe our model of the real equilibrium path for the economy, and in section 4.5 our model of the factor-adjustment processes. Section 4.6 completes the discussion of disequilibrium dynamics by detailing SAM's model of capacity utilization and inventory fluctuations. A description of our estimation procedure (section 4.7) is followed by the results (section 4.8) and an equation-by-equation analysis of those results, including some of the implied static properties (section 4.9). The chapter concludes with a consideration of sector properties through partial simulations of response to two forms of demand shock (section 4.10).

4.2 SAM's Production Technology and Output Equation

We specify a production technology with a nested constant-elasticity-of-substitution (CES) form. A bundle of capital and energy with CES structure is combined with labour in a second CES structure:

```
EQ52UFS   UGPC = AF10*{ AF05*AF08*[AF04*(CAPU*J2A(KCT))**AF06 +
                 (1-AF04)*ENC**AF06]**(AF07/AF06) + (1-AF05)*
                 (LC*PRODL)**AF07} **(1/AF07).
```

This equation says that output over the period depends on (i) the average capital stock, J2A(KCT), modified by an index of intensity of its use (or capacity utilization), CAPU, (ii) the use of energy, ENC, and (iii) the efficiency-unit input of labour, LC*PRODL. To obtain efficiency units of labour, actual employment, LC, is modified by an index of labour-embodied (Harrod-neutral) technical progress (or productivity), PRODL.

Equation EQ52UFS is a production function in the sense that in steady state, with CAPU at its long-run level, CAPUSS, the function describes the technological constraint faced by firms. This constraint is binding on the long-run optimizing decisions of those firms.

The output equation

Equation EQ52UFS also provides the short-run output equation in SAM. Modellers have provided a variety of answers to the question of how short-run output is influenced by the long-run technology. A useful general statement has the form:

$$u = f(c \cdot k,\ e,\ h \cdot p \cdot l) \cdot r, \tag{4.1}$$

where u is output; c is capacity utilization in the sense described above; h is an index of hours; p is trend productivity; k, e, and l are the three factors; and r is a residual 'total productivity' term. We ignore the possibility that energy can be used with varying technical efficiency. The key distinction here is between r and either c or h, which appear inside the function f. Variation in c and h can represent short-run response to the business cycle, but such variation in factor services is specified to influence output according to the same technology that applies to variation in k and l. The index r represents the extent to which output is not restricted in the short run by the function f.

Many modellers have taken the view that the technology is not binding except in the long run, or that a different technology is operative in the short run than in the long run (systematically different, that is, not simply the result of fixed or quasi-fixed factors).[2] The practical implication is that a model, generally ad hoc, is specified for the residuals, in the sense of r, from (4.1). Firms are not seen as constrained by a technology in the short run, but move off the technology in response to the cycle. For SAM we have not included a structural model for r. Rather, we assume that log(r) is a random disturbance. Hence, output is stochastic. Except for this factor, however, the technology in the sense of f is presumed to be binding in the short run as well as the long run. In this sense firms do not move off the technology, but this is largely a matter of semantics. In SAM firms do choose to vary the level of capacity utilization and change the nature of the short-run relationship between inputs and output. This is moving off the long-run technology according to a particular rule. In effect there is a technology relating inputs and outputs for each level of c. The choice of c determines the operative short-run technology. The same would be true of h, but SAM does not explain variation in hours.[3]

In summary, EQ52UFS embodies both the long-run technology and the structural short-run output equation in SAM. It is important to note, however, that this equation cannot provide a complete model of short-run fluctuations in output. It must be complemented with the behavioural equations determining CAPU and the levels of factor use.

It is important to understand that CAPU is an endogenous choice variable in SAM. A model is specified and fitted to data on capacity utilization in a simultaneous estimation of this CAPU model, the technology/output equation, and equations describing short-run factor use. Our measure of CAPU is the Bank of Canada series B60029 based on the capital/output ratio in goods-producing industries, excluding energy.[4] Our use of these data as a direct measure of capital-stock utilization works better than either treating it as a measure of utilization of the capital/energy bundle or treating it as an overall factor-productivity measure (i.e., a measure of r in equation (4.1)). But there is some residual correlation in our estimated output equation, and possibly some room to improve the historical explanation of very short-run fluctuations in output. Our focus on the medium to long term leaves us less concerned with residual correlation than we would be if the model were to be asked to provide short-run forecasts. It would be easy to add a time-series or a structural model of r.

---

[2] The MACE model (e.g. Helliwell et al., 1982) provides a good example.

[3] A model with an hours dimension would require a more complex model of labour supply than that presented in Chapter 3, which limits discussion to the participation decision. Our focus on the medium to long term led us to opt for simplicity in the labour market and to concentrate our modelling of short-run output fluctuations on capacity utilization.

The nested CES technology

Almost all econometric models built before the 1980s contain Cobb-Douglas (CD) technology, which is analytically simple and at the time seemed roughly consistent with the data. In particular, the CD prediction that the (trend) income shares of capital and labour are constant did not seem grossly at odds with the facts. CD technology imposes unit elasticities of substitution on factors. When attention was limited to capital and labour as inputs, this seemed an acceptable property; but as interest in energy and other resources as factors of production was heightened by the OPEC price increases, it became clear that CD would have to be abandoned. In multi-factor worlds, factors can be complements. Indeed, some micro evidence suggests that this might be the case for certain factors, notably energy and capital. Clearly, the unit elasticity of substitution restriction of CD was unacceptable. Many researchers moved to flexible functional forms (e.g., the translog function), especially for micro work. We prefer to use a function that is well behaved for all possible values of the inputs, and not just in a particular region of approximation. The extra flexibility of the CES form removes the most objectionable properties of CD, but does not make analysis intractable. We view the CES form as the natural extension in the multi-factor world of the traditional CD form of macro models. It is the simplest form that allows us to deal with the data of the multi-factor world and, under particular testable restrictions, degenerates to CD.

---

[4] See Table H6 (formerly Table 56), Bank of Canada Review (monthly) and the May 1980 issue for a discussion of the methodology used in constructing the data as well as comments on appropriate interpretation.

The particular form we have chosen, with nested structure, was first suggested by Sato (1967). AF08 and AF10 are scale parameters. They depend only on the units chosen for measurement and are of little economic interest. AF04 and AF05 are 'share' parameters. Inside the capital-energy bundle AF04 describes the weight applied to capital and (1-AF04) the weight on energy. Similarly AF05 and (1-AF05) are weights applied to the 'bundle' and to labour in efficiency units, respectively. AF07 and AF06 determine factor substitutability.

The Hicks-Allen partial elasticity of substitution (output and other factor prices fixed) between labour and either capital (SLK) or energy (SLE) is given by

$$\mathrm{SLK} = \mathrm{SLE} = 1/(1-\mathrm{AF07}). \tag{4.2}$$

The similar elasticity of substitution of capital for energy can be written as

$$\mathrm{SKE} = 1/(1-\mathrm{AF07}) + [1/(1-\mathrm{AF06}) - 1/(1-\mathrm{AF07})]/\mathrm{ShKE}, \tag{4.3}$$

where ShKE is the expenditure share of the capital/energy, KE, bundle. Note that the Hicks-Allen partial elasticity of substitution of capital for energy is constant only so long as the expenditure share stays constant, despite the use of the CES form for the KE bundle.

Sato (1967) suggests that more highly substitutable factors (in the direct partial elasticity of substitution sense) be grouped in a bundle. He offers no reason and admits that the function is mathematically well behaved the other way around. It is clear, however, that his argument comes from an 'aggregation' perspective. He feels that since in the limit very close substitutes are indistinguishable and easily aggregated, close substitutes belong together. But, since the partial elasticity of substitution between bundles is necessarily positive with this form (i.e., bundles are substitutes), only within bundles can complementarity emerge. Thus, contrary to Sato, we find it intuitively appealing to define bundles by associating factors that are most likely to be complements or weak substitutes. It has been argued that, capital and energy are not very substitutable at least in the short run, given that energy requirements for a given vintage of capital are relatively fixed. Moreover, some researchers have found empirical support for the proposition that energy and capital are complements. Thus, we feel it appropriate to bundle energy and capital.[5] Our estimates suggest that although energy and capital are less substitutable than is labour for the other factors, they are nevertheless more substitutable than many have argued. We have not, however, investigated alternative bundling formulations empirically.

EQ52UFS contains two restrictions that give the technology the property of constant returns to scale (CRTS). The first is that the capital/energy bundle has a CRTS structure. This requires the common exponent on capital and energy, AF06, as well as the inverse exponent 1/AF06 outside the bundle. The second restriction is that the capital/energy bundle combine with labour in a CRTS structure. This requires the common exponent on the bundle and labour, AF07, and the inverse exponent 1/AF07 applied to the entire expression in { }. Note that CRTS applies to efficiency units for labour and capital, that is, labour adjusted for productivity and capital for the utilization rate.

---

[5] Our energy/capital bundling is identical to that chosen by Helliwell and his colleagues for the MACE model. They, however, originally chose the CD form for labour and the bundle (e.g., Helliwell et al., 1982). The most recent version of MACE (Helliwell et al., 1984) has adopted an external CES form similar to that chosen for SAM. Our test (section 4.9) clearly rejects the restriction to CD form.

behaved for all possible values of the inputs, and not just in a particular region of approximation. The extra flexibility of the CES form removes the most objectionable properties of CD, but does not make analysis intractable. We view the CES form as the natural extension in the multi-factor world of the traditional CD form of macro models. It is the simplest form that allows us to deal with the data of the multi-factor world and, under particular testable restrictions, degenerates to CD.

The particular form we have chosen, with nested structure, was first suggested by Sato (1967). AF08 and AF10 are scale parameters. They depend only on the units chosen for measurement and are of little economic interest. AF04 and AF05 are 'share' parameters. Inside the capital-energy bundle AF04 describes the weight applied to capital and (1-AF04) the weight on energy. Similarly AF05 and (1-AF05) are weights applied to the 'bundle' and to labour in efficiency units, respectively. AF07 and AF06 determine factor substitutability.

The Hicks-Allen partial elasticity of substitution (output and other factor prices fixed) between labour and either capital (SLK) or energy (SLE) is given by

$$\mathrm{SLK} = \mathrm{SLE} = 1/(1-\mathrm{AF07}). \qquad (4.2)$$

The similar elasticity of substitution of capital for energy can be written as

$$\mathrm{SKE} = 1/(1-\mathrm{AF07}) + [1/(1-\mathrm{AF06}) - 1/(1-\mathrm{AF07})]/\mathrm{ShKE}, \qquad (4.3)$$

where ShKE is the expenditure share of the capital/energy, KE, bundle. Note that the Hicks-Allen partial elasticity of substitution of capital for energy is constant only so long as the expenditure share stays constant, despite the use of the CES form for the KE bundle.

Sato (1967) suggests that more highly substitutable factors (in the direct partial elasticity of substitution sense) be grouped in a bundle. He offers no reason and admits that the function is mathematically well behaved the other way around. It is clear, however, that his argument comes from an 'aggregation' perspective. He feels that since in the limit very close substitutes are indistinguishable and easily aggregated, close substitutes belong together. But, since the partial elasticity of substitution between bundles is necessarily positive with this form (i.e., bundles are substitutes), only within bundles can complementarity emerge. Thus, contrary to Sato, we find it intuitively appealing to define bundles by associating factors that are most likely to be complements or weak substitutes. It has been argued that, capital and energy are not very substitutable at least in the short run, given that energy requirements for a given vintage of capital are relatively fixed. Moreover, some researchers have found empirical support for the proposition that energy and capital are complements. Thus, we feel it appropriate to bundle energy and capital.[5] Our estimates suggest that although energy and capital are less substitutable than is labour for the other factors, they are nevertheless more substitutable than many have argued. We have not, however, investigated alternative bundling formulations empirically.

EQ52UFS contains two restrictions that give the technology the property of constant returns to scale (CRTS). The first is that the capital/energy bundle has a CRTS structure. This requires the common exponent on capital and energy, AF06, as well as the inverse exponent 1/AF06 outside the bundle. The second restriction is that the capital/energy bundle combine with labour in a CRTS structure. This requires the common exponent on the bundle and labour, AF07, and the inverse exponent 1/AF07 applied to the entire expression in { }. Note that CRTS applies to *efficiency* units for labour and capital, that is, labour adjusted for productivity and capital for the utilization rate.

---

[5] Our energy/capital bundling is identical to that chosen by Helliwell and his colleagues for the MACE model. They, however, originally chose the CD form for labour and the bundle (e.g., Helliwell et al., 1982). The most recent version of MACE (Helliwell et al., 1984) has adopted an external CES form similar to that chosen for SAM. Our test (section 4.9) clearly rejects the restriction to CD form.

4.3 Technical Progress: DHPRL, PRODL

One fundamental determinant of the growth of potential output in an economy is the rate of technical progress. Economists have put considerable effort into understanding the process of technical change, especially as an issue in economic development. But in models of developed economies the trend rate of technical progress is typically an exogenous variable and cannot be altered by standard macro policy levers. Some work has been done that considers the possibility that government spending, especially spending on 'infrastructure', changes the productivity of private sector factors. Economists have also considered the possibility that governments can influence the rate of technical progress by subsidizing research or creating private rents by protecting innovations (e.g., patents).[6] But none of these discussions have led to systematic modelling of technical progress at the macro level.

In SAM we follow standard practice. We add an exogenous function of time to represent technical progress. We estimate the parameters of this function simultaneously with the other technological and behavioural parameters that affect output and factor use, but we offer no economic explanation of technical change. Note that the simultaneous estimation involves the imposition of cross-equation restrictions -- both factor demands and output are influenced by technical progress.

We specify that technical progress is Harrod neutral or embodied in labour. Except for the special case of the CD function, the nature of technical progress matters. In general, a steady-state growth path does not exist unless technical progress is labour embodied. There is no logic that can show that the equilibrium growth path must have steady-state properties -- e.g., a fixed capital/output ratio and fixed factor efficiency price ratios -- but those properties are certainly convenient. With Harrod-neutral technical progress, labour and wages can be measured in efficiency units (the efficiency wage is simply the measured wage deflated by the index of productivity, PRODL) and a clear interpretation can be given to steady state. Output, capital, and energy use all grow at the rate of labour force growth plus the rate of technical progress. All the proceeds of technical progress accrue to labour in the sense that the real wage grows at the rate of technical progress. These propositions do not hold if technical progress is other than Harrod neutral. Moreover, the intertemporal household-optimization problem (Chapter 3) relies on Harrod-neutral progress, and becomes much more complex (possibly intractable) under alternative assumptions.

---

[6] See Stuber (1985) for a survey of recent work on technical progress.

An exogenous trend need not be constant. Indeed, one of the most prominent puzzles in applied macroeconomics in the past decade has been the apparent decline in the rate of technical progress since the first OPEC price increase. Some (e.g., Helliwell et al, 1984) have argued that there has been no shift in the underlying trend. Others disagree. We have allowed for a shift of the trend rate of technical progress, DNPRL, in 1973 and for the corresponding 'kink' in the evolution of PRODL.

$$\mathrm{DNPRL} = \mathrm{AF09} - \mathrm{AF11}*(\mathrm{TIME}>1972) \qquad (4.4)$$

$$\mathrm{PRODL} = \mathrm{J1L}(\mathrm{PRODL})*\exp(\mathrm{DNPRL}). \qquad (4.5)$$

Our estimates show a reduction in the trend rate of technical progress beginning in 1973. Although the reduction is not significant at the usual confidence levels, it is large enough and significant enough (80% confidence level) that we retain it in the model. For simulation over future periods the value of DNPRL is adjusted to an assumed steady-state value of 1.3% per annum. This value is a model parameter and can easily be changed.

4.4 The Steady-State Model of Aggregate Supply

4.4.1 Introduction

At SAM's heart is a description of an equilibrium growth path for the economy. The properties of this path depend on many things. Some of these are exogenous variables, such as the world energy price. Some are based on optimal behaviour that is not explicitly derived. For example, firms are presumed to choose desired levels of inventory stocks. We specify an equation, but do not provide a formal model of this choice. The essence of the model, however, comes from a formal static optimization by firms (profit maximization) with two market restrictions imposed on the solution: that the perfectly competitive system will generate equilibrium with zero excess profits and that there will be full employment of labour.

We feel that for a model designed for medium- to long-term policy analysis it makes sense to suppose that markets clear, at least eventually. In our view, there has been no convincing model developed that displays rational behaviour while permiting permanent disequilibria. Without market clearing one must ask every time the model is used how much the results depend on arbitrary rigidities and irrational behaviour. For SAM we permit some unexplained rigidities and irrational behaviour in the short run, but we do not permit these to prevail and block a transition to a full-equilibrium steady state. The world may not be best characterized by market-clearing processes, but experiments done under this assumption have clear interpretations. Policy conclusions based on long-term arbitrary rigidities or unexplained irrational behaviour are unclear, at best. So market clearing is imposed as a fundamental property of our steady-state growth path.

Decisions of firms and other economic agents in SAM are forward looking. Current decisions are influenced by expected future values and in most cases the expectations are influenced by the steady-state properties of the model. As the model's solution converges on steady state, expectations become fully Muth-rational. We do not, however, impose consistency of expectations in the sense of letting agents know and use as information the dynamic paths generated by the model from states of disequilibrium.

SAM can be characterized as having supply determination of output in the long run plus mechanisms guaranteeing that demand will become consistent with supply. This characterization is an over-simplification. The model is fully simultaneous and there is no sense in which output is uniquely determined by supply considerations. In particular, in response to permanent real shocks to aggregate demand, there are real interest rate, real exchange rate, and real wage responses that change the participation rate, the labour supply, and the optimal output/labour ratio. These responses change potential output. But quantitatively these effects are small. Essentially, the long-term level of output is determined by the available supply of the scarce resource, labour, and by the existing technology. This level is determined in the model. The long-run rate of growth of output is determined by the rate of population (and hence labour force) growth and the rate of technical progress, both exogenous to the model.

Note that the technology itself cannot determine the long-run level of output. We have a constant-returns-to-scale production function. In the absence of resource restrictions, goods could be produced at the same long-run average cost regardless of the level of output. The supply restriction comes from available resources. Energy is assumed available under conditions of perfectly elastic supply in the world market at the going price. SAM is not designed to deal with the interesting questions that arise in economies with scarce natural resources. Thus, although energy has to be paid for, there is no sense in which its supply limits output in SAM. Capital is subject to restrictions of a sort. To put capital in place, consumption must be foregone or debts to foreigners incurred to finance imports. But, fundamentally, the level of the capital stock can be set without restriction in the long run. It does not constrain output -- it is chosen along with other things according to certain rules of optimal behaviour and the market restrictions of full employment and zero excess profits. It is labour that is the fundamentally scarce resource in the long run in SAM.

Most models that consider the long run have elements of supply determination of output. Yet these models generally specify some variant of expected sales to generate the scale for 'long-term' firm behaviour. SAM gives aggregate demand an important role in the short run, but breaks from this approach in the specification of both long-term decisions and adjustment processes. The supply dimension is given greater emphasis than in other models in that the market restrictions of full employment and zero excess profits are imposed on the supply system and hence the long-term path to which the system adjusts. Hence, the fundamental assumption that markets clear is given practical importance — we take the view that this process occurs over a time period short enough to be useful in the modelling of adjustment processes.

4.4.2 Steady-state energy prices and related measures: PENRSS, TTEN, PRELN, PRELEN, REN, PENR, PEN, PENW

In this section we describe the data and equations in SAM that relate to energy prices. The long-term plans of firms in SAM are based on expected future factor prices, and so we need to describe our concept of a steady-state or 'normal' real energy price, PENRSS. In a sample that contains the influence of OPEC and the early phases of the National Energy Program, there is no obvious definition of normality. Nevertheless, for empirical work we need a measure of PENRSS.

We assume that Canada's production of energy, although of great importance domestically, is too small to influence the world price. Because energy has many distinct forms and because its composition in domestic consumption differs from its composition in external trade, it is not possible to identify a single price that appropriately values all aspects of energy in the model. Moreover, the composition of Canada's energy trade varies over time and its value is not easily linked to specific world prices. These difficulties are avoided by defining a world energy price for the model, PENW, using domestic trade-price indexes, PXEN (exports) and PMEN (imports):

$$\mathrm{PENW} = [(\mathrm{XEN}*\mathrm{PXEN}+\mathrm{MEN}*\mathrm{PMEN})/(\mathrm{XEN}+\mathrm{MEN})]/\mathrm{PFX}, \tag{4.6}$$

where XEN and MEN are the constant-dollar energy export and import series, respectively, and the division by PFX (price of foreign exchange) converts the price to U.S. dollars. Equation 4.6 is not a model equation, just a data definition. PENW is exogenous to the model. In model equations, PENW*PFX is used for domestic valuation of energy trade.

The problem of the systematic difference between the domestic energy price, PEN, and the average trade price, PENW*PFX, is addressed through the introduction of an exogenous relative price, PRELN. For the period before 1972, this relative price rises roughly linearly:

$$\underset{}{PEN/(PFX*PENW)} = \underset{(98.6)}{1.011} + \underset{(12.3)}{.0135*QTIME}. \qquad (4.7)$$

Sample 1955-71 RSQ = 0.91 DW = 1.3

Although there was some price support for domestic oil producers during this period, the relative price and its trend reflect primarily changes in composition of production and trade and not policy. After 1972 things are more complicated. There are both substantial changes in the composition of energy production and trade and a major policy initiative by the federal government to limit the domestic consequences of the OPEC price increases. For the discussion here, however, we can ignore the source of the relative price change.[7] PRELN is defined as the fitted values from (4.7) up to 1972 and is set to values close to the actual ratio for the rest of the sample. We allow some foresight by firms in that from 1976 we specify that they anticipate a gradual rise in domestic prices and use as a 'normal' price a value slightly higher than the current value. As Figure 4-1 makes clear, however, we tie the perception of normal closely to the actual domestic price. By 1983, the larger part of the remaining gap between domestic and world prices is a composition effect. The decline in world prices in 1982-83 removes most of the policy-induced gap.

---

[7] In the data base and in the model we identify a split of PRELN into pure relative price effects, PRELEN, and policy effects, REN, where PRELN=PRELEN*REN. The split is not based on precise study of the underlying relative prices and policies. It is a rough approximation based partly on data and partly on judgement. The split is introduced so that we have a point of departure for experiments in which domestic-price policies are altered.

To define the normal or steady-state real energy price we introduce the variable TTEN, the terms of trade for energy. In equilibrium, TTEN is given by PRELN*(PFX*PW/P)*(1+RINDT). The relative price term, PRELN, is multiplied by the real exchange rate (with respect to the world price level, PW). The (1+RINDT) adjustment reflects the fact that we measure real energy prices at factor cost -- the adjustment removes indirect taxes from the market price deflator, P. We do not, however, adjust the energy terms of trade one-for-one with every short-term fluctuation in the real exchange rate. Instead we specify a partial adjustment process:

```
EQ09EGR   TTEN/PRELN = J1L(TTEN/PRELN) + AA04*
                        J1L(PFX*PW*(1+RINDT)/P-TTEN/PRELN).
```

The adjustment equation is written so that permanent policy changes and relative price shifts pass immediately into TTEN (via PRELN). Changes to the real exchange rate or the indirect tax rate pass through gradually. The adjustment speed, AA04, is imposed at 0.5. This value appears to provide reasonable smoothing of the historical movements in the real exchange rate. Note that whether any effect passes through to TTEN when RINDT changes depends on what happens to P. If indirect tax changes pass fully into P, then there is no effect on TTEN.

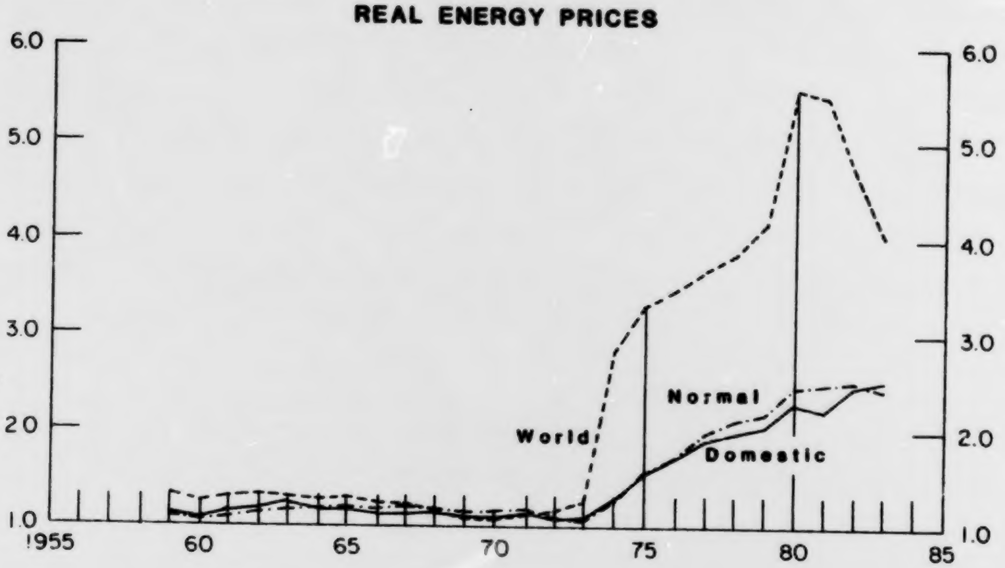
The normal or steady-state real energy price is given by the product of the world real energy price and TTEN:

```
EQ10EMP   PENRSS = TTEN*PENW/PW.
```

This steady-state price, the actual real domestic price, and the domestic equivalent of the world (trade) price are shown in Figure 4-1. All measures are at factor cost. The difference between the 'world' and 'normal' values reflects PRELN.

So far we have discussed only the target price. We also specify a short-run reaction function whereby the actual real energy price (at

Figure 4-1



factor cost), PENR, adjusts towards PENRSS, the steady-state price, according to a first-order process. The estimated model is:

```
EQ81EGR    PENR = (1-AG81)*J1L(PENR) + AG81*J2A(PENRSS).
                   .1731               .8269
                                       (6.3)

           Sample 1961-81         RSQ = 0.980        DW = 1.03
```

Note that the short-run reaction function indicates quite rapid adjustment towards the target value.

Finally, the energy-price system is closed with the identity that describes the nominal market-price rule consistent with all of the above:

```
EQ88EMI   PEN = PENR*P/(1+RINDT).
```

In summary, three points are noteworthy. First, energy prices are set by policy choice, influenced by the world market (before OPEC the policy is to adjust rapidly to the world price). Second, the policy target is specified in real terms. Inflation is not permitted to erode the real cost of energy. Third, changes in the real exchange rate will change the real cost of energy; we do not specify endogenous policy response to offset such effects.

4.4.3 Steady-state capital costs: CCRESS, CCRISS, RRKSS

User cost of capital: CCRESS

Some modellers try to measure what they consider to be the actual cost of capital, combining interest rates and measures of the cost of equity finance. Influences such as cost advantages to financing through retained earnings and the complications of depreciation rules and other fiscal incentives are often introduced. It is sometimes claimed that this type of measure replicates real-world institutions and the actual constraints faced by firms and hence has the best chance of explaining fluctuations in investment demand. Its empirical success has, however, been limited.

Although we are interested in explaining investment demand, our approach begins with specification of a desired capital stock. Our capital-cost signals must be those appropriate for longer-term decisions. We explain adjustment relative to the steady-state path, not relative to targets based on short-run rigidities. As described in section 4.5, the adjustment process gives plenty of scope for cycle influences, but the focus for adjustment is always the long-term, full equilibrium position. Thus, our long-term, factor-price signals do not give much weight to current costs.

We begin with a Jorgensonian user cost of capital. Essentially, the user cost per unit of expenditure on real capital is the opportunity cost of the resources committed (a real interest rate) plus the resources consumed (a depreciation rate) modified according to the tax rules and fiscal incentives extant.[8] In practice, many researchers have used a long-term interest rate for the opportunity cost. But current measures of such rates are not necessarily appropriate long-term signals. Clearly, given the way these costs are used in SAM we must try to specify a measure that eliminates short-term fluctuations in interest rates. Any effects from short-term fluctuations must come from the disequilibrium part of the model.[9] Of course, perceived changes in equilibrium real interest rates must influence the equilibrium cost of capital (via the opportunity-cost term).

In an open economy, world capital markets are important in determining the long-term user cost of capital for a small country. There is dispute over the extent of this role, but there is some evidence (e.g., Murray, 1982) that after-tax returns to equity are equated in world markets. This does not mean that user costs are equated. Countries can alter user costs through fiscal incentives and hence influence the level of the capital stock, even when market valuation of capital moves to equate equity returns. We allow for such a wedge.

Our equation for the steady-state, efficiency cost of producing capital is:

EQ14KML CCRESS = (CCRW+CCRDIF)*CCTAX1*PRELIP/CAPUSS.

We divide by CAPUSS because we are measuring the price of an efficiency unit of capital. Each unit of capital is equivalent to CAPUSS units of efficiency capital. We multiply by PRELIP because our data values capital goods using an index, PI, that is distinct from the output price, P. The relative price plays no role in simulation; it is held fixed (see Chapter 7). Note that no indirect tax rate appears in EQ14KML. In Canada, all registered companies are exempted from indirect taxes on machinery and equipment. Not all construction goods are exempt, but we ignore this and assume that all capital formation is exempt. Thus, we specify that firms purchase capital goods at factor cost so that deflation by a factor-cost price index leaves only a pure relative price, PRELIP, that is independent of the rate of indirect tax. The term CCTAX1 reflects fiscal influences on user cost. It includes consideration of investment tax credits, interest deductibility, depreciation rules, and profits taxation. Because of the erratic movements in these values, CCTAX1 is specified to be a substantially smoothed version of the actual historical values. This smoothing removes particularly large fluctuations caused by temporary changes in the rules for depreciation. All components of CCTAX1 are exogenous to the model. In simulation, the structure of the equation that defines the CCTAX1 data can be used to design appropriate changes for given fiscal-policy initiatives.

---

[8] See Boadway (1980) for a detailed derivation of a user-cost formula that respects Canadian fiscal institutions.

[9] We tried various methods of introducing such short-run fluctuations into our model of investment, but had no empirical success. See section 4.5.

The main term in EQ14KML, CCRW+CCRDIF, represents the Jorgensonian opportunity cost plus depreciation. No relative capital-price appreciation is possible in SAM because there are no distinct capital goods and PI is tied to P. The first term, CCRW, represents the world before-tax-and-fiscal-incentives user cost. The second term, CCRDIF, represents the differential between the equivalent Canadian value and CCRW. It represents the real premium Canadian firms must pay to borrow in world capital markets.[10] This premium is endogenous in SAM. It is part of the overall system of adjustment to full equilibrium. See Chapter 1 for a general overview of the role of real interest rates in long-term adjustment. See also section 4.4.10.

User cost of inventories: CCRISS

In addition to producing capital, firms hold stocks of inventories that have associated costs that are different from those of producing capital. Our equation for the user cost of inventories is:

---

[10] For estimation we did not use direct measures of world capital cost. Instead, CCRW+CCRDIF was replaced by a domestic measure. For simulation of this version of SAM we link CCRW to world values by passing through any hypothetical changes in the world capital cost. We also use an alternative version in which domestic capital costs are linked to domestic equity returns, with any links to world real interest rates entering through the financial system. How domestic user costs of capital are determined in open economies is a subject requiring further research.

```
EQ11KML    CCRISS = (CCRW+CCRDIF-DELK)*CCTAX2.
```

There are several differences between EQ11KML and EQ14KML. First, since inventories are valued in units of the output good, there is no relative price term. Second, no efficiency-units transform is required. Third, inventories do not depreciate as does producing capital. We assume that inventories do not depreciate at all and deduct the depreciation rate, DELK, from the basic user cost. Finally, the tax treatment of inventories is different, and we compute a separate adjustment factor, CCTAX2.

Rate of return to firm capital: RRKSS

The notional firm in SAM has physical assets in the form of producing capital and inventories. Equity claims on firms are claims to the bundle of producing capital and inventories and to the profit stream they generate. There are no claims to individual types of capital. In steady state, the real capital stock of the firm is KCD + INVCD. Although each type of capital has its notionally distinct return, CCRESS*CAPUSS and CCRISS respectively, from the perspective of owners there is just one average return, RRKSS:

```
EQ98KML   RRKSS = (KCD/(KCD+INVCD))*CCRESS*CAPUSS
                + (INVCD/(KCD+INVCD))*CCRISS.
```

To measure the current excess-profits rate, this steady-state average return to physical capital can be deducted from RRK, the actual gross profit rate, calculated as the residual factor return from the income distribution (equation EQ97KMI, Appendix B). We use such an excess-profits measure in several model equations describing disequilibrium adjustment.

4.4.4 Firm behaviour, market restrictions, and the equilibrium wage

Overview

The static-optimization problem solved by firms involves maximization of real profits subject to the production-technology constraint. Firms are viewed as atomistic competitors. They take all prices as given, and do not consider the form of the aggregate labour supply function or the aggregate demand function when making their plans. Markets force firms to act as if they are forward looking and base their effective long-term plans on steady-state factor prices, not current prices. It is here that the first hint of a market restriction enters the problem. SAM is a model of a world in which all markets clear over a horizon short enough that dynamics are sensibly specified relative to desired positions as defined by the equilibrium path. There is no short-run optimization with fixed factors and no consideration of the possibility of permanent disequilibrium. A long-term, full-employment, zero-excess-profits path is computed and adjustment dynamics specified relative to that path and not to constrained short-term targets.

Notionally, firms choose the rates of use of labour and energy, the stocks of producing capital and inventories, and the long-term rate of capacity utilization. We do not, however, explain the long-term rate of capacity utilization. It is specified as an exogenous variable (see section 4.4.9). We do specify a model for desired inventory stocks. Inventories are treated as facilitating trade but not contributing directly to the production process. The decision on the desired level of inventories is assumed separable from the decision on the three factors of production. The solution has the form:

$$ID = h \cdot U, \tag{4.8}$$

where ID is the desired stock and U the rate of output. The proportionality constant, h, depends on the user cost of inventories, but given a constant value for this cost the output required to sustain the stock of inventories (i.e., user cost times stock) can be written as $k \cdot U$, where k is the user cost of inventories, multiplied by h (see section 4.4.8 for details).

Given the solution for inventories, the problem can be concentrated such that firms choose[11] K, E, and L so that they

$$\begin{aligned} &\text{maximize} \quad (1-k)U - w \cdot L - r \cdot K - q \cdot E \\ &\text{subject to} \quad U = f(K,E,L), \end{aligned} \tag{4.9}$$

where w, r, and q are the real costs of labour, capital, and energy, respectively. Problem (4.9) is equivalent to

$$\begin{aligned} &\text{maximize} \quad U = w^* \cdot L - r^* \cdot K - q^* \cdot E \\ &\text{subject to} \quad U = f(K,E,L), \end{aligned} \tag{4.9a}$$

where each * price is the original price divided by (1-k).

The technology in SAM exhibits constant returns to scale (homogeneity of degree 1). As a result, the partial derivatives of the technology with respect to the factors are homogeneous of degree zero. The conditions for profit maximization include the first-order conditions -- that each real factor cost is equated to the relevant marginal product. Since the marginal-product functions are homogeneous of degree zero, the three conditions are not independent. Rather, each condition can be written as a function of two factor ratios -- say the capital/labour and energy/labour ratios:

$$w^* = f_3 \ (K/L, E/L, 1) \tag{4.10}$$

$$r^* = f_1 \ (K/L, E/L, 1) \tag{4.11}$$

$$q^* = f_2 \ (K/L, E/L, 1). \tag{4.12}$$

---

[11] For simplicity we temporarily ignore technical progress and capacity utilization. The reader should think of all units as efficiency units.

Given any two conditions, say (4.11) and (4.12), unique solutions for K/L and E/L can be obtained in terms of r* and q*. If the w* condition (4.10) is to hold, then there follows a restriction that w* is equal to f_3, evaluated at the K/L and E/L solutions. Since these ratios are functions of r* and q*, there is a restriction on the equilibrium factor prices. If profit maximization is to hold then one factor price must be written in terms of the other two (and the parameters of the technology). We choose to embody this restriction in an equation for w* as a function of r* and q*, but since all real factor prices are endogenous in the model, this normalization is arbitrary. Note that CRTS plays a crucial role here. Without it, (4.10), (4.11) and (4.12) would be independent equations and no restriction on equilibrium factor prices would arise from profit maximization, per se. Because of CRTS, however, firms cannot satisfy the conditions of profit maximization with arbitrary values for w*, r* and q*.

One might ask how individual firms could act to satisfy a restriction on factor prices. The issue here, however, is not how factor prices are set, but rather how firms' plans are formulated. Suppose that the real wage computed from the first-order conditions above, conditional on current best guesses as to normal values for the real capital cost and real energy price, is below the current market real wage. If the firms is correct about capital and energy costs, and if the current real wage is sustained, the firm will fail. A rational firm will not make long-run plans based on such assumptions. We presume that firms make long-run plans based on factor prices that satisfy conditions for their continued existence (i.e., the computed wage above). One can think of this as a micro-consistency restriction on the planning of firms.

Satisfaction of all three marginal-product conditions (4.10-4.12) is necessary for profit maximization and to ensure that there are zero excess profits in the sense of a gross profit rate equal to the gross marginal product of capital. It is in this sense that the equation described above, which embodies the restriction on factor prices, represents the zero-excess-profits restriction. Without it the three marginal product conditions are not simultaneously satisfied and therefore the conditions of Euler's theorem for exhaustion of output by factor payments do not hold. Owners of capital have the residual claim on firms' income; for this residual income to be equal to the gross marginal product of capital (zero excess profits) the restriction on factor prices must hold.

Satisfaction of the zero-excess-profits condition in the sense of Euler's theorem is not sufficient, however, to guarantee full equilibrium in the system and a profit rate at its long-run sustainable level. In particular, it does not ensure that desired net savings by households are just sufficient to provide for equilibrium capital growth. In SAM, there is one rate of return at which households will forego just enough current consumption to provide for equilibrium capital formation. This, in turn, will dictate the equilibrium cost of capital and the steady-state 'normal' profit rate. To achieve this normal profit rate, we need a market process that provides the right level for all factor prices. Given the restriction on factor prices for zero excess profits, and given a real energy price, we need one further equation to determine the levels of the real wage and real cost of capital.

Since the technology is CRTS, we cannot determine a unique level of output that will satisfy the conditions of profit maximization from the firm problem alone. The scale of activity must come from elsewhere. We impose the market restriction that the equilibrium solution must yield full employment of labour. Given the wage, we can compute from the labour supply function the quantity of labour that must be employed. The optimal K/L and E/L ratios from profit maximization then give us the optimal levels for K and E at the full-employment L. Output then follows from the technology. We use an alternative coding of the same results. We derive output from the labour quantity using the profit-maximizing output/labour ratio and the optimal levels of K and E from profit-maximizing factor/output ratios. See Armstrong (1984) for derivations of the factor-price restriction and the equations for steady-state factor use and output.

The rest of this section describes the restriction on the equilibrium wage in detail. Sections 4.4.5 and 4.4.6 provide the equations that determine steady-state output and factor use.

All these results are conditional calculations in the sense described above -- they are correct in the macro sense only when real factor prices are at appropriate levels. In the short run, firms can make long-run planning errors. The process described in section 4.4.10 ensures that full equilibrium will eventually hold and that firms' long-run plans are realized.

Real efficiency wage: WRESS

Condition (4.10), evaluated at the solution of (4.11) and (4.12) for K/L and E/L, yields an equation expressing the wage in terms of the steady-state costs of energy, PENRSS; producing capital, CCRESS; inventories, CCRISS; the parameters of the technology, AF04-AF10; and the parameters of the desired inventory equation, AF01-AF03 (see section 4.4.8):

```
EQ15LML  WRESS = (1-CCRISS*(AF01-AF02*CCRISS+AF03*D6670))*
                 ((AF10*((1-AF05)**(1/AF07)))/((1-(((AF04*
                 (1+((1-AF04)/AF04)*(((PENRSS*AF04)/(CCRESS
                 *(1-AF04)))**(AF06/(AF06-1)))))**((AF07*
                 (1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07/
                 (1-AF07)))*(((AF08*AF05)/(1-AF05))**(1/
                 (1-AF07)))*((CCRESS/(1-CCRISS*(AF01-AF02*
                 CCRISS+AF03*D6670)))**(-(AF07/(1-AF07))))*
                 (1-AF05)*((AF04/(AF04*(1+((1-AF04)/AF04)*
                 (((PENRSS*AF04)/(CCRESS*(1-AF04)))**(AF06/
                 (AF06-1)))))+(1-AF04)/((1-AF04)*(1+(AF04/
                 (1-AF04))*(((CCRESS*(1-AF04))/(PENRSS*AF04))**
                 (AF06/(AF06-1))))))**(AF07/AF06))*((AF10*
                 (1-AF05))**(AF07/(1-AF07)))))**((1-AF07)/AF07))
                 -WREADJ.
```

Note that the value of WRESS determined by EQ15LML does not depend on any of the parameters of the labour supply function. Although this is an interesting structural point, it must be carefully interpreted. The equation is not a reduced form; it contains the endogenous capital-cost variables. The solutions for these variables depend on household preferences generally and the labour supply function in particular.

The equilibrium wage is essentially what is left of the full-employment, zero-excess-profit output per worker after all other factors, including inventories, have received their shares.[12] The equation is complex only because of the inherent complexity of the nested CES technology. The inventory parameters and inventory cost, CCRISS, enter EQ15LML because of the real costs of maintaining the desired stock of inventories (the term (1-k) in the simplified discussion above).[13]

The diligent reader will discover in EQ15LML an extra term, WREADJ. In the estimation of the complete supply system we found that we were unable to impose all the theoretical restrictions and still explain all variables with acceptable average errors. In particular, our model measure of the steady-state real efficiency wage (conditional on the parameter estimates) was consistently higher than the measured real efficiency wage. To insulate the rest of the system from this obvious bias we allow one free constant to enter the system in estimation. The free constant, AF57, can be interpreted as an adjustment to the dollar value of the equilibrium wage. WREADJ is the constant converted to the WRESS dimension -- real efficiency units at factor costs (i.e., WREADJ=AF57*(1+RINDT)/(P*PRODL)). Thus, over time the adjustment declines in WRESS units, because of the deflation by P*PRODL. By the end of the sample the adjustment is small, and WREADJ is reduced to zero for simulations over future periods. For the historical period WREADJ is treated as an exogenous variable and is not permitted to influence the model's response to shocks.

A graphical illustration of the equilibrium wage, computed using the estimates described in section 4.8, appears in Chapter 3, Figure 3-1.

Nominal market wage: WS

The transformation from WRESS to WS involves four terms:

EQ18LML   WS = WRESS*PS*PRELC*PRODL/(1+RINDT).

---

[12] Note that energy must be paid for in this accounting because the output measure is not a value-added measure, but is gross of the value of energy inputs.

[13] At the time of estimation our derivation of WRESS did not correctly account for the influence of inventories. The effect of the error is very small and should not influence the results significantly.

First, we multiply by PS to convert the real wage to a nominal wage. Second, we multiply by PRELC, the relative price measure that represents the ratio of the product price to the consumption price. This ratio is endogenous in SAM (see Chapter 7). The conversion is necessary because WRESS is defined as a real wage to the producer, and PS applies to the consumption price. Hence PS*PRELC is the equilibrium producer price appropriate for the conversion of WRESS. Third, we multiply by PRODL, because WRESS is the efficiency wage; workers receive PRODL*WRESS in steady state. Finally, we divide by (1+RINDT). WRESS notionally includes the portion of real output that goes to government in the form of indirect taxes. The division essentially removes the portion of output not available for actual nominal payments to labour, owing to labour's share of indirect taxes.

4.4.5 Steady-state employment and output: LSS, LCD, UGPCSS

Given the equilibrium efficiency wage, we can compute the equilibrium value of human wealth, VNSS (equation EQ16IHL, described in Chapter 3). Given VNSS, the equilibrium price level, PS, and the equilibrium nominal wage, WS, we can compute the equilibrium participation rate and supply of labour:

```
EQ19LHL   LSS/NPOP = AH00 - ((AH01*AH02*((1+NK )**AH03)
                     *((V/PS+VNSS)/NPOP))/((1-RMTAX
                     + AH02*(1-RATAXN)*((1+NK)**AH03))
                     *(WS/PS)*(1-RNAT+AG01*RNAT*
                     ((1-RMTAX)**(QTXRFM-1))))).
```

The equation for WS is provided in section 4.4.4, above. The equation for PS, the form of EQ19LHL, and all other variables it contains are described in Chapter 3. EQ19LHL is the regular labour supply function, evaluated under conditions of full equilibrium.

To obtain steady-state output we need two more steps. First, we specify that given exogenous energy sector employment, LEN, and steady-state government sector employment (an exogenous, policy determined proportion, LGST, of LSS), equilibrium employment in the private non-energy sector is the rest of the labour offered (up to the natural rate of unemployment):

```
EQ20LFL   LCD = (1-RNAT-LGST)*LSS-LEN.
```

It is here that we impose the market condition that equilibrium is characterized by full employment. All subsequent computations (output and other equilibrium factors) use condition EQ20LFL, and so the scales of output and all factors are consistent with full employment once the other requirements of equilibrium have been satisfied.

Given factor prices, the static firm-optimization rules provide us with an optimal output/labour ratio. Given LCD, we can determine steady-state output, UGPCSS, by multiplying LCD by the optimal output/labour ratio. The form of the optimal output/labour ratio is derived in Armstrong (1984). The equation is:

```
EQ21UFL   UGPCSS = LCD*PRODL*AF10*(((1-AF05)*(1+((AF04*
                   (1+((1-AF04)/AF04)*(((PENRSS*AF04)/
                   (CCRESS*(1-AF04)))**(AF06/(AF06-1)))))**
                   ((AF07*(1-AF06))/(AF06*(1-AF07)))))*(AF04**
                   (AF07/(1-AF07)))*(((AF08*AF05)/(1-AF05))**
                   (1/(1-AF07)))*((WRESS/CCRESS)**(AF07/
                   (1-AF07)))))**(1/AF07)).
```

Steady-state output changes in proportion to both productivity and the quantity of labour. Ceteris paribus, a rise in the real efficiency wage, WRESS, increases the desired output/labour ratio and hence increases UGPCSS. This is not very interesting, however, since WRESS cannot change independently. It is a function of the same parameters and factor prices that appear in EQ21UFL.

Consider a change in PENRSS or CCRESS. If we perform the ceteris paribus experiment, holding LCD and WRESS fixed, then a rise in either other factor price lowers the desired output/labour ratio and output.

Moreover, given that AF06 and AF07 must both be negative, the partial derivatives of WRESS with respect to CCRESS and PENRSS are necessarily negative. This reinforces the output/labour ratio effect. The fall in WRESS will lower labour supply, LSS, and hence lower LCD and UGPCSS. The only possibility of a reversal arises if LSS (and hence LCD) is highly negatively responsive to the real wage. Then the fall in WRESS could, in principle, offset the other effects. This possibility is not relevant in SAM. When either CCRESS or PENRSS rises, UGPCSS and WRESS fall. The effect of the OPEC shocks, for example, would be downward pressure on real wages and on output in the non-energy sector. Of course, for the economy as a whole, there could be offsetting influences such as higher energy output and/or more real income from energy exports.

4.4.6 Steady-state capital and energy use: KCD, ENCD

The static-optimization problem solved by firms determines both desired factor ratios and desired output/factor ratios. Given LCD and UGPCSS, we could use either perspective on optimality to provide equations for the equilibrium capital stock, KCD, and the equilibrium level of energy use, ENCD. We have chosen to write the equations using UGPCSS and the desired output/factor ratios. These are EQ22KFL for KCD and EQ23EFL for ENCD. They can be found in the model listing in Appendix B but are not repeated here because of their complexity, which comes solely from the inherent complexity of the nested CES technology. The equations are simple in principle. In each case, the desired level of the factor is written as UGPCSS divided by the optimal output/factor ratio. This optimal ratio is dependent on the technology and factor prices, and in the case of capital the steady-state rate of capacity utilization.

4.4.7 Average expected sales: SALN, SALES

The SALES variable in SAM is a smoothed measure of actual sales:

EQ78UMD $$\text{SALES} = \text{CON} + \text{GEXPNW}/\text{PG} + \text{XNEID} - \text{MNEID} + \text{IC} + \text{IEN} + \text{DNUCSS}*\text{INVCD}.$$

It consists of actual sales to domestic consumers, CON-MNEID; to domestic governments, GEXPNW/PG; to foreigners, XNEID; and sales to firms in both sectors for gross investment, IC+IEN. It is smoothed only through the use of 'normal' inventory growth, DNUCSS*INVCD, which is used instead of actual changes in inventories because of the use of inventories to buffer shocks. If demand falls unexpectedly, then inventory stocks will rise. It is not appropriate to count this as 'sales' when deriving a measure of flow excess demand. It is, however, appropriate to count the growth in desired inventories, because such growth is part of the demand that must be satisfied from potential output. Hence, we add a term that represents the steady-state increase in inventory stocks — the level of desired inventories, INVCD, times the steady-state real growth rate of the non-energy sector, DNUCSS.

Our measure of current excess demand is log(SALES/UGPCSS). This is a useful measure, but in some cases the desire to allow behaviour to be forward looking leads us to prefer a measure based on the anticipated future path of SALES, and not simply on its current value. We therefore introduce the variable SALN.

SALN is defined to be UGPCSS grossed up by the discounted expected average excess demand over all future time:

$$\mathrm{SALN} = \mathrm{UGPCSS} * \int_0^\infty \theta e^{-\theta\tau} E[\mathrm{SALES}(\tau)/\mathrm{UGPCSS}(\tau)]\,d\tau. \qquad (4.13)$$

The forward average uses exponentially declining weights, reflecting both pure discounting of the future and a notional distribution of decision horizons across firms. The solution to the integral equation (4.13) depends on the nature of the path expected for SALES.

For a correctly anticipated, constant proportional shock that lasts one period, we have simply:

$$\mathrm{SALN} = \mathrm{UGPCSS} * \left[\int_0^1 \theta e^{\theta\tau} \frac{\mathrm{SALES}}{\mathrm{UGPCSS}}\,d\tau + \int_1^\infty \theta e^{-\theta\tau}\,d\tau\right]$$

$$= (1-e^{-\theta})\mathrm{SALES} + e^{-\theta}\mathrm{UGPCSS}. \qquad (4.14)$$

The nature of the weighting is somewhat arbitrary, but a value for θ of about 0.25 seems reasonable. This would imply that about 8% of the weight is given to expectations of the future beyond ten years. With such a value the weight on current SALES is about 0.22. Thus, for a shock of 10% (i.e., SALES/UGPCSS=1.1) the excess demand measure would be 0.095 using log(SALES/UGPCSS), but only 0.022 if the more rational, forward-looking expectations are considered and the gap measured as log(SALN/UGPCSS). Without explicit dynamic theory it is impossible to specify, based on optimization, how cycle information would be used in current decisions. It is clear, however, that because of adjustment costs optimal response to a one-period shock to sales cannot be the same as optimal response to the same shock with longer duration. Our approach is to specify a signal that is sensitive to the expected future path of SALES.

For a shock that approaches UGPCSS exponentially, such that $E_t(\mathrm{SALES}(\tau)/\mathrm{UGPCSS}(\tau)) = 1+(\mathrm{SALES}/\mathrm{UGPCSS}-1)e^{-\gamma\tau}$, we obtain

$$\mathrm{SALN} = (\theta/(\theta+\gamma))\mathrm{SALES} + (\gamma/(\theta+\gamma))\mathrm{UGPCSS}. \qquad (4.15)$$

The faster the return to UGPCSS in the shock, the larger is γ and the smaller is the weight on current sales. For $\gamma = 0.167$ (a rate of convergence of about 17% per annum) and $\theta = 0.25$, we obtain a weight of about 0.6 for SALES.

For a linearly declining shock the result is more complex, but an exact solution to (4.13) is available. It is

$$\mathrm{SALN} = (1-(1-e^{-\theta T})/\theta T)\mathrm{SALES} + ((1-e^{-\theta T})/\theta T)\mathrm{UGPCSS}, \qquad (4.16)$$

where T is the time over which the shock is dissipated. For $T = 9$ and $\theta = 0.25$ we obtain a weight on SALES of about 0.6. Thus, even though the myopic value of 1.0 is not so bad as it is for the one-period shock, it still overstates considerably the weight appropriate for SALES when the future course of the shock is considered.

Note that in all three examples; i.e., equations (4.14), (4.15), and (4.16), the solution to equation (4.13) can be written as a linear combination of SALES and UGPCSS,

EQ24UFE SALN = AF40*SALES + (1-AF40)*UGPCSS,

with the value of AF40 a constant that depends on the shock. Only in special cases will it be possible to derive such a linear combination with constant weights. But, in general, EQ24UFE provides an approximate solution to equation (4.13), and is used as a default specification, with AF40 set at 0.6. Users are free to change the equation or the parameter value as appropriate for particular experiments.

4.4.8 Desired inventories: INVCD

In the notional accounts of SAM only two goods are produced by the domestic private sector -- energy and everything else. Inventories of both goods are held by producing firms. Energy inventories, INVENT, are exogenous to the model, but inventories of the non-energy good, INVCT, are explained and play a key role in the dynamics of adjustment and response to shocks. The role of inventories in disequilibrium is described later in this chapter (section 4.6). Here we consider the model of the desired level of inventory stocks, INVCD, to which the average of end-of-period actual stocks, J2A(INVCT), must converge in full equilibrium.

In the real world, not all goods can be stored. The standard example is a service. Moreover, of goods that can, in principle, be stored, not all are held to any great extent in inventories. There are many such produce-to-order activities. For these goods and services excess demand shows up in different ways -- the size of unfilled orders, the size of queues, or the delay in receiving a good or service. These issues are not dealt with formally in SAM, but the impact of cycles on such activities does influence measured capacity utilization. The notional average firm in SAM varies both inventories and capacity utilization in responding to shocks.

We specify the desired stock of inventories as follows:

```
EQ25UFL   INVCD = (AF01-AF02*CCRISS+AF03*D6670)*SALN.
                   .268  .512          .019
                  (28.6) (5.4)         (5.0)
```

Given a constant user cost of inventories, CCRISS, the model states that desired inventories are some fixed proportion of SALN. In steady state SALN equals UGPCSS, and the model generates a constant stock/sales ratio. It has been widely observed, however, that inventories are highly pro-cyclical. The pure buffer-stock notion, that inventory stocks fall when demand rises, is empirically valid only for surprises. Longer, predictable demand cycles are positively associated with inventories (i.e., stocks are increased when demand is high). We specify that the scale variable is SALN, which moves with the cycle but is not as volatile as current SALES.

The shift in the desired stock/sales ratio for the period 1966-70, introduced via the dummy variable D6670, is a response to what we interpret to be an excessively large cost sensitivity, AF02, in the version without such a shift. Without the dummy, the estimate of AF02 is considerably higher. We interpret the run up of aggregate inventory stocks in the late 1960s as associated with influences other than costs or the demand cycle. Similarly, the decline in the stock/sales ratio since 1975 may indicate other sources of inventory movement; for example, changes in the mix of aggregate output.[14]

One difficulty in estimating the parameters of EQ25UFL is that SALN depends on INVCD (via SALES) and on UGPCSS, which cannot be known until the parameters of EQ25UFL are known (they appear in the WRESS function). This problem does not arise if EQ25UFL is embodied in the simultaneous estimation problem. We have rejected this possibility because of the large increase in simultaneity involved and the consequent difficulty in identifying the parameters. Instead, we specify a proxy for SALN in estimation, trying two measures. In one, we specify SALES as the proxy and renormalize to remove the dependency of SALES on INVCD. In the other we simply use output, UGPC, as the proxy. The results are very similar. In the reported estimation, the latter approach is used. In this estimation the actual average stock is used as a proxy for INVCD, and we control for residual autocorrelation.

---

[14] Since 1981, inventory stocks and stock/sales ratios have declined dramatically, more than can be explained by increases in carrying costs. We allow for this when using the model beyond the estimation sample. See Montador (1985) for a discussion of these issues.

Figure 4-2 shows actual average inventory stocks, J2A(INVCT) and the INVCD values generated from the estimated model.

4.4.9 Steady-state capacity utilization: CAPUSS

We do not explain the normal level of capacity utilization, CAPUSS, although in principle it could come from the theory of the firm. The data for the actual level of capacity utilization, CAPU, exhibit a marked decline after 1974. It seems impossible to determine whether this represents unusual behaviour either before or after the decline. In the simultaneous estimation we found that the model prefers a CAPUSS that declines over the sample. We tried, and were unable to see any great differences between, a smooth downward trend and a step function (a level shift in the mid-seventies). We settled on a downward trend to 1974 with a level shift to a constant value, 0.834, from 1975 to 1981. The historical trend up to 1975 is given by:

$$\text{CAPUSS} = .8735 - .00292 * \text{QTIME}. \qquad (4.17)$$

The CAPU data and our CAPUSS specification are shown in Figure 4-3.

There is some speculation that the fall in CAPU is one result of an overestimation of the capital stock after the OPEC oil price increases. If so we should see a gradual rise in the level of CAPU in the 1980s. For now, however, we treat the decline as permanent and hold CAPUSS at its 1981 value for simulations over future periods.

Figure 4-2

INVENTORIES

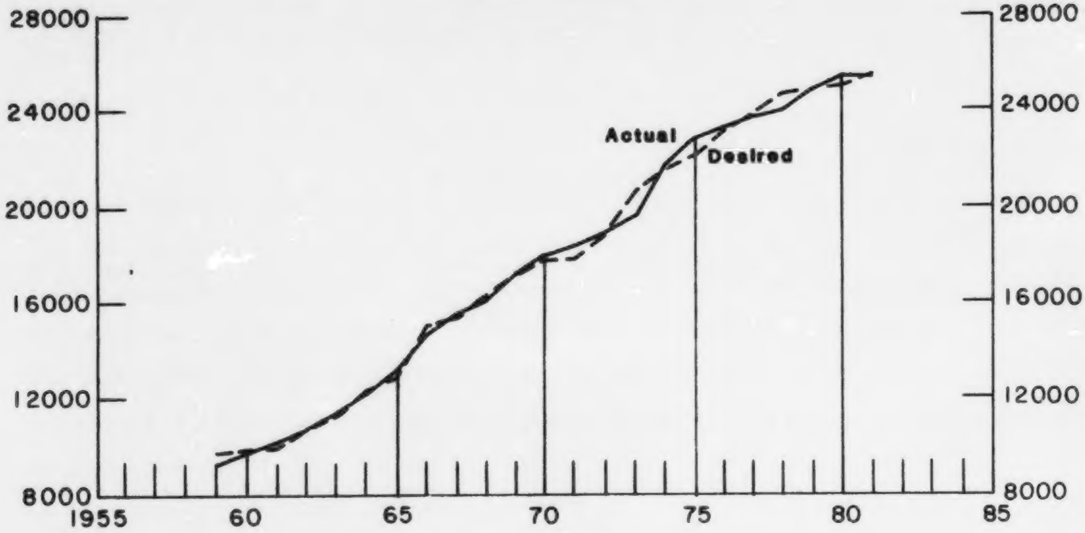
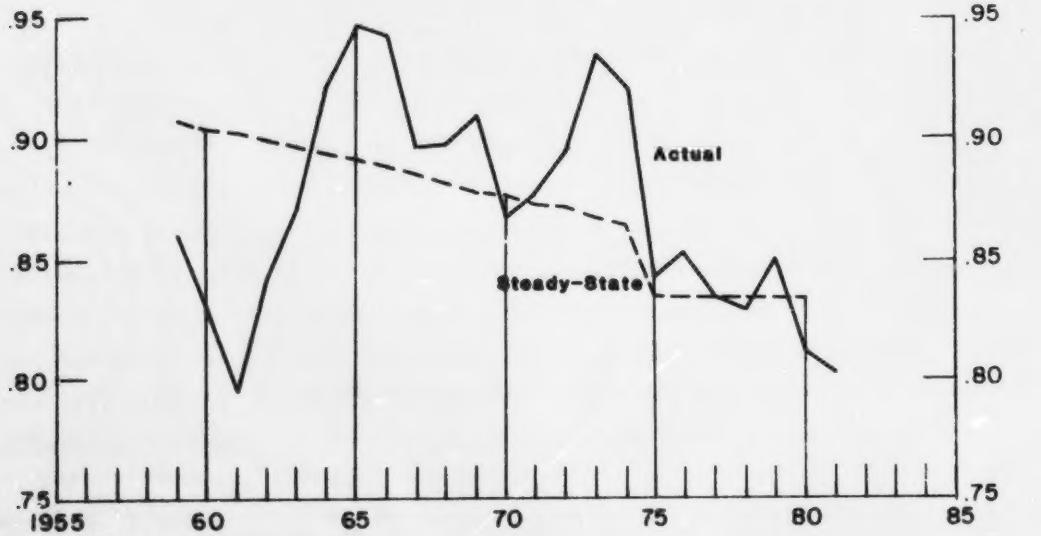


Figure 4-3

CAPACITY UTILIZATION RATE



4.4.10 The determination of the steady-state discount rate and capital-cost differential: RDRH, CCRDIF

In Chapter 1, section 1.3, we review in general terms the mechanisms that guarantee the existence of steady state in SAM and how the model works to ensure that aggregate demand and aggregate supply come together. Here, we provide more detail on these mechanics and in particular describe the role of the general level of real interest rates in the adjustment process.

Recall that, broadly speaking, there are four variables that adjust (the real exchange rate, the real wage, the price level, and the real interest rate) and four conditions to be satisfied (external balance in the sense of stock equilibrium in asset transactions with foreigners; aggregate supply/demand balance, or saving/investment balance, or labour supply/demand balance; zero excess profits; and money supply/demand balance). Here we do not discuss the real exchange rate and the external balance conditions; we return to them in Chapter 5. We also ignore relative real interest rates (other asset equilibrium conditions); we return to them in Chapter 6.

In some models, aggregate supply/demand balance is achieved through adjustment of the nominal wage, and price level is calculated to provide the correct real wage (and zero excess profits) through a cost mark-up equation. The interest rate then comes from the money demand/supply condition. In some other models, supply/demand balance is achieved using an adjustment mechanism for the price level, and the nominal wage is calculated to satisfy zero excess profits. Again, the interest rate is calculated to give money demand/supply balance. Note that in both cases, by Walras' law, clearing the labour and asset markets and satisfying the zero-excess-profits condition (assuming the model is based on perfect competition) is sufficient to guarantee clearing of the output market. Note also that although the variable selected for the adjustment equation differs, both models can generate the same long-run solution. Moreover, the models are simultaneous and have important feedback affects.

In SAM we describe the same fundamental adjustment system in a somewhat different manner. We make the real interest rate the variable that adjusts to ensure aggregate demand/supply balance. The real wage is calculated to satisfy the zero-excess-profits condition (the WRESS equation in secton 4.4.4), and the price level is calculated to provide money supply/demand balance (the PS equation in section 3.3.3). Independent dynamics for the nominal wage and the price level are added to allow for disequilibrium cycles and other aspects of the adjustment path to full equilibrium (see Chapter 7). These are purely descriptions of dynamics, however. All labour-market and output-market gaps could be removed from the wage and price dynamics and the model would still attain full equilibrium. This is not the case for the alternative models sketched above.

There are some important advantages to SAM's characterization of the equilibrating system. First, the model's price dynamics are directly related to the nominal process at work and not to a real equilibrium condition. This permits us to focus clearly on the links between money and prices. Second, our calculation of the real wage necessary for a full-equilibrium, zero-excess-profits solution brings to the fore the role of relative factor prices in determining that solution, including the functional distribution of income. This allows us to identify different forms of macro disequilibrium and to exploit this information in identifying income-distribution effects in the model's dynamics. Our focus on interest rates in the fundamental adjustment equation permits us to make a clearer distinction between real and nominal processes than would otherwise be possible and makes explicit the fact that equilibrium real interest rates are determined by real forces.

We specify the fundamental link from aggregate supply and demand to interest rates via the discount rate used by households in evaluating future income:

EQ99UHP $$RDRH = J1L(RDRH) + AH99*\log(SALES/UGPCSS).$$

We interpret this as a market process in the most general sense. Although RDRH is not a market rate, it provides a critical interface between interest rates and aggregate demand (through consumption). One alternative is to link RDRH to the measured actual return on the portfolio, and let that return go to an equilibrium and hence force RDRH to the appropriate place. The problem with this approach is that all measured rates are highly volatile, and there is no sense in which the discount rate should follow. See Chapter 3 for evidence on this point. Our approach is to avoid the difficulty of finding a reasonable direct filter for ex post rates and to specify an adjustment process that reflects the underlying fundamental market forces. It is ad hoc, but it captures the necessary link.[15]

An equation like EQ99UHP could be estimated, but only as part of a full-model, simultaneous estimation. At a minimum, we would need the supply block and the consumption demand/labour supply equations. Even if we were to use a highly smoothed series based on real market rates for RDRH, there would be simultaneous equations bias in estimates of EQ99UHP. When actual real rates are high, aggregate demand and sales will tend to be low, and so there is a difficult identification problem. The feedback from fundamental structural excess demand to the level of interest rates is probably not identifiable from the market data. We have not tried. We impose a small coefficient, 0.005 or less, for AH99. This does not mean the effect is small. RDRH has enormous leverage in the calculation of human wealth. See Chapter 3 for details.

As noted in section 4.4.4, there is a logical link between the rate at which households can trade present for future consumption and the user cost of capital. We force this restriction on the model by linking CCRDIF to RDRH in simulation. The equation is

EQ05KMP   CCRDIF = J1L(CCRDIF) + AH98*J1L(RDRH+DELK+AH96-CCRW-CCRDIF).

---

[15] A more recent version of SAM does the job differently. We introduce a variable to represent the long-term average return on the portfolio. This variable is made sensitive to the state of excess demand. A direct link is provided to RDRH and actual portfolio returns are made to adapt to the required equilibrium. In turn, through the implied real return on equity, capital cost is tied explicitly to the asset-market solution. Readers who prefer a more explicit representation of adjustment in terms of markets might prefer this alternative.

RDRH is like a real interest rate. To compare it with a user cost we must add the depreciation rate, DELK. The parameter AH96 is the historical mean differential between RDRH+DELK and CCRW+CCRDIF. It plays no other role in the adjustment process. Equation EQ05KMP prevents an arbitrary wedge from developing between the marginal productivity of capital and the discount rate used by households to notionally exchange goods over time. The parameter, AH98, that determines the closeness of the temporal link (but not the ultimate value of CCRDIF), is imposed. We currently set it at 0.8, so adjustment of CCRDIF to movements in RDRH is relatively rapid.

4.5 Factor-Adjustment Processes

4.5.1 Introduction

Factor-adjustment processes in SAM can be thought of as generalized partial-adjustment models. They are partial-adjustment processes because they all have terms that, ceteris paribus, assure that in each period actual factor use adjusts part of the way to close any gap between actual and long-term desired use. They are 'generalized' because other terms are added to the equations to enhance the model's dynamic properties. In all cases these extra terms must be zero in full equilibrium, but in states of disequilibrium they operate such that factor use can move away from long-term desired values for a time in a business cycle. We also generalize the structure of partial adjustment itself to allow for a distinction between shocks that influence desired (equilibrium) factor use and other shocks that affect current actual use without changing the long-term equilibrium. The speed of adjustment to equilibrium is made higher for the latter — the pure partial adjustment applies only to the former.

The desired values are those derived from the formal static-optimization theory — the values firms would select given steady-state factor prices and the market restrictions of full employment and zero excess profits[16] Thus, in SAM we place great emphasis on the long-term supply restrictions when specifying market dynamics. Unlike many other models, we do not specify distinct short-run targets based on demand restrictions or fixed factors. Instead, we emphasize the adjustment to long-run equilibrium, assuming that markets generate such adjustment over a time horizon short enough to make such a specification empirically interesting. The extra dynamic influences are designed to introduce the flexibility required, (e.g., cycle-amplifying effects) given the long-run focus of the partial adjustment itself. An advantage of this approach is that consistency of short- and long-run behaviour is assured. Regardless of what happens in the short-run cycles, adjustment to the long-run equilibrium path is ultimately assured by the partial-adjustment terms, assuming that other necessary conditions of equilibrium are established.

The factor-adjustment equations describe the results of market processes, but can be interpreted as reflecting the short-run demands of firms. The supply curve for labour provides a long-run restriction, but is not binding in the short run. Firms determine employment and workers provide a buffering function by moving off their supply curves. But excess or deficient unemployment, relative to the natural rate of unemployment, is always involuntary in the long-run sense.

4.5.2 The basic adjustment model

In this section we present the simplest form of our combination of partial adjustment and other factors and describe the transformation from continuous to discrete time. The example we use involves the capital stock, but similar arguments apply to the other factors.

The basic model in continuous time can be written as follows:

$$D\log(k) = g - \alpha\log(k/kd) + \beta x, \qquad (4.18)$$

---

[16] Recall, however, that the real capital cost and real exchange rate consistent with full equilibrium cannot be computed exactly. Hence, the computed desired values for the factors are based on estimates of long-run factor prices. These estimates converge on correct values in simulation without noise over future periods as the solution approaches a full equilibrium.

where g is the equilibrium growth rate, kd is the desired stock, x is some disequilibrium influence (x=0 in equilibrium), α and β are parameters, and k is the actual stock. Essentially we follow the Bergstrom-Wymer-Sargan procedures, but with some modifications of our own.[17] If we integrate the continuous process (4.18) over annual intervals we obtain:

$$\log(k_t/k_{t-1}) = g - \alpha\int\log(k) + \alpha\int\log(kd) + \beta\int x$$

$$\log(K_t/K_{t-1}) = g - (\alpha/2)\log(K_t+K_{t-1}) + \alpha\log(KD) + \beta X, \qquad (4.19)$$

where K_t is an end-of-year stock (equal to k_t at those points, but k_t exists everywhere), KD is the average desired stock for the year, and X is the average of the disequilibrium values, x, for the year. For estimation the equation must be renormalized to:

$$\log(K_t) = [2/(2+\alpha)][(1-\alpha/2)\log(K_{t-1}) + \alpha\log(KD) + \beta X + g]. \qquad (4.20)$$

Equation (4.20) is not exact. There are two types of approximation error. The first involves integral approximations. The $\int\log(k)$ is proxied by the average of logs of end-of-period values. This proxy is exact when the growth rate is constant. The approximation that $\int\log(kd)$ is $\log(\int kd)$ or log KD is not exact, but given typical real growth rates this error will be very small. A second type of error in the functional form depends on how the variable x evolves through time. The conditions under which this second type of error becomes important (see Armstrong and Rose, 1983) do not seem to arise for any of SAM's factor adjustments.

---

[17] See, for example, Bergstrom (1976). For a more complete discussion of the technical aspects of discrete representations of continuous-adjustment processes see Armstrong and Rose (1983).

Equation (4.20) has the property that, in the absence of growth and X disequilibrium, $\log(K_t)$ is a linear combination of $\log(K_{t-1})$ and log(KD). Note that the weights sum to 1.0 and for $\alpha>0$ (required for stability) and $\alpha<2$ (required for the approximation to be valid), the weights are both positive and less than unity. When growth is added, the linear combination must be modified as shown in equation (4.20). In full equilibrium, when X is zero and J2A(log(KCT)) equals log(KCD), the process collapses to J1D(log(KCT)) = g, where g is the equilibrium growth rate. Note that once the system is in equilibrium, the process of growth itself does not disturb the equilibrium. Such a property is not automatic and is not always satisfied in functional forms chosen for adjustment equations in economic models. For long-term analysis such details are important.

When the capital stock is not at its desired level the growth process is modified in the standard partial-adjustment manner such that net investment is higher (lower) when the desired stock is higher (lower) than the current average value. The speed of adjustment, α, is a parameter to be estimated. This process is modified by other properties of the cycle. Suppose, for example, that aggregate demand has fallen and resulted in a negative value for X. If the capital stock is already too high, the extra term will magnify the reduction in investment that would otherwise occur. Conversely, if the capital stock is too low, the process of adjustment back to equilibrium will be delayed. The extent of this infuence, β, is a parameter to be estimated.

For estimation we do not make g constant. The postwar period was one of very unusual population growth and technical progress, when seen from a long-term perspective. For estimation g is specified as DNPOP (a smoothed version of the actual profile of annual adult population growth) plus DNPRL, the rate of technical progress estimated jointly with the rest of the supply side (see section 4.3). The sum appears as the variable DNUCSS in the model equatious described below. For default values for the future steady-state path we assume a population growth rate of 1.1% per annum and a technical progress rate of 1.3% per annum, giving an overall real growth rate of 2.4% per annum. These assumptions are easily changed.

4.5.3 An extension to the basic model: adjustment to transitory shocks

We now introduce a complication to the above analysis, designed to provide the model with a more flexible dynamic response structure and, in particular, an adjustment profile that can differ for transitory and permanent shocks. Equation (4.20) allows us to consider permanent shocks, those affecting equilibrium behaviour through KD (e.g., factor-price changes), as well as other transitory influences (including the response to demand shocks) that enter through X variables. In this form, however, the model implies a strong restriction. Consider two shocks, one which lowers KD such that a 5% gap is created relative to K_t, and the other which (through X) raises K_t by 5% but then shuts off (X returns to zero). In both cases K_t is 5% above KD and, according to the adjustment model, the same profile of adjustment back to equilibrium will result. In other words, once the disequilibrium terms are 'off' the process of adjustment does not depend on the source of the disequilibrium.

We extend the model by specifying an adjustment process that permits a more rapid approach to equilibrium when the source of disequilibrium is a transitory shock. In continuous time we write

$$D(\log(k) - \gamma\beta x) = g-\alpha[\log(k)-\gamma\beta x-\log(kd)] + (1-\gamma)\beta x. \qquad (4.21)$$

The key change is that the adjustment process is written in terms of k, net of a proportion γ of the influence of the transitory effects, βx. The gap does not compare the actual k with kd, but rather k adjusted for the deviation caused by the disequilibrium, x. Equation (4.20) is the special case where γ is zero, where it does not matter why k is different from kd. In the other polar extreme, where γ is one, transitory influences have no long-lasting effects. If γ is one, then if we start in equilibrium and consider a shock where x is 'on' for only one period, then in period 2 we would be right back on the equilibrium path. Our specification permits the identification of some intermediate position wherein a proportion $(1-\gamma)$ of transitory influences results in longer adjustment processes, but the rest is treated as purely transitory and does not extend the disequilibrium.

In discrete time the equation becomes

$$\log(K_t) = [(2-\alpha)/(2+\alpha)]\log(K_{t-1}) + [2/(2+\alpha)][g+\alpha\log(KD)]$$
$$+ [1/(2+\alpha)][(1+\alpha\gamma+\gamma)\beta X_t + (1+\alpha\gamma-3\gamma)\beta X_{t-1}]. \qquad (4.22)$$

In equation (4.22) the X variables are written as end-of-period values. To reconcile (4.22) with (4.20) under the restriction $\gamma=0$, one must recognize that X in (4.20) is an average. If the average is written as $(X_t+X_{t-1})/2$ the reconciliation is exact.

This extension provides one reasonably important side benefit by permitting us to handle fairly easily the links between stocks and flows that would otherwise be a source of complexity. Consider, for example, the effect of inventory stock disequilibria on factors of production. A firm may wish to use extra variable factors in the production of enough extra output to close the inventory gap. But, in the model where γ is zero, as stock equilibrium is regained the factors have not returned to their long-run equilibria (owing to partial adjustment) and so there is overshooting of the inventory stock targets and considerable cycling can result. One approach to removing this automatic overshooting would be to use a much more complicated measure of inventory disequilibrium. Standard control theory applied in such cases suggests that a level, a rate of change, and an integral measure will be required to permit a smooth approach to equilibrium. In particular, a complex calculation of the forward integral of extra output from excess factors would be required. Our extension of the simple model appears to provide reasonable dynamics using only the simple gap measure. We still get some overshooting, but it is quite controlled quantitatively.

The example of the capital-adjustment process provides us with the least convincing case for the extension. If capital is indeed at all a fixed factor then we would expect γ to be relatively small, as even transitory response results in stock that cannot easily be eliminated. For more variable factors we could expect larger values for γ. In estimation this is indeed what we find. For capital, γ is very small and insignificantly different from zero, but for both energy and labour the γ values are significant in both the economic and statistical senses.

4.5.4 Disequilibrium influences on factor adjustment: the equations for estimation

Capital: KCT

Four disequilibrium variables are included in the equation that describes movements in the private sector non-energy capital stock. These are a measure of the real excess demand for the product, a measure of the level of nominal disequilibrium in the economy, a measure of the market value of capital relative to its replacement cost, and a measure of the deviation of the cost of capital from its long-run equilibrium value.

```
EQ45KFD   log(KCT) = [(2-AF12)/(2+AF12)]*J1L(log(KCT))
                     + [2/(2+AF12)]*[DNUCSS+AF12*log(KCD)]
                     + [(1+AF16*(1+AF12))/(2+AF12)]*
                     { AF34*log(SALN/UGPCSS)+AF20*log(PS/PC)
                     + AF19*CCGAP + AF56*log(TOBQ/TOBQSS) }
                     + [(1-AF16*(3-AF12))/(2+AF12)]*J1L{...}
```

The speed-of-adjustment parameter, α in equation (4.22), is AF12. The coefficient that allows for more rapid return to equilibrium after temporary shocks, γ in equation (4.22), is AF16. The four special disequilibrium factors appear in { }. They also appear with an additional lag in the final term, J1L{...}.

The first disequilibrium term, the gap log(SALN/UGPCSS), gives the investment model something akin to an accelerator mechanism. If demand is high relative to steady-state levels of output, investment responds. Recall that SALN is a forward-looking, average-expected sales measure. The term is thus influenced by the current state of excess demand, but is somewhat smoothed owing to the expected eventual return to equilibrium. For estimation, because of the difficult identification problem associated with the dependency of SALN on unknown technology parameters (through UGPCSS), we proxy SALN with the SALES data. Recall that the SALES variable is itself somewhat smoothed in that disequilibrium movements in inventories are removed.

The second disequilibrium term, log(PS/PC), represents the extent to which current prices are out of equilibrium. If PS exceeds PC firms anticipate higher prices and put capital in place now. Note that this is consistent with consumer behaviour in that consumption demand will be unusually high in such circumstances according to our estimated behaviour. Note further, that although PC is the consumer's price index, and is influenced by import prices, the adjustment of PC towards PS influences the producer's price directly (see Chapter 7 for equations). This gap variable is important for nominal (monetary) shocks, because it creates a mechanism for a short-term nominal-demand influence independent of the real cycle.

We tried to introduce cost-of-capital effects, especially fiscal policy effects, but had no empirical success. The term CCGAP represents several measures tried in estimation. We report estimates based on the measure log(CCRESS/CCRE), where CCRE is a Jorgensonian user-cost measure similar in structure to CCRESS, but using current values of component variables. In all cases we obtained small, insignificant coefficients. We do not use this variable in the standard model. Similarly, we could not find any significant effect of profits on investment.

A related effect, through asset prices, is included via the TOBQ (Tobin's q) term (EQ66AMI, Appendix B). The term log(TOBQ/TOBQSS) reflects the market valuation of capital relative to its equilibrium valuation. When the valuation of capital is high, relative to replacement cost, firms respond by increasing the rate of capital formation. This provides another important financial/real link. For example, given an unanticipated increase in the money stock, asset markets will generate a rise in the valuation of capital and hence investment. The mechanism works through the valuation term and not through the discount rate in steady-state capital cost (and hence KCD). This seems to be reasonable, since no permanent real-interest-rate effects follow from a monetary shock.

Gross investment, IC, is determined from KCT by adding capital consumption to net investment:

$$\text{EQ49KFI} \quad \text{IC} = \text{J1D(KCT)} + \text{DELK*J2A(KCT)}.$$

In this formulation, investment in the current year is depreciated at half the normal rate.

Energy: ENC

The energy-use equation has three disequilibrium variables, which differ from those used in the capital equation. In the energy equation we use two measures of the real cycle -- one in the flow dimension (CAPU gap), the other in the stock dimension (inventory gap) -- as well as a disequilibrium energy price term:

$$\begin{aligned}\text{EQ46EFD} \ \log(\text{ENC}) = {} & [(2-\text{AF23})/(2+\text{AF23})]*\text{J1L}(\log(\text{ENC}))\\ & + [2/(2+\text{AF23})]*[\text{DNUCSS}+\text{AF23}*\text{J2A}(\log(\text{ENCD}))]\\ & + [(1+\text{AF13}*(1+\text{AF23}))/(2+\text{AF23})]*\\ & \{\text{AF29}*\log(\text{PENRSS}/\text{PENR})+\text{AF33}*(\text{CAPU}-\text{CAPUSS})\\ & + \text{AF26}*\log(\text{INVCD}/\text{J2A}(\text{INVCT}))\}\\ & + [(1-\text{AF13}*(3-\text{AF23}))/(2+\text{AF23})]*\text{J1L}\{\ldots\}.\end{aligned}$$

The structure of this equation is similar to that of EQ45KFD. The speed of adjustment is AF23 and the γ parameter from equation (4.22) is AF29. Note, however, that since ENC is the energy use over the period, and not a rate of use at the end of the period, the two equations have a somewhat different continuous-time source. As in EQ45KFD the three disequilibrium effects appear in { } and, although not explicitly repeated, in J1L{...}.

The first disequilibrium term, log(PENRSS/PENR), measures the extent to which current real energy prices are out of equilibrium. Historically, the term helps explain why actual energy use stays above desired use for extended periods after the OPEC shocks. One can think of this term as reflecting the temporary price advantage in energy enjoyed by domestic producers during the transition period. In general, given a positive AF29 and delayed response of actual real energy costs, PENR, to a permanent shock to equilibrium real energy costs, PENRSS, this term will operate to delay response to the price shock. Alternatively, if the shock is a temporary change in real energy costs, which does not change PENRSS, this term provides some temporary response in energy use.

The next two terms in { } are the real-cycle measures. We capture the flow excess-demand effect with the deviation of CAPU from CAPUSS, rather than with the output gap directly. In estimation the CAPU-gap formulation proved a bit more successful. The stock gap reflects the fact that firms are unwilling to let inventory stocks deviate permanently from desired levels. If actual stocks rise above desired, in response to a period of less than normal demand, firms react to the disequilibrium by reducing variable factor usage as long as the stock gap persists. Lower factor use reduces output and causes inventory stocks to fall back towards desired levels.

Labour: LC

The equation describing the use of labour in the non-energy private sector is identical to the equation for energy use with two exceptions: the disequilibrium price effect is specified with respect to the wage rate, and the real growth rate for labour is the trend adult population growth rate, DNPOP:

```
EQ47LFD log(LC) = [(2-AF45)/(2+AF45)]*J1L(log(LC))
                  + [2/(2+AF45)]*[DNPOP+AF45*J2A(log(LCD))]
                  + [(1+AF51*(1+AF45))/(2+AF45)]*
                  { AF50*log(WS/W)+AF52*(CAPU-CAPUSS)
                  + AF48*log(INVCD/J2A(INVCT))}
                  + [(1-AF51*(3-AF45))/(2+AF45)]*J1L{...}.
```

The speed of adjustment is AF45 and the γ coefficient from equation (4.22) is AF51. The real-cycle disequilibrium effects are identical to those in the energy equation and have the same rationale. The wage gap, log(WS/W), provides for short-run sensitivity of employment to the wage rate. If we assume that AF50 is positive, then the higher the wage, ceteris paribus, the lower is employment. This is not a marginal-productivity demand response in the usual sense -- that effect is embodied in the LCD function for changes in the equilibrium wage -- but its operation is similar.

4.5.5 Modifications in simulation

For simulation we make a few changes to the estimated adjustment equations. It is well known that the partial adjustment model is not capable of generating a complete return to equilibrium -- arbitrarily close but not complete. This is not important for short-run analysis, but for longer-term simulation we would like to see convergence to full equilibrium. There are three economic points here. First, the partial-adjustment model itself does not generally describe optimal behaviour. It can be considered a first-order approximation to the true adjustment process, capturing the most important structure for short-run analysis (i.e., impact effects), but generally understating the effective speeds of adjustment over longer periods (e.g., simple partial adjustment never generates complete convergence). Second, because the historical data reflect more or less continuous shocks and measurement errors, and because any errors in the measures of equilibrium factor prices pass into the measures of desired factor use, we tend to add noise to the main gap measures and hence underestimate response to shocks. Third, in the short run we would expect a certain amount of confusion about the nature of shocks; for example, to what extent an observed change in a factor price was permanent. A simulation is a counterfactual experiment and can be designed to reveal response to a shock with average historical properties. Usually, however, the hypothetical shocks are more specific and researchers are looking for what would happen in an artificial world where the shocks appear in isolation from other normal noise. In such cases, we would expect rational agents to respond more strongly as the signal becomes clearer.

For these reasons, we think it unnecessary to restrict the adjustment processes in simulation to the estimated partial structures. Our normal operating procedure is to use the estimated model for the first few periods (up to five years, say) and to increase adjustment speeds thereafter. To do so we modify the basic adjustment equations using a variable called STIME. This stands for simulation time and simply means the number of years from the introduction of a shock. It must be set for each particular experiment. The reader can see an example of this approach by comparing the factor-use equations reported above to those reported in Appendix B.

4.6 Mechanisms for Short-Run Buffering: Inventories and Capacity Utilization

Any model of disequilibrium response to shocks must answer several basic questions. How much price movement will there be in the short run? How much will the flow of inputs generating the output flow be altered (e.g., how much unemployment is created in response to a negative demand shock)? To what extent can flow shocks be buffered by changes that do not seriously disrupt the income flows to factors, particularly labour (e.g., capacity utilization, inventories)? How will the composition of these effects be determined?

For SAM we specify that all these forms of response to shocks play a role. We test for, and find significant links between, excess demand and prices (Chapter 7). We find that factor use responds significantly to demand shocks (sections 4.5 and 4.8). But the main mechanisms for response, at least initially, are variations in capacity utilization to alter flow output, and variations in inventory stocks. Given our estimates, the two mechanisms absorb roughly equal shares of the adjustment to a shock to a full equilibrium. In general, the relative shares of stock and flow response depend on the initial conditions and the size of the shock.

4.6.1 Inventories

Formally, inventories are both economically and technically residual in SAM. That is, we solve for inventories using the identity that what is produced and not sold goes into inventories:

EQ80UMI J1D(INVCT) = UGPC-(CON+IC+IEN+GEXPNW/PG+XNEID-MNEID).

We take the view that inventories are held precisely because of their buffer-stock role. Demand is stochastic and subject to business cycles. It is costly to use flow adjustment and price variation to clear markets. Inventories are also costly to hold, but holding some is superior to simply accepting the consequences for factor inputs and prices of transitory demand fluctuations. Furthermore, although firms do not 'want' inventory stocks to deviate from desired levels, they willingly choose to let this occur. Such changes may or may not be fully unexpected, depending on the nature of autocorrelation in the demand shocks, but even truly unexpected stock changes are unplanned only in the limited sense that the exact shocks are not known, ex ante Firms plan ex ante to let inventories vary as required to respond to demand (and supply) innovations and cycles. This is the purpose of a buffer stock. Inventory stocks are permitted to deviate systematically from long-term targets not because it is costless to do so but because there are greater costs associated with changing the flow of output (e.g., by adjusting factor usage).

Holding inventories is costly, however, and the more they rise relative to desired levels the less willing firms will be to let them rise further. Similarly, as stocks decline the resulting lack of freedom to respond normally to fluctuations in sales will lead firms to resist further reductions. So stock disequilibrium must affect behaviour elsewhere. If inventories are too high then less labour and energy are used. Output falls and inventories are brought back towards desired levels, ceteris paribus (see section 4.5). Variation of capacity utilization also helps restore inventory equilibrium (see section 4.6.2).

4.6.2 Capacity utilization: CAPU

Although inventories are selected as the residual variable proximately determined by identity EQ80UMI, variation in capacity utilization is given an important role in the short-run buffering of shocks. All demand is satisfied and any differences with flow output are reconciled through inventory fluctuations, but the size of a residual inventory fluctuation depends on how much flow output is adjusted in response to a shock. Some such response is generated by varying actual factor use, but the main source of short-run output response is variation in the degree of capacity utilization.[18]

We specify a stochastic behavioural equation for CAPU. Although we provide no formal derivation, we view this equation as describing firm behaviour based on optimal response to shocks and cycles, given various costs of adjustment.[19]

$$\text{EQ51UFS} \quad \mathrm{CAPU} = \mathrm{CAPUSS} + \mathrm{AF43}*\log(\mathrm{SALES}/\mathrm{UGPCSS}) + \mathrm{AF38}*\log(\mathrm{INVCD}/\mathrm{J2A}(\mathrm{INVCT})) \\ + \mathrm{AF14}*\log(\mathrm{RRK}*(\mathrm{J2A}(\mathrm{KCT}) + \mathrm{INVCD})/(\mathrm{RRKSS}*(\mathrm{KCD} + \mathrm{INVCD}))).$$

The equation states that actual CAPU is set to its steady-state value, CAPUSS, unless: (i) there is excess demand,[20] (ii) profits are abnormally high, or (iii) inventories are not at desired levels.

---

[18] See section 4.2 for a description of the output equation, how it is influenced by CAPU, and the source of our CAPU data.

[19] Formally, however, we do not make explicit a resource cost for adjustment. In particular, firms can alter the rate of capacity utilization without changing the depreciation rate.

[20] For estimation we add two lags of the excess demand gap. Their coefficients are AF36 and AF37, respectively. The second-order process is meant to capture the average cycle properties in the sample. In a simulation of a demand shock with very different autocorrelation, the average historical structure may not be relevant. To leave the equation as estimated is to say that the shock is (falsely) interpreted as similar to the historical average. This is a different experiment from a correctly perceived shock. For this reason our standard control model does not contain the lagged terms. We feel this is a more neutral specification. The estimation results are provided for users wishing to retain the lags.

Parameters AF43 and AF38 are of great importance in determining the proportion of a demand shock that will be buffered initially by inventories and the proportion that will be buffered by an output response. The larger is AF43, the greater the initial response of CAPU to a demand shock, and the smaller the required residual buffering through inventories. The larger is AF38, the greater the output response to correct a given deficiency in inventories, and the faster, ceteris paribus, that gap closes. The inventory gap is also important because it helps ensure that inventory stocks will return to desired levels. A CAPU response reinforces the effects of movements in the variable factors to correct an inventory imbalance.

The other term in EQ51UFS measures gross income accruing to capital (residual, after other factors have been paid) relative to the equilibrium gross income. In other words, it measures short-run excess profits.[21] A positive coefficient would indicate that firms tend to produce more, in a cycle sense, when profits are high. We note, in passing, that since (CAPU-CAPUSS) appears as a disequilibrium influence in the labour- and energy-adjustment equations, profits will have an indirect influence on variable factor use in simulation. We attempted to find a direct influence of profits on investment, but were unsuccessful.

Why estimate a CAPU equation and let inventories be residual rather than the other way around? Our view is that one should identify the fundamentally residual category in terms of firm behaviour, and let this be determined by identity EQ80UMI. In simulation, what is determined by the identity and what by the behavioural rule does not matter; in estimation it does. The dependent variable in the equation chosen for estimation should be that for which the addition of a random error makes the most sense. If something can be identified as the behaviourally residual component of a system, then its 'errors' reflect the sum of the direct errors on the chosen variables and as such it is appropriately treated as the technically residual variable in estimation. We feel more comfortable arguing that firms choose operating levels and let inventories be set residually; therefore we estimate a stochastic CAPU equation. But whatever the choice for estimation, we are still specifying a system of response that simultaneously determines both CAPU and inventory behaviour.

---

[21] This formulation was established for the simulation model. The equilibrium part of the measure cannot be known until the estimates are available (KCD is unknown). For estimation, we use a proxy — the deviation of profits from a historical trend.

4.7 The Estimation Procedure

The output equation, the three factor-use equations, and the CAPU equation constitute a five-equation system with a complex set of cross-equation restrictions. In addition, there are a large number of identities that must be respected by the estimator. The cross-equation restrictions can be respected efficiently only with a systems estimator. We estimate our system using a modified maximum-likelihood procedure that incorporates a purpose-designed combination of the Davidon-Fletcher-Powell and Nelder-Mead algorithms of GQOPT2.[22]

Our estimator is not a standard application of maximum likelihood. We shall not distract the reader with technical arguments not central to the model; separate papers are available that consider the estimator in more detail.[23] A limited discussion of the two unusual features of our estimator seems necessary, however.

First, we do not specify a full covariance matrix for the system. Instead, we estimate under the assumption that the disturbances are independent. Armstrong (1985a) presents the properties of this estimator and the argument that in many applications in economics it is more appropriate than the general maximum-likelihood estimator.

Second, we modify the usual maximum-likelihood criterion by introducing a penalty function. This function reduces the value of the objective function of the estimator depending on the extent to which certain qualitative properties are not satisfied by the estimated results. We refer to properties that must hold asymptotically, but not necessarily in small samples; for example, that desired and actual factor usage be the same on average. All steady-state values such as WRESS, desired inventories, and desired levels of factor use, are model-based concepts, dependent on parameters that must be estimated. The normal criterion maximizes the likelihood of the data for the stochastic variables, utilizing if necessary any ancillary identities, but it does not permit the imposition of asymptotic properties (e.g., that on average actual and desired stocks must be the same). In small samples there is no presumption that such average gaps should be zero. In fact, in a sample that includes the OPEC price increases, it seems particularly inappropriate to impose such a condition.[24] In free estimation there was, however, a tendency for the model to produce unreasonable desired and steady-state values, usually for very little gain in terms of the objective function. We argue that as long as not too much weight is given to the penalty the efficiency of the small-sample estimator is increased by penalizing deviations from the asymptotic properties. These properties can be thought of as parametric restrictions, expressible as inequality restrictions on the means and variances of deviations of actual outcomes from the (parametric) desired values.

---

[22] GQOPT2 is a library of subroutines for function maximization made available by Goldfeldt and Quandt of Princeton University.

[23] Armstrong (1985a), Armstrong (1985c).

One interesting case is the gap between actual and desired inventories. In the formal model the CAPU equation is treated as stochastic and inventories as residual. But in reality we wish to specify the joint choice as to how shocks are buffered by variation in CAPU and inventories. A change in the parameters that marginally improves the fit of the CAPU data, while at the same time destroying the relationship between actual and desired inventories, is not necessarily a preferred point according to the complete model. Yet the concern about the relationship of actual and desired inventories is not reflected in the likelihood function for the stochastic equations. The use of a penalty on the deviation of inventories from desired levels provides an effective way to introduce this concern.

---

[24] Helliwell and his colleagues (e.g., Helliwell et al., 1982) argue that in the short run firms are not constrained by the production function, and therefore that such technologies cannot be directly estimated from small samples. They get parameters of the technology by imposing that actual and desired factors be equal on average over the sample. This is the antithesis of our viewpoint. We would argue that in small samples there is no reason for the average factor gaps to be zero. On the contrary, such an approach is liable to introduce considerable bias, especially given the large relative factor-price changes in this particular sample. Our compromise is to imbed the technology in the equation describing short-run movements in output and to estimate that output equation (and with it the technology) simultaneously with the factor-demand and CAPU equations -- without imposing that any gap have a zero mean over the sample.

4.8 Estimation Results

4.8.1 Parameter estimates

In Table 4.1 we report the simultaneously estimated parameters along with their asymptotic t-ratios. The asymptotic statistics were calculated using numerical second derivatives generated by the GRADX routine in GQOPT2 to obtain the appropriate Hessian.[25] The sample period for the estimation was 1959-81.

The search for these results was far from straightforward. We encountered many local solutions or flat regions. We have done considerable testing using different starting values, however, and are reasonably confident that the solution presented is a global maximum point. The use of the penalty function helped considerably in directing the search in appropriate directions. In the end, however, the actual penalty value is very small, relative to the value of the likelihood function.[26]

In all cases where there is a clear expected sign for a parameter we obtain that sign. Moreover, most parameters are statistically significant at the usual confidence levels. It should be noted, however, that such t-ratios can be misleading. The statistics correctly reflect the curvature of the objective function at the point of solution, but there could be many points that fit almost as well where groups of parameters

---

[25] The t-ratios available from the Davidon-Fletcher-Powell procedure are based on an approximation to the Hessian built up during the search procedure. We find these statistics virtually meaningless (i.e., bad approximations) in most runs. We therefore switch to GRADX for the statistics.

[26] We are not sure whether the global optimum would change significantly if we removed the penalty function and restarted from the reported point. We intend to investigate this issue further in subsequent work. A preliminary discussion of our use of a penalty function is available in Armstrong (1985c).

Table 4.1

PARAMETER ESTIMATES: OUTPUT, CAPU, AND FACTOR EQUATIONS

Equation	Variable	Parameter	Point estimate	Asymptotic t-ratio
Output (Technology)	K weight	AF04	0.9409	247.1
	L weight	AF05	0.5923	20.7
	Exponent	AF06	-0.3304	59.3
	Exponent	AF07	-0.2199	7.2
	Scale	AF08	3.1546	4.5
	Scale	AF10	1541.8	4.5
Output (Technical Progress)	Constant	AF09	0.0207	11.4
	1973 Shift	AF11	0.0049	1.2
Capacity Utilization	Sales Gap	AF43	0.5857	1.6
	"(Lag 1)	AF36	1.0077	2.4
	"(Lag 2)	AF37	-0.6513	1.7
	Inventory Gap	AF38	0.7766	1.6
	Profit Gap	AF14	0.2259	2.6
Capital	Adjustment	AF12	0.1291	2.9
	Gamma	AF16	0.0936	0.8
	Cap.Cost Gap	AF19	-0.0275	0.3
	Price Gap	AF20	0.1064	2.4
	Sales Gap	AF34	0.2205	4.4
	TOBQ Gap	AF56	0.0210	1.8
Energy	Adjustment	AF23	0.0531	2.0
	Gamma	AF13	0.3015	3.0
	En. Price Gap	AF29	0.1138	0.8
	Inventory Gap	AF26	0.9968	3.6
	CAPU Gap	AF33	0.2929	2.3
Labour	Adjustment	AF45	0.2589	1.7
	Gamma	AF51	0.4101	3.3
	Wage Gap	AF50	0.1590	2.2
	Inventory Gap	AF48	0.2348	1.2
	CAPU Gap	AF52	0.4038	5.1
	WREADJ	AF57	1272.6	12.4

are more distant from these point estimates than their individual standard errors seem to indicate. But the parameters do seem reasonably robust to small changes in the definitions of variables and to changes in the detail of the disequilibrium specification.

4.8.2 Analysis of individual equation results

In Table 4.2 we provide some individual equation statistics. The fits are all fairly good. It is important to remember that these results are generated by a maximum-likelihood systems estimator. The fit statistics are of interest as properties of the results, but are not of paramount concern to the estimator. We can report, for example, that application of the maximum-likelihood principle in our system leads to a deterioration in the fit of the output equation. Single-equation estimates produce an RSQ value greater than 0.9998. Furthermore, some important changes in the qualitative properties of the steady-state model come from the systems estimator. This is the point of the exercise and we consider the joint estimation of the supply-side equations an important contribution of SAM. It is also important to remember that although high values of RSQ are almost automatic for equations with strong trends and lagged dependent variables, there are no free constants or trends in these equations. Moreover, the coefficients of the lagged dependent variables in the factor equations are subject to within-equation restrictions.

Table 4.2

INDIVIDUAL EQUATION STATISTICS
(Sample, 1959-81)

Equation	RSQ	Residual autocorrelation
Capital	0.9998	0.215
Energy	0.9974	-0.039
Labour	0.9980	0.505
Capacity Utilization	0.7821	0.437
Output	0.9985	0.627

Figure 4-4

PRIVATE SECTOR (NON-ENERGY) OUTPUT
(Billions of 1971 Dollars)

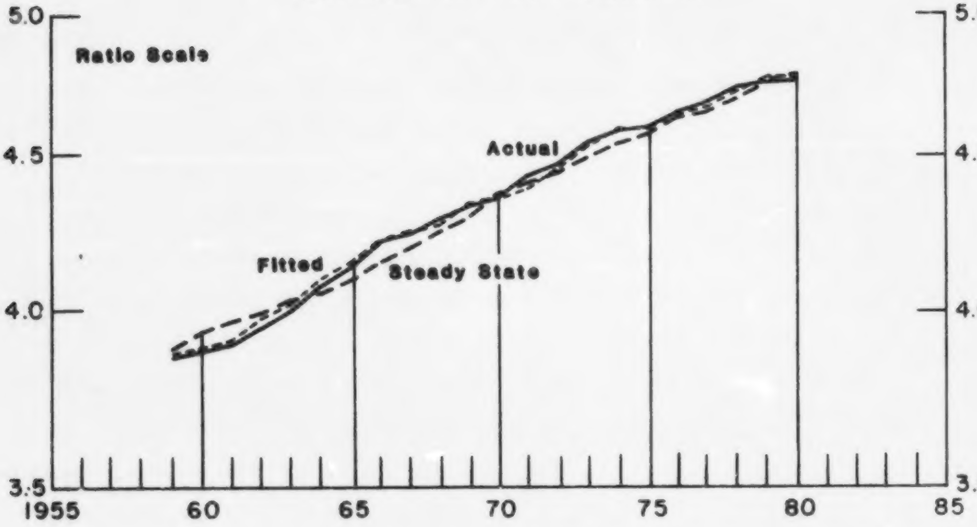


Figure 4-5

CAPACITY UTILIZATION

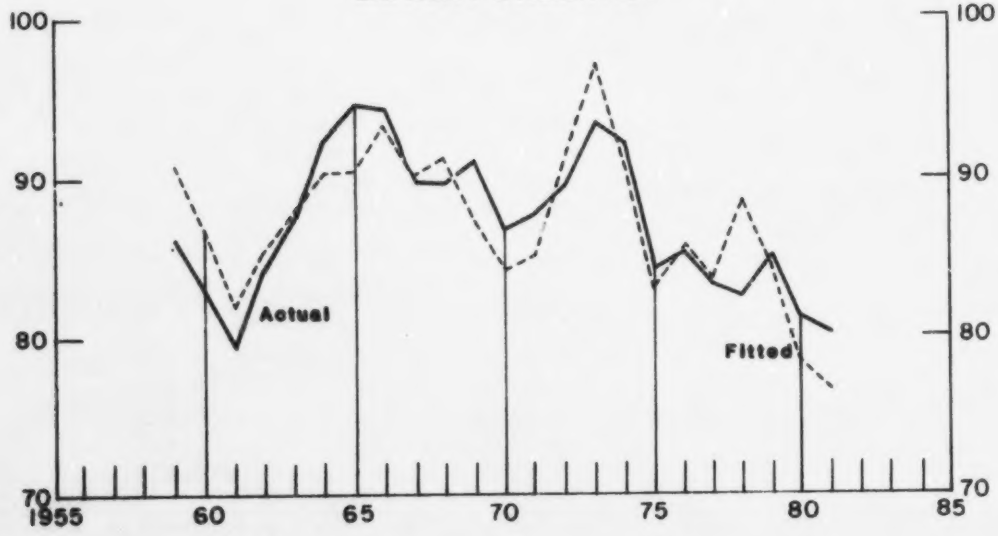


Figure 4-6

PRIVATE SECTOR (NON-ENERGY) CAPITAL STOCK
(Millions of 1971 Dollars, One Year Difference)

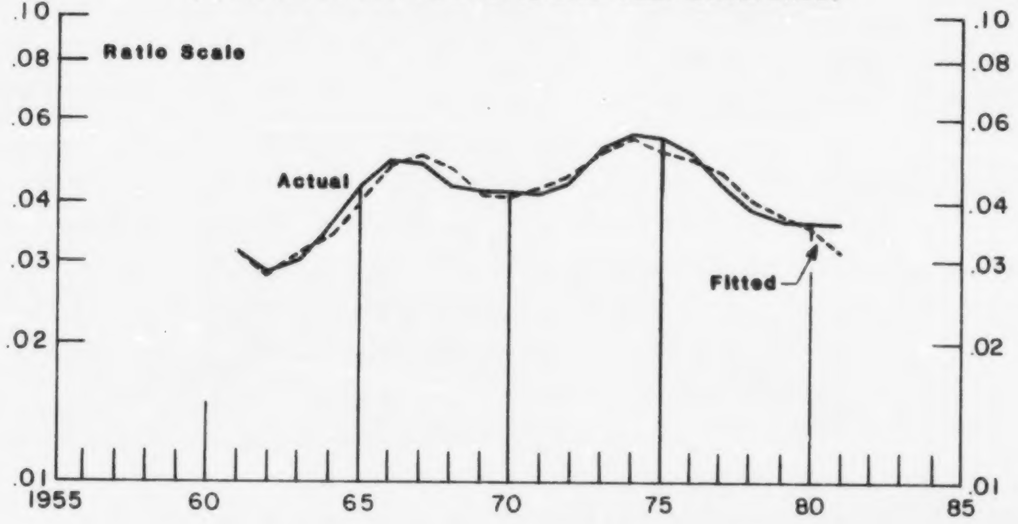


Figure 4-7

PRIVATE SECTOR ENERGY USE
(Millions of 1971 Dollars)

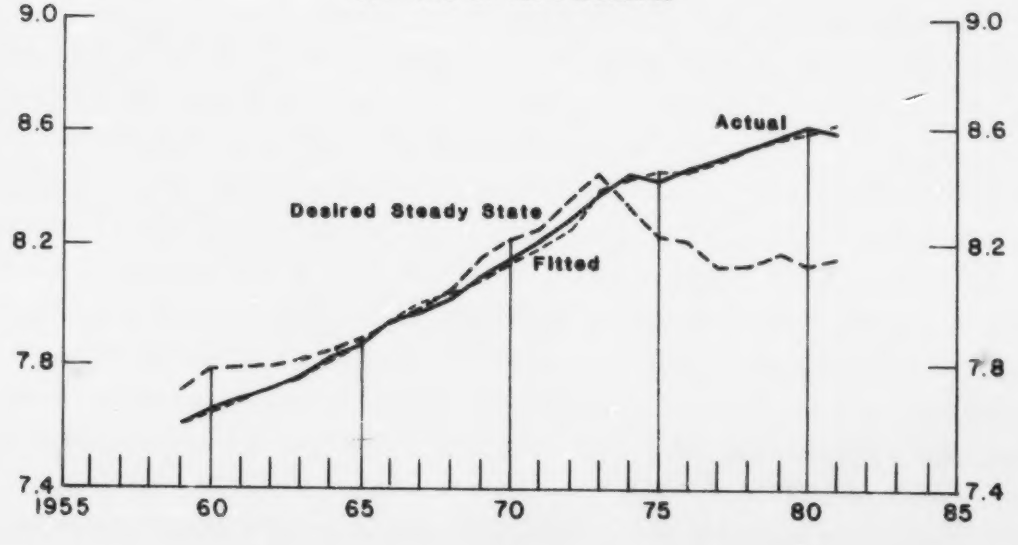
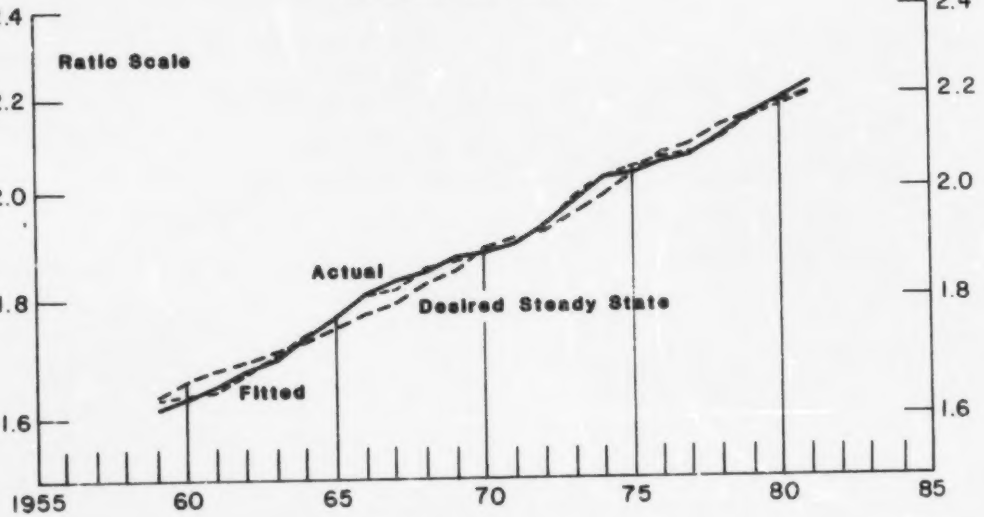


Figure 4-8

PRIVATE SECTOR (NON-ENERGY) EMPLOYMENT
(Millions of Persons)



The equations also fit reasonably well in first-difference form. For example, for the period 1961-81 the squared correlation coefficient for the fitted and actual values of the growth in capital (i.e., relative net investment) is 0.92.

The residual correlation statistics reported in Table 4.2 are first-order autocorrelation coefficients. The usual single-equation tests do not apply to results from a systems estimator, but we report the autocorrelation statistics as a general guide to the properties of the estimated residuals. There appears to be important structure, especially in the output equation. Although it is not a formally valid test, we can apply the benchmark standard error, $1/\sqrt{n}$, where n is the sample size. On this basis the labour, capacity utilization, and output statistics are significantly large. Given that there are no free constants in these equations, these results are understandable, but if the model were to be used for short-run analysis some attention would have to be given to the error structure.

The fitted values for each endogenous variable are shown in Figures 4-4 to 4-8, along with the actual data and the desired or steady-state values, where appropriate. For capital, we show the fits for the growth rate (first difference in logs). The figures show that the model is generally quite successful in tracking the historical data. This is true for the more cyclical variables (CAPU, net investment) as well as for the levels. Note, however, that although we can 'explain' the energy-use data, after the OPEC price increase an enormous gap develops between actual use and the model's computed desired long-term use. This is discussed in the section on the energy equation, below.

We now turn to an equation-by-equation analysis of the parameter estimates. The output equation is not discussed here. We are concerned not with its parameters, per se, but with the implied properties for the steady-state supply system. These are described in section 4.9.

Capacity utilization

The parameters that proximately determine the way demand shocks are absorbed through changes in inventories and capacity utilization are AF38 and AF43. It is perhaps not too surprising that neither is well enough determined to be statistically significantly different from zero at the usual confidence levels. Both, however, are significant at the 89% confidence level (t-ratios about 1.6). Our estimates show that, ceteris paribus, if inventories are 10% below desired levels, CAPU will be set 0.078 above where it would otherwise be. This represents about 9.3% of the steady-state value for CAPU. This strong response considerably limits the extent to which inventories will be used to buffer demand shocks. In section 4.10 we report some partial-simulation evidence on this point. We can anticipate those results, however, and report that if all factors are held fixed, then for demand shocks of about 9.5% (under initial conditions of equilibrium) there will be roughly equal contributions in the first period from inventories and from output response via CAPU variation. Both the size of the shock and the initial conditions matter in determining the exact split.

We find a significant effect from excess profits through AF14. If profits are 10% above normal, our estimate implies that CAPU will rise by about 0.022 and provide an important short-run output effect. Although we must recognize the danger of simultaneity here, our results indicate that the course of profits can significantly modify firms' response to demand cycles.

Finally, we have the cycle parameters AF36 and AF37. Although they play some role in estimation, controlling for the historical demand cycle, we do not retain these lags for the standard simulation model. We interpret the particular lag structure found as relevant for average historical experience, but not necessarily for particular simulation experiments. The joint contribution of the lags is only marginally significant, and in preliminary historical dynamic simulations we found that the model tracks inventories, CAPU, and output slightly better if we exclude the lagged gap terms.

Capital

Parameter AF12 represents the short-run speed of adjustment to a capital-stock disequilibrium caused by a change in KCD. If the capital stock is 10% above the desired level, our estimate implies that, ceteris paribus, the growth rate of capital would be reduced by about 0.013, about one-third of its average value in the sample.

The small and statistically insignificant value for AF16 tells us that when capital deviates from desired levels, firms cannot easily return to equilibrium when the source of the disturbance disappears, even if that source is a transient effect. Instead, they must adjust gradually back to equilibrium much as they respond to shocks to the desired capital stock itself. In this sense, capital is by far the most 'fixed' factor.

Parameter AF19 reflects the influence of the deviation of capital cost from its steady-state value. We cannot give this parameter a clear expected sign. From the perspective of fiscal policy we were looking for a positive sign -- indicating that when tax policy temporarily reduces the user cost investment will pick up. Indeed, we tried several specific measures of such fiscal policies in place of (or in combination with) the variable used in the reported run. In no case did we find a significant fiscal effect, and the sign was usually incorrect for the fiscal-effect interpretation. Similarly, we tried an excess-profits measure, the same variable that is used successfully in the CAPU equation, but found no significant effect on investment. The variable used for the Table 4.1 results could be given another interpretation, however. If current cost is below steady-state cost, then the signal that costs are expected to rise could capture negative forward-looking effects not adequately captured by the static model with simple partial adjustment. This is what we find, but the coefficient is very small and has a large standard error. It adds nothing to the fit and does not affect the other results. We set AF19 to zero in the standard model. Perhaps the most important information here is that any effects of short-term fiscal incentives on investment will have to be imposed in simulation. We could not find any such effects in estimation. Recall, however, that permanent changes in capital cost induced by fiscal policy will have a powerful effect on investment through KCD.

Parameter AF20 represents the effect of product-price disequilibrium. Our estimate suggests that if the output price is high relative to the sustainable equilibrium, firms will reduce investment. This could be interpreted as a response to expectations of lower profitability. The effect is statistically significant.

Parameter AF34 provides a direct short-run response to excess product demand -- an accelerator mechanism. The effect is large and highly significant in the estimation. For real shocks, this effect and the price effect through AF20 will tend to work in opposite directions, with the accelerator effect dominant. For monetary shocks, however, these influences will tend to reinforce each other. Finally, we have the disequilibrium TOBQ effect via AF56. This is small and only marginally statistically significant, but it provides the model with an extra dimension of short-run real/financial linkage. Note that a monetary shock will both raise the equilibrium price level and generate a positive equity-valuation effect. The combined impact on investment gives the model a short-run link between money and real activity, independent of the direct accelerator mechanism.

Energy

The estimated speed of adjustment, AF23, is quite low, but energy has such a small weight in the technology that even large persistent energy-use disequilibria do not seem to cause much difficulty in terms of output disequilibria or other distortions. Nevertheless, we feel that this is one coefficient we might well adjust using the STIME variable in simulations.[27] Clearly, the estimated model treats the response to OPEC energy-price changes as indicative of a long, slow adjustment process. It is important to remember, however, that these results are based on a PENRSS that lies above the observed real price throughout the OPEC period. That may be a reasonable ex post view, but there was considerable uncertainty at the time as to whether OPEC would survive and what domestic pricing policy would be. If we based our model on 'expected' steady-state prices we could introduce a temporary but important difference to the energy price signal, raise ENCD, and probably raise the estimated speed of adjustment. Our results may indicate slow response to the correct ex post price signals because firms were aiming at different targets owing to temporary expectations errors. If this is true we would obtain misleading results in simulation for other types of shocks affecting ENCD and for correctly perceived changes in PENRSS. For the time being we have retained the estimated coefficient for the standard core model.

When the disequilibrium is caused by factors other than a change in ENCD, adjustment to equilibrium will be much more rapid. Our estimate of AF13 indicates that roughly 30% of disturbances in response to transitory effects can be immediately reversed. This may understate the effect for correctly perceived, short-run disturbances. It is likely that response differs, depending on the profile of the shock. The disequilibrium adjustment to temporary but long-lasting shocks would likely resemble permanent change and lead to slower adjustment to full equilibrium. However, very short-lived shocks that are correctly perceived (e.g., a one-period increase in demand) would lead to different types of disequilibrium response that are easier to reverse. Our model cannot capture this distinction in estimation unless we try to identify specific details about the nature of the shocks in the historical data. We prefer, at this point, to estimate average responses and to change the coefficients as seems appropriate in particular simulations.

---

[27] See section 4.5.5 for a discussion of the use of STIME to change the longer-term properties of factor-adjustment processes.

The estimated response of energy to inventory disequilibrium, AF26, seems extraordinarily high. We can offer no insight as to why this result is obtained. Although the standard error of the parameter estimate is not small, even the low end of a normal confidence interval would leave a powerful link between inventories and energy use. Since the output elasticity of energy is low, however, the feedback to inventories through output would be small even with our estimated coefficient.

The disequilibrium energy-price term, with parameter AF29, has the anticipated sign, but is not well determined.

Finally, for energy, we have the link to CAPU. The estimated coefficient, AF33, shows a reasonable and significant link. When combined with the somewhat stronger link to labour usage, this gives the model a very clear and powerful set of factor responses to demand shocks through CAPU. We show in partial simulations (section 4.10) that these effects are strong enough to cause the model to generate cyclical approaches to steady-state paths.

Labour

The point estimate of the speed of adjustment to the labour gap, AF45, is 0.26, the highest such coefficient. This is large enough for us to say that labour responds fairly rapidly to changes in desired values. We also find a substantial value for AF51, indicating that labour can be used to react to temporary disturbances with considerable flexibility for rapid reversal when those disturbances disappear. Both results suggest that labour is the most 'variable' factor. The interpretation of AF51 is influenced by the same arguments presented above for AF13 in the energy equation; namely, that it represents average response to historical cycles and is probably too low for transitory shocks and too high for long-lasting disequilibria.

The disequilibrium wage term (parameter AF50) provides a significant result. This gives the model more labour-demand sensitivity to wage variations in disequilibrium. If wages decline owing to deficient product and labour demands, then relatively more labour will be employed, limiting the rise in unemployment.

The inventory response for labour, AF48, is not well determined. It is much smaller than the parallel coefficient for energy, but because labour has a much greater weight in the technology, the feedback effects are considerably greater than those from the energy equation. From our simulations we conclude that AF48 is, if anything, too high. The model as estimated tends to produce overshooting in response to demand shocks. This is almost inevitable in models with such stock-flow linkages.

We also find a powerful link between demand cycles and the labour market from AF52 and the CAPU gap. This effect is not only quantitatively strong but also seems quite well determined, with an asymptotic t-ratio of over 5. The combined effects of the CAPU gaps and the inventory gaps in the labour- and energy-use equations provide the model with a complex system of short-run adjustments, and strong links between aggregate demand and output via the variable factors.

The final coefficient in Table 4.1, AF57, does not belong solely with the labour equation -- it affects all the equations. It is the coefficient that determines the level of WREADJ, the historical adjustment to the level of the computed equilibrium wage (see section 4.4.4). The effect of the adjustment at the end of the sample is to reduce the computed equilibrium real wage by about 5%.[28]

4.9 The Technology and Static Properties of the Supply System

The parameters in the first block of Table 4.1 describe the technology. They are all quite well determined. The individual parameter estimates need no detailed discussion, but we are concerned with some of the implied properties of the steady-state model.

---

[28] Refinements to the model subsequent to this estimation, combined with the correction of some errors in construction of the data, have reduced the size of the WREADJ adjustment. We have not re-estimated the entire supply system. It is possible that in re-estimation we will be able to eliminate the WREADJ term.

Parameter AF07 determines the elasticity of substitution of labour for both capital and energy (see equation 4.2). The implied point estimate is 0.820 with an approximate 95% confidence interval (0.77, 0.87). This is significantly different from the Cobb-Douglas value of 1.0 in both the statistical and economic senses.

The elasticity of substitution of capital for energy depends on the parameters AF06 and AF07 and on the expenditure share of the bundle (see equation 4.3). It therefore varies over the sample. The mean of the calculated values is 0.631, with a range of 0.599 to 0.642. No trend is evident over the sample. Evaluated at the mean of the expenditure share of the K/E bundle, the approximate 95% confidence interval for the estimated elasticity of substitution is (0.56, 0.70). These results are striking in two ways. First, they are quite stable. The large relative factor-price changes have not led to any appreciable change in the estimates of the elasticity of substitution. Neither is there evidence of any other source of systematic variation. Second, our estimates of substitutability are high relative to many other estimates. In the literature there has been debate about whether or not capital and energy are substitutes at all; some researchers have even found complementarity. We find relatively high substitutability. In all previous versions of SAM we have found substitutability, but never an elasticity above 0.3. It seems that the formal introduction of the capacity utilization transform in the current version substantially affects the results. Recall that CAPU declines markedly at the same time that the big relative energy-price changes are experienced. Apparently, this conjuncture permits the estimator to interpret history somewhat differently than is the case without the CAPU effect. If we exclude the CAPU effect from the model, the estimator must confront the fact that factor proportions do not change very much, despite the large increases in energy prices; the conclusion is that there is little substitutability. When CAPU is integrated into the model, however, and the data for the 1970s are interpreted to reflect, in part, a decline in CAPUSS, then the cost of an efficiency unit of capital rises (see section 4.4.3) at the same time as energy costs (i.e., after OPEC). The resulting decline in the size of the relative price change enables the estimator to conclude that there is more substitutability. Although this explanation is speculative, in that we have not searched exhaustively for alternatives, the relatively high estimate of substitutability was striking enough to lead us to try various reformulations of our model and of the way data are defined. The elasticity result seems robust and we can suggest no other explanation. It is worth noting, however, that the Hicks-Allen elasticity of substitution we report does not describe a feasible trade-off. If there is a change in the relative price of energy and capital, there are necessarily changes in the steady-state wage, labour supply, and potential output. When these are taken into account, the effective substitutability of capital and energy is considerably lower. We provide some numerical evidence on the point later in this section.

It is often useful to know how much effect on output a given change in a factor will induce, ceteris paribus. Such information is helpful, for example, when looking at output implications of factor-market disequilibria. Our estimates provide the following output elasticities for capital, energy, and labour, respectively: 0.295, 0.063, .642. These figures are the means of quite stable time series for each elasticity.

In Table 4.3 we report mean values of the factor-price elasticities (output and other prices fixed) of the equilibrium factor-demand equations. The largest temporal variations are observed for the energy-price elasticities of capital and labour. The KCD response to

Table 4.3

FACTOR-PRICE ELASTICITIES OF FACTOR DEMANDS
(Mean values, 1959-81)

	WRESS	PENRSS	CCRESS
LCD	-0.295	0.053	0.243
ENCD	0.525	-0.711	0.187
KCD	0.525	0.041	-0.565

PENRSS *rises* over the OPEC period from 0.039 to 0.046. The LCD response over the same period also rises from 0.051 to 0.059. The other values, including the own- price elasticity of ENCD, are very stable. Note that of all the factors energy use is the most sensitive to its own price and labour use the least sensitive.

The elasticities in Table 4.3 provide useful information, but they suppress the model's restriction on relative steady-state factor prices. To analyze effective sector properties, it is necessary to recognize that link. In Table 4.4 we provide elasticities of response of various endogenous variables to changes in the steady-state real energy price, PENRSS, and the steady-state capital cost, CCRESS.[29] These elasticities are computed holding the output price and all demand influences fixed, but particular, changes in the steady-state wage, human wealth, labour supply, and potential output are considered.[30]

Table 4.4

ELASTICITIES OF RESPONSE, EQUILIBRIUM SUPPLY
(Mean values, 1959-81)

	PENRSS	CCRESS
KCD	-0.116	-1.284
ENCD	-0.867	-0.533
LSS	-0.007	-0.030
UGPCSS	-0.094	-0.434
WRESS	-0.118	-0.543

---

[29] There is no sense in which the real wage is more endogenous than other real factor prices. Yet there is a restriction on the relationship among real factor prices for full equilibrium, embodied in our WRESS equation. Because it is often necessary to consider shocks coming from world energy prices or world real interest rates (or capital cost), it is useful to know the partial supply elasticities with respect to changes in PENRSS and CCRESS.

[30] When human wealth changes (because of the change to WRESS), there will be a change in the equilibrium price level (assuming a given money stock) that will alter the real value of nominal assets and financial wealth. This small effect, which would reinforce the human-wealth effect on labour supply, is ignored in the calculations.

First we consider the wage and labour supply responses. The equilibrium real efficiency wage falls if either PENRSS or CCRESS increases; the elasticities are -0.12 and -0.54, respectively. Evidently, wages are much more sensitive to a change in capital cost than to a change in the price of energy. The elasticity of response of the steady-state labour supply with respect to WRESS, taking into account the effect of the change in WRESS on human wealth, is only 0.055. This limits the labour supply response to changes in PENRSS and CCRESS; the elasticities are -0.007 and -0.030, respectively. To summarize, if either of the other factor prices changes, the real wage moves in the opposite direction, but the resulting change in the steady-state labour supply is quite small.

Next we consider the capital and energy responses. The reader can contrast the results in Table 4.4 with those in Table 4.3 to see that the output effects dominate the direct price effects. For example, we see from Table 4.3 that, if output and other prices (including the wage) could remain fixed, then the result of a higher PENRSS would be lower energy use and higher labour and capital use. But both WRESS and UGPCSS will decline if PENRSS rises. When we take these changes into account, the net result (shown in Table 4.4) is lower capital and labour use. The output effect dominates. The elasticities of response of output are -0.09 and -0.43 for PENRSS and CCRESS, respectively. It is also worth noting the very much stronger KCD response to its own price in Table 4.4. The wage response to a capital-cost change is fairly strong. As a result there is a powerful output effect that reinforces the pure price effect shown in Table 4.3, more than doubling the elasticity of response. Capital cost matters a lot in SAM.

Technical Progress

The trend rate of labour-embodied technical progress is represented by parameters AF09 and AF11. From AF09 we see that for the period up to 1973 we have a trend rate of productivity growth of about 2.1% per annum. The point estimates suggest that this trend growth rate falls to about 1.6% per annum after 1973, a value just above our assumed steady-state value of 1.3% per annum.[31] However, the decline of just under 0.5 percentage points is not statistically significant at the usual confidence levels.[32] We nevertheless retain the shift and for the future assume that the value moves gradually towards 1.3% per annum.

4.10 Sector Properties: Response to Demand Shocks

In this section we examine the response of the supply system to two types of demand shock. First, we consider a shock of about 10% to SALES that declines roughly linearly to zero over nine years. This is not an exact replication of a historically average shock, but it is much more so than the other shock considered: a 10% shock to SALES for only one period. We ignore any labour supply effects and induced factor-price changes in the analysis here, so that the demand shock must eventually completely disappear. The linearly declining shock is long-lasting and represents what might happen in the model as a permanent specific shock (e.g., a permanent shift in export demand) is eroded by the endogenous response of prices, the exchange rate, and so on. But it must be understood as a partial simulation exercise. In the full model there would be supply response and changes in relative factor prices (unless the shock was purely nominal). Our purpose here is to illustrate the relative roles of inventories, capacity utilization, and the factors in responding to a demand shock, under the partial-simulation assumption that no long-term supply response is generated.

---

[31] Although the shift is exogenous to the model we can speculate on the cause of the decline. From a long-term perspective, real growth rates were very high in the sixties. Part of this may be historically high labour productivity growth, so that in the seventies we merely returned to more normal growth. It is also no accident, in our view, that the drop appears to be closely associated with large energy-price changes. Although SAM supposedly captures normal substitution in response to relative factor-price changes, there is no reason why the rate of technical progress itself could not be affected by energy costs. However, we would not choose to emphasize this point, but rather that the large energy-price changes may have made some capital obsolete or led to 'retrofitting' investments that, in terms of the published data, gave too high a measured capital stock. One implication may be an apparent decline in labour-productivity growth that in reality is a shift down in the level of productivity.

[32] Although this is true of our final point, we did find significant shifts in several results for slightly different models. The shift was often larger as well. There seems to be some relationship between the size and significance of AF11 and the result for the elasticity of substitution of energy and capital, primarily through AF06.

In addition to contrasting the results for the two types of shock we consider evidence on the sensitivity of the results to particular aspects of the specification of behaviour. In particular, we contrast what might be called myopic response to the shocks with something closer to a perfect-foresight solution. The discussion is not meant to be comprehensive. We merely want to communicate some initial impressions of SAM's supply system and how it can be adapted to particular experiments.

4.10.1 A linearly declining shock

We begin with a simulation in which we make absolutely no changes to the parameters from estimation. Furthermore, other than eliminating the sample-specific lags in the CAPU equation, we simulate using the data definitions of estimation. In particular, we define SALN as SALES. We interpret this experiment as myopic behaviour or behaviour without information on the form of the shock. In using SALN=SALES we do not permit agents to look ahead to future states of excess demand. In this case the error is not grievous, since the shock persists for a long time. Nevertheless, there is an element of myopia in responding only to the current state of excess demand, especially given that the capital stock is slow to adjust back to equilibrium.

We present the results of the shock in three charts. Figure 4-9 shows the shock to SALES and the model's supply response. Figure 4.10 illustrates the factor responses, and Figure 4.11 shows the inventory and CAPU responses. All measures are percent shock minus control except for CAPU where we report an absolute difference. In the first year about one third of the extra demand is satisfied from inventories and two thirds from extra output, generated roughly equally by an increase in CAPU and increased factor use. In year 2 supply response increases such that more than enough extra output is generated to satisfy demand. Indeed, by the end of year 2 about half the lost inventories have been replaced. It is the average gap that affects behaviour in SAM, however, and as Figure 4-11 shows that average gap is larger in year 2 than in year 1. The process continues for two more years, by which time the inventory gap has been

Figure 4-9

AGGREGATE SUPPLY AND DEMAND
Shock Minus Control %

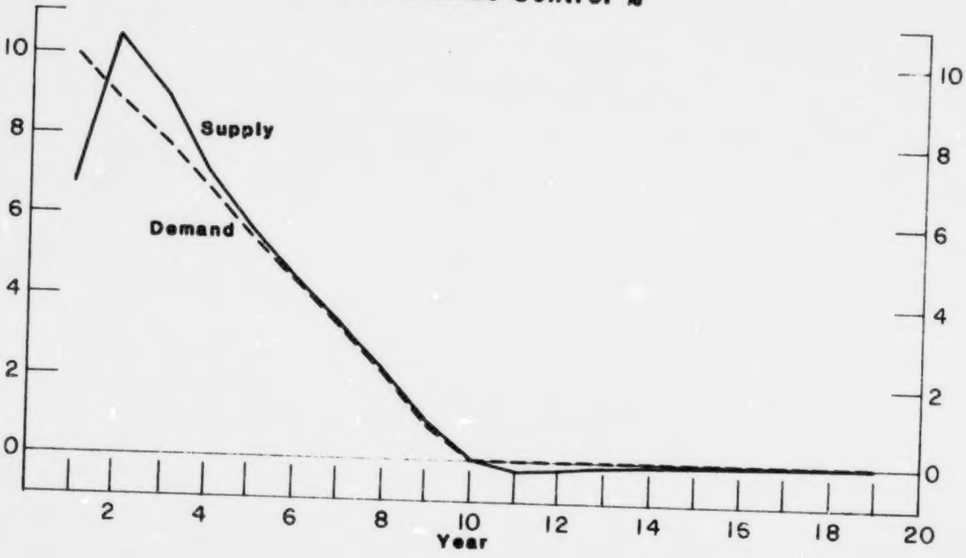


Figure 4-10

FACTOR USAGE
Shock Minus Control %

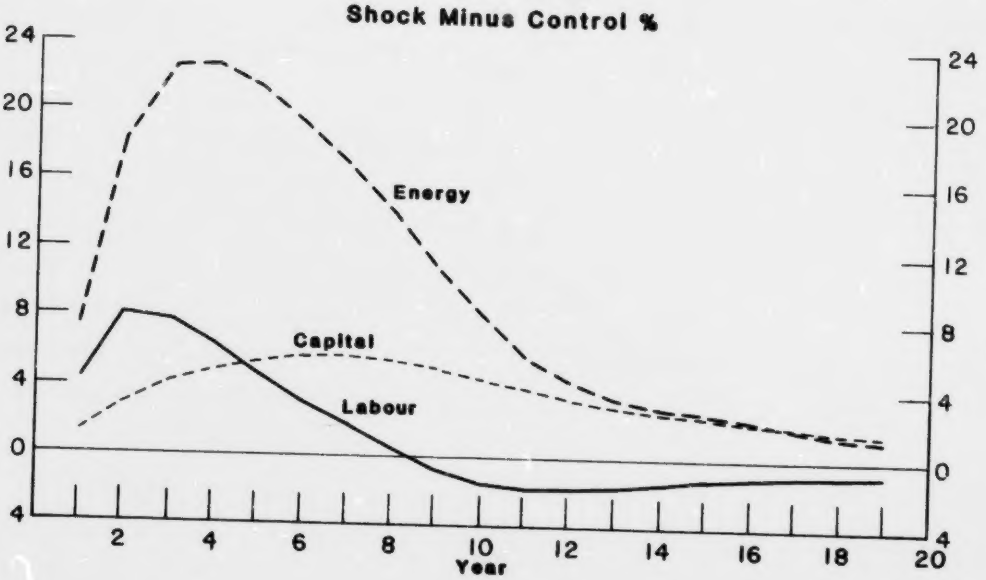
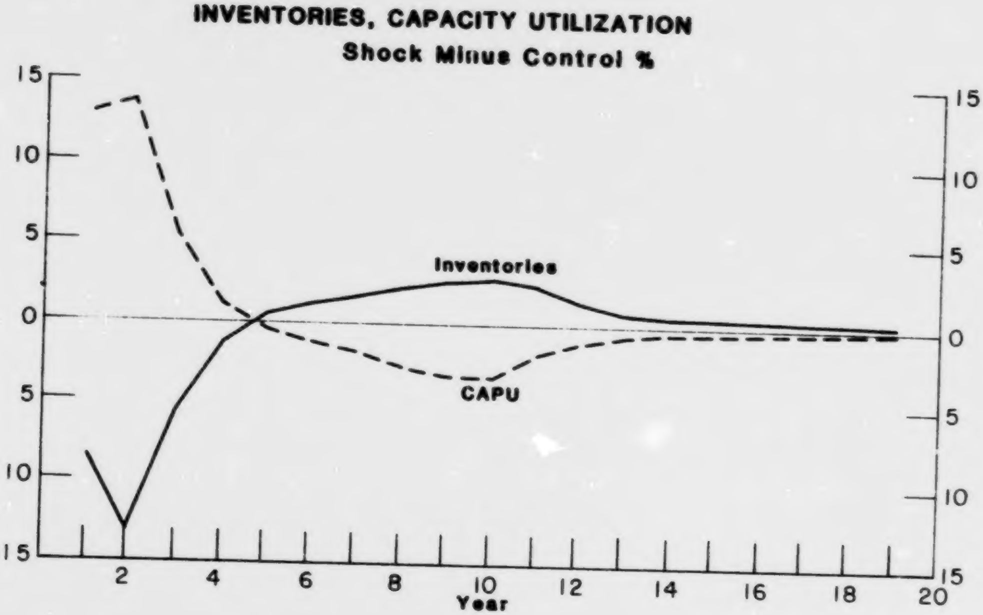


Figure 4-11



closed. All factors are still above control levels at that point, so inventories overshoot their equilibrium level. To control for the excess flow supply and to keep the overshoot to reasonable levels, firms must operate below normal capacity. At the extreme, CAPU is 0.029 below equilibrium (about 3.5%), while inventory stocks simultaneously peak at about 2.7% above control (equilibrium). Thereafter, everything adjusts towards full equilibrium. An important point is that labour demand eventually overshoots as well, reaching just under 2.0% below equilibrium before turning back up. Thus, for this version of this experiment, CAPU and eventually labour bear the brunt of the excess use of capital and energy and the overshoot of inventories.

Several aspects of these results might be construed as problematical. First, the very large energy response seems excessive. By year 4 energy use is almost 23% above control. Second, the capital stock is still moving up and away from equilibrium in year 7, even though the demand shock has almost disappeared and capital is 5.7% above control.

The energy response is due to the very large coefficient on the inventory gap, AF26, which has a relatively large standard error (just under 0.3). If we reduce AF26 by one standard error, we substantially blunt the simulated increase in energy use. We feel, however, that a more substantial reduction is required.[33] A value of about 0.4, almost two estimated standard errors below the point estimate, seems to provide acceptable results for a variety of experiments.

The capital-stock response reveals two important features of SAM common to many macro models. First, there is a significant investment response to the demand shock. It tends to be long lasting and leads to the buildup of a considerable gap with respect to the equilibrium stock. As a result, the initial boom is followed by a lengthy period of less than normal investment. Recall that our estimate of AF16, the parameter describing the amount of 'disequilibrium' investment that can be rapidly reversed, is very small. This means that the effects of temporary deviations of the capital stock from equilibrium are especially long lived. Therefore, it is very important that we fully understand the mechanism operating to create the initial capital response.

The source of the capital response is the excess flow demand itself. In the experiment above, firms respond to a gap measured as log(SALES/UGPCSS). We have called this myopic, because firms respond only to the current excess demand and ignore the fact that SALES will return to UGPCSS as the system adjusts. To perform the analysis without such myopia we turn to our simulation equation for capital adjustment (EQ45KFD, section 4.5.4) that allows for forward-looking expectations about the course of SALES, through SALN. See section 4.4.7 for a derivation of SALN. Recall that SALN is usually expressed as a linear combination of current SALES and steady-state sales, UGPCSS. For a nine-year, linearly declining shock the weight on SALES is about 0.6, considerably below the 1.0 used in the experiment above.

---

[33] Even reduced by one estimated standard error, AF26 is large enough to generate large energy response to demand shocks. Moreover, as inventories tend to overshoot, the energy response tends to become highly cyclical. We find such results inconsistent with observed historical fluctuations in energy use. Although we are not sure why, we think that the point estimate of AF26 is too high.

The following are some highlights of the results when the experiment is repeated with a lower AF26 and SALN substituted for SALES. When AF26 is reduced to 0.4, energy peaks at about 13% above control, cutting 10 percentage points from the previous result. In the first few years of the shock a bit more labour is employed and CAPU is set at slightly higher levels to compensate for the lower energy use. Both overshoot less in the later years. When the SALN measure is introduced, the biggest impact is on investment. Whereas previously the capital stock peaked at 5.7% above control, under the SALN rule it peaks at about 3.5% above control. An important implication is that again more labour must be used. Indeed, whereas in the first experiment the overshoot for labour reached almost 2% (i.e., 2% below control), with the two changes the maximum overshoot is about 1.4%.

We find these results reasonable. The use of SALN is not really a change to the simulation model. Recall (section 4.5.4) that the use of SALES was just an expedient for estimation. The experiment with SALES is meant to show how the estimated model would behave in simulation and to further emphasize the importance of forward-looking expectations. We return to this point in the next section. With a one-period shock the difference becomes more dramatic. We impose the change to AF26 on the model for simulation. We find the estimated value implausible and we much prefer the properties of the model with a lower value.

4.10.2 A one-period shock

For this experiment, we increase sales by 10% for one period and then remove the excess demand completely for the rest of the simulation. This is the type of shock for which myopic behaviour could produce strange results. With the myopic version, the initial response to the one-period shock is identical to the response if the shock is quasi-permanent. This seems doomed to generate overshooting and cycles. Indeed this is the case, although the results are not as extreme as might be expected. In Figures 4-12 to 4-17 we show either two or three curves. In each case there is a record of the variable's response to the shock using both the

Figure 4-12

SUPPLY REPONSE: ONE-PERIOD SHOCK
Shock Minus Control %

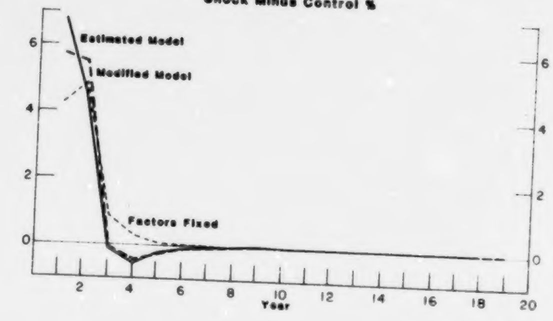


Figure 4-13

CAPACITY UTILIZATION
Shock Minus Control %

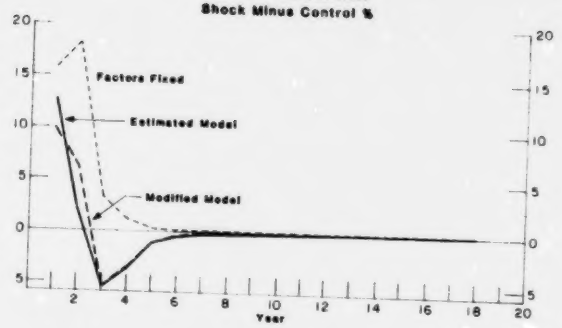


Figure 4-14

INVENTORIES
Shock Minus Control %, Year Average

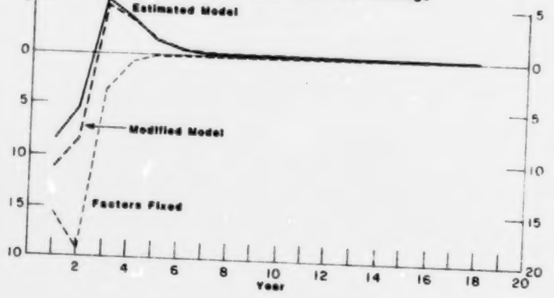


Figure 4-15

CAPITAL
Shock Minus Control %

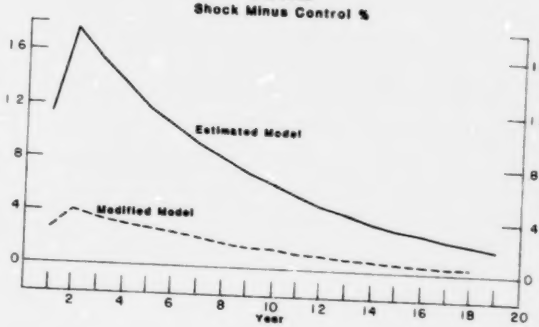


Figure 4-16

ENERGY
Shock Minus Control %

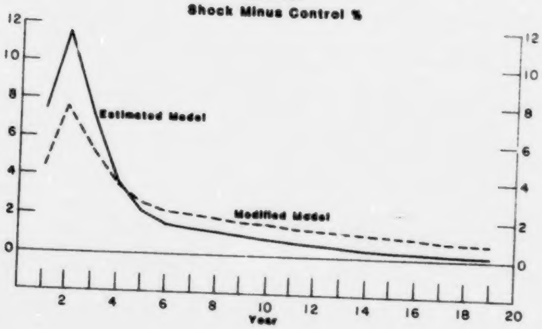
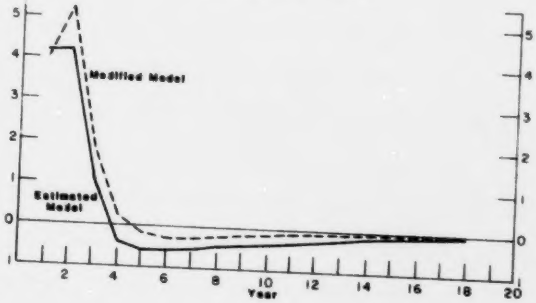


Figure 4-17

LABOUR
Shock Minus Control %



estimated model and the modified model (AF26 = 0.4 and SALN, with AF40 set to 0.22 to reflect the one-period shock). For all variables except the factors we show a third line that represents the model's response when the factors are held at control levels. There is some appeal to an argument that says that for typical shocks experienced under conditions close to equilibrium and lasting only one year there should be little or no factor response, with inventories and CAPU bearing the burden of response. We have no formal theory to settle the issue, and since the shock under consideration is not typical of historical experience it is dangerous to rely blindly on empirical evidence in making a choice. Thus, although we do not want to argue that it is correct to impose that there be no factor response to a one-period shock, we offer the results of such an approach as providing useful information about the model's operation under extreme but plausible conditions.

Figure 4-12 shows the supply response of the model under the three assumptions. The first-year response is largest under the model as estimated, where just under 70% of the 10% demand shock is satisfied from supply. With the estimated model, this and all other features of the impact effect are identical to those for the more permanent shock. Modification of the model by introducing SALN and changing AF26 lowers the initial supply response to about 5.8%. Removal of the factor variation reduces this to about 4.2%. Of course, the smaller the output response, the larger the inventory response. For the fixed-factor model, inventories fall by about 16% in the first year, roughly double the response for the estimated model. Note, however, that the CAPU response does not move monotonically in the same fashion. It is higher when the factor response is cut off, but the introduction of SALN reduces the size of the disequilibrium signal and leads to lower CAPU response on impact than for the myopic model. This reinforces the switch to reliance on inventories.

Note that in all cases the supply response shows virtually complete adjustment to the shock by year 3. There is, however, some overshooting in the experiments with factor response, whereas the approach is monotonic in the fixed-factors experiment.

We now consider the factor responses (Figures 4-15 to 4-17). With the myopic model, the capital stock peaks in year 2 at about 1.8% above control and then returns slowly to equilibrium. Introduction of the forward-looking expectations through SALN cuts the peak response to about 0.4%. For energy, mainly owing to the change to AF26, the modified model produces a peak of about 7.5% above control in the second year, down from over 11.5% in the myopic-response experiment. The modified model shows longer-lasting energy disequilibrium, however, although the differences are small (about 0.5 percentage points). The labour response is very interesting. To compensate for the lower capital and energy and the bigger initial inventory decumulation, labour demand responds more in the modified model after the first period. Although overshooting still occurs, it is very small, only about 0.25% at the extreme.

Aside from the first few periods the CAPU (Figure 4-13) and inventory (Figure 4-14) responses for the estimated and modified models are very similar. Both overshoot by virtually identical amounts. The fixed-factor experiment produces quite different results, however. The initially greater inventory decumulation must be removed by operating above equilibrium capacity. There is no overshooting and the initial disequilibria are substantially larger and longer lived. Yet return to equilibrium occurs over the same horizon, about six years.

Figure 4-14 shows the average inventory gap. In fact, by the end of year 2 inventories have overshot by about 5% in the estimated model. Despite the change to capital, inventories overshoot by as much when the information on the nature of the shock is used. Although the first- and second-year effects are different, from year 3 to the end the modifications do not change the results for inventories and CAPU. Instead, energy and labour respond to take up the slack (Figures 4-16 and 4-17). In the longer term, energy substitutes for capital; in the shorter term, labour substitutes for both.

The results for capital (Figure 4-15) are striking. It seems implausible that a one-period demand shock, whether perfectly forseen or not, would generate such a large and long-lasting disequilibrium for capital. Clearly, the results from the estimated model must be interpreted as an ex post incorrect response, based on the assumption that the shock would last longer (i.e., reflect an average historical demand cycle). This highlights the importance of the assumptions in conditioning the results of a simulation experiment. The model cannot give a clear indication of the effects of a shock unless specific assumptions are made about how the shock is interpreted by economic agents. There is no escaping this uncertainty. Expectations matter in SAM, and behaviour based on incorrect interpretation of signals can cause disequilibrium or perpetuate disequilibrium cycles.

Chapter 5

LINKS TO THE WORLD IN SAM: IMPORTS, EXPORTS, DIRECT INVESTMENT, AND THE SUPPLY OF NET FOREIGN ASSETS

5.1 Introduction: Balance of Payments Identities

5.1.1 The current account: XBAL

In SAM we aggregate in such a way that two goods are produced by the domestic private sector: energy and everything else. Both goods are traded. In addition a foreign good is imported, and this imported good is an imperfect substitute for the domestic good. As such, the imported good has a price that is notionally distinct from the domestic good price and the export good price.

The current account in SAM has three components: a trade account, a service account, and net transfers to foreigners,

```
EQ56UAI    XBAL = {PXNEID*XNEID-PMNEID*MNEID+PENW*PFX*(XEN-MEN)}
                  +{PFX*[RACUS*J2A(FHT)+RACUSG*J2A(FGBT)]
                  -RAC*J2A(LGFT)-FOPRO}-TRANSF.
```

The non-energy trade balance is a combination of exports of the non-energy domestic good, XNEID, and non-energy imports of the foreign good, MNEID, valued at PXNEID and PMNEID, respectively. Net energy exports are valued at a 'world' price, PENW, that we measure using Canadian energy-trade price indexes. Combined as the first term in {} in EQ56UAI, they define the balance of trade in SAM. This is a more general concept of trade than 'merchandise trade' in the balance-of-payments statistics, because we include travel, freight and other services as part of trade. Only the 'investment income' part of the official service account is included in SAM's service account, which is given by the second term in {} in EQ56UAI. These services consist of the payments made as a result of foreign ownership of domestic capital and the net claims of domestic governments and households on foreigners (i.e., all payments for capital services).

Foreigners own a portion of domestic capital and earn a flow of profits, FOPRO, on that capital. This constitutes a balance-of-payments outflow. FOPRO is defined inclusive of profits that stay in Canada. Only actual payments made to foreigners are recorded in the official balance-of-payments records. We count the full profit and treat any retained profits as new capital inflows.

The domestic government sector holds stocks of foreign assets, FGBT, that pay interest in U.S. dollars. These bonds bear a coupon rate, RACUSG, that is linked to the yield on U.S. bonds.[1] The domestic government receives interest on the average bond stock outstanding over the year, so RACUSG*J2A(FGBT) measures the interest flows from these bonds in U.S. dollars. The domestic government sector can be, and generally is, a net debtor in the foreign-pay asset by issuing bonds identical to the foreign bond. In this case FGBT is negative and the interest payments constitute an outflow.

The domestic government also issues a domestic-pay bond, some of which is held by foreigners. Foreign holdings of this bond are measured as a liability of the domestic government, LGFT. It is assumed that the foreigners' portfolio of these bonds has the same average maturity structure as the domestic residents' portfolio, and hence these bonds bear the average domestic coupon rate, RAC. The interest outflow on foreign holdings of domestic-pay bonds is then RAC*J2A(LGFT).

The domestic government also holds official reserves, FGRT. In fact, most such reserves (60% in January, 1985) are now held in interest-bearing forms. Despite this, if we ignore interest on official reserves we get a closer reconciliation of the data on net interest flows in government intruments with the hypothetical flows derived from our data on interest rates and stocks. Because of this, and also to avoid introducing a separate interest rate, we treat FGRT as non-interest-bearing. Therefore, although the stock changes appear in the model's capital account, there is no corresponding flow entry in the service account.

---

[1] See Chapter 6 for details on these link equations and more information on our data. Net drawings on standby credits are included as a liability in the measurement of FGBT. If the funds drawn were not spent, an offsetting entry would appear in official reserve holdings, FGRT.

Domestic private sector agents hold stocks of the foreign asset, FHT. The average coupon rate on FHT is RACUS. This rate differs from the average coupon rate applied to the domestic governments' stock of foreign assets because the maturity structures of the two portfolios are not necessarily the same. The interest flow associated with the domestic private sector's holdings of foreign assets is RACUS*J2A(FHT). Like the domestic government, domestic private sector agents can become net foreign-pay debtors by issuing a bond identical to the foreign asset.

The final term in EQ56UAI, TRANSF, represents consolidated transfers to foreigners. We could identify more precisely transfers by government and by the private sector, but to keep the accounting simple we represent only the public policy role of such transfers. Private sector transfers are treated as transfers by government. An offsetting data adjustment is made to government transfers to households so that we do not distort government financing requirements.

5.1.2 The capital account and the balance of payments

Capital outflows in SAM consist of changes in public sector net foreign assets, PFX*(J1D(FGRT)+J1D(FGBT)), changes in private sector net foreign assets, PFX*J1D(FHT), the negative of changes in the public sector's domestic-pay liabilities to foreigners, -J1D(LGFT), and the negative of net direct investment by foreigners, -PI*J1D(KWT). Both FGRT and FGBT are foreign-pay concepts and their quantities are measured in foreign currency units. Consequently, for our capital account we convert to Canadian dollars using PFX. KWT is the end-of-period stock of domestic capital to which foreigners have a claim. This claim is valued using the deflator, PI.

The total capital outflow in Canadian dollars is thus given by the expression PFX*(J1D(FHT)+J1D(FGRT)+J1D(FGBT))-J1D(LGFT)-PI*J1D(KWT). The balance-of-payments identity requires that any current account surplus (deficit) must be matched by an accumulation (decumulation) of assets.

Thus, the above capital flow must equal XBAL. We normalize the balance-of-payments identity on the accumulation of net foreign assets by the domestic private sector and write:

```
EQ57AAS   PFX*J1D(FHT) = XBAL + PI*J1D(KWT)
                         - PFX*(J1D(FGRT)+J1D(FGBT)) + J1D(LGFT).
```

The variable chosen for normalization in the identity is somewhat arbitrary. There is no implication that FHT is constrained only by EQ57AAS. Indeed, in SAM there is an asset-demand function for FHT that must be taken into account as well as direct feedback through interest flows and through PFX via XBAL. But the balance-of-payments identity must be imposed, and in SAM this identity provides the supply equation for net foreign assets.

5.1.3 External balance

In this section we extend the discussion of sections 1.3.3 and 4.4.10 to describe the role of the exchange rate in the long-run equilibrating process. Recall (from section 4.4.10) that we need one macro adjustment equation in a model to ensure aggregate demand/supply balance, and that in SAM this equation applies to the real interest rate. Recall, further, that in some models this fundamental adjustment appears in the wage or price equations. We now complete the discussion by showing how various models handle open-economy aspects of the long-run process. We add one further class of model -- those based on perfect substitutability of assets in world markets.

We begin with the perfect-substitutes case. In such models real interest rates are determined in world markets and the small open economy must accept the world real rate. Neither real-wage nor interest-rate adjustment can provide the long-run equilibrating mechanism. The interest rate mechanism is a non-starter by assumption, and the real wage mechanism (whether through the nominal wage or the price equation) is ruled out because with the fixed interest rate the full-equilibrating real wage is constrained by the technology via the zero-excess-profits condition. An obvious candidate for the fundamental adjustment process is the real exchange rate. A rise in the real price of foreign exchange will increase net exports and hence remove any deficiency in flow aggregate demand.[2] Indeed, the natural extension of section 4.4.10 is to add a fourth possible adjustment equation -- one that provides for real depreciation when there is deficient aggregate demand.[3] Although this may be sufficient to ensure flow equilibrium at a fixed level of national wealth, steady state requires a particular level of national wealth and rate of accumulation of net claims on foreigners. For full equilibrium we require that real national wealth grow at the steady-state real growth rate. This in turn requires that real net foreign assets grow at that same rate and puts a level restriction on net foreign assets and national (as opposed to domestic) wealth. If, for example, at the current level of wealth, flow equilibrium in the product market requires a surplus on current account greater than sufficient to provide equilibrium growth in net foreign assets, then national wealth will likewise grow faster (than in equilibrium) as will domestic demand and import demand. This will, in turn, change the real exchange rate consistent with flow equilibrium. Then, assuming the process is stable, a full equilibrium will be attained with a particular combination of net foreign assets and real exchange rate. Although the stock and flow equilibrium conditions are simultaneously linked, it is useful to think of the stock condition being satisfied by an adjustment of the distribution of wealth across nations and the flow condition being satisfied by adjustment of the real exchange rate.[4]

---

[2] We assume that the model satisfies generalized Marshall-Lerner conditions such that a depreciation will in fact be stabilizing. We ignore for this discussion problems of dynamic instability such as those identified by Martin and Masson (1979).

[3] Note that this makes absolute purchasing-power parity untenable, although a relative purchasing-power-parity restriction will hold (i.e., the nominal exchange rate will move continously to reflect any inflation-rate differentials, but not to maintain a particular level of relative prices).

[4] This argument can be recast in terms of income flows rather than wealth. The key distinction is between national and domestic income or wealth. As the level of national wealth adjusts so does the level of national income via changes in interest payments.

In the perfect-substitutes model there can be no restriction from preferences on the distribution of wealth across countries or steady state will not in general exist.[5] In the long run a country must accept a market-determined level of ownership of domestic wealth by foreigners. Moreover, the equilibrating response to real shocks may require substantial changes in the national wealth or the real exchange rate.

We have noted several times that interest rates have the potential to play a powerful role in the equilibrating process. Even slight relaxation of the perfect-substitutes paradigm that permits small deviations of domestic real rates from their world counterparts (i.e., high but not perfect substitutability) can have marked effects on the quantitative properties of an adjustment process. For example, small changes in the real rates can exert a powerful influence on consumption demand in SAM through revaluation of financial assets and changes in human wealth (present value of future labour and transfer income). This leverage for real interest rates means that seemingly small movements can produce quite different results for equilibrium values of real national wealth, share of trade and so on.

Any of the methods of closing a macro model reviewed in section 4.4.10 can be used if assets are not perfect substitutes internationally. Some models use a wage-adjustment process without specifying a preference restriction on net foreign assets. Some other condition must then be provided to pin down the real exchange rate. Usually this is an absolute purchasing-power-parity condition. Although such models are complete, we find them less interesting than those that determine the real exchange rate consistent with preferences of economic agents. In SAM, such preferences enter as a demand function for net foreign assets (see Chapter 6). The real exchange rate, the level of domestic real interest rates, the domestic price level, and the level of net foreign assets are all active in the equilibrating process.

---

[5] If there are preferences on the level of net foreign assets then a solution can exist in general only if government policy assures that result (e.g., a rule for government non-wage expenditures). In some models, for example multi-generation models, a solution can be found through changes of the distribution of income or wealth across the generations. An alternative is to make the marginal utility of wealth variable by making the aggregate time-preference rate a function of the level of national wealth.

5.2 The Trade Equations

5.2.1 Imports

In the simplest theories of international trade, imports are viewed as substitutes for the domestically produced good in domestic consumption. Typically, such models include a domestic income or wealth variable and the price of imports relative to domestic goods. Usually, such a model is completed with the small-open-economy assumption that import prices (in foreign currency) are independent of domestic demand. Thus, essentially unlimited quantities of imports are available at given world prices and the quantity of imports is demand determined.

Such a model can be formally derived from SAM's household preferences.[6] But in reality imports reflect the decisions of firms and governments in addition to those of households. Moreover, there is evidence that the cyclical properties of import decisions made by the various sectors are different. We therefore decided to leave the import equation free of any parametric or functional-form restrictions directly associated with the utility function.

It seems to us that the simple relative price and wealth model is an adequate equilibrium model. To add more realism to the model of imports under conditions of less than full equilibrium, however, we need to add some influence from real-side disequilibria. In this regard, we follow a long tradition in RDX models by giving imports a role in directly buffering shocks to the product market. When there is excess demand (supply) in the domestic market, agents tend to buy more (less) from foreigners than they normally would. These quantity effects can occur independently of relative price changes and can precede any such price changes in a cycle. We capture this notion through a term that measures the extent to which domestic capacity utilization differs from the normal level. This seems to work quite well, empirically.

---

[6] See Masson, et al. (1981).

The equation expressing the demand for non-energy imports has the following form:

```
EQ55UHD    log(MNEID) = AT01+AT02*J3A(log(P/(PMNEID*(1+RTAR))))
                        +AT04*log(UGPBSS)+AT03*(CAPU-CAPUSS)
                        +AT05*DUMCAR.
```

Coefficient AT02 measures the response of non-energy imports to changes in the relative price of imports. This relative price is specified as the ratio of the average price of domestic output, P, to the domestic market price of imports.[7] A value of AT02 greater than zero is necessary for a downward-sloping import-demand curve, and a value greater than unity is necessary for import revenues to move in the same direction as quantities in response to a relative price change.

The next two terms represent the domestic income and disequilibrium cycle effects. Note that our scale variable is a measure of domestic potential output and not actual income. We have separated the permanent and transitory determinants of imports into distinct terms. In steady state the CAPU gap will be zero and the level and growth of imports will be determined primarily by domestic potential. Away from steady state, however, the CAPU gap plays an important role.

We add one special factor to our simple model of imports. The changes in trade volume associated with the Automobile Pact with the United States were large and unrelated to the fundamentals of the trade model.[8] We therefore add a dummy variable designed to control for this structural change.[9]

---

[7] The import price is scaled up by the tariff rate, RTAR, to put it on a market-price basis consistent with the measure of P.

[8] It has been suggested that trade in automobiles should be modelled separately since it is sufficiently different from trade in other commodities. In SAM, however, automobiles are not singled out either in production or consumption, and hence the theoretical argument for singling them out in international trade is not compelling.

[9] DUMCAR is not a simple dummy (binary) variable. As in all RDX models it describes a structural shift over several years. It is defined to be 1.0 in 1965, 2.0 in 1966, 3.0 in 1967 and 4.0 thereafter. This seems to capture adequately the transition to higher levels of trade. Developments in automotive trade after our estimation sample may require renewed attention to this special factor.

5.2.2 Exports

Exports of the non-energy good

If we extend the discussion of a small open economy and imagine Canadian exports to be closely substitutable for foreign goods in foreign consumption, then we are led to a supply view of what determines the level of exports. In such a world a small country can sell practically unlimited quantities of domestic goods in world markets without disturbing prices. Actual sales would therefore be determined by the decisions of domestic suppliers. Practically speaking, this would mean that domestic potential output would be the appropriate scale variable in an exports equation.

If there is any significant product differentiation in world markets, or if the otherwise small country is an important supplier of particular products, then the pure supply view is not tenable. Indeed, the lower the price sensitivity of foreign demand for its products, the more its export sales will depend on the level of activity in world markets.

For good reason, most empirical models of Canadian exports have focused on the demand constraints on our sales. There is absolutely no question that demand influences play a crucial role in the short run. A pure supply model is not consistent with the data. The real question is, What determines exports in the long run? The critical experiment is, What happens if foreign and domestic real growth rates differ systematically for long periods?

Suppose the world is growing faster than is Canada. If exports are fundamentally demand determined then they will be under demand pressure to grow faster than domestic potential. In fact, foreigners cannot obtain an ever-increasing share of domestic output without convincing domestic residents either to switch to imports or to produce more, or both. In any case, the steady-state path for the domestic economy, if it exists, will be characterized by continuously changing relative prices, including the real exchange rate. Neither absolute nor relative purchasing-power parity will hold for the real exchange rate.[10] If exports are structurally supply determined, however, the differential in growth rates will not put continuing pressure on the export share and the real exchange rate can, in principle, be constant on the steady-state path.

In general, one would expect the ideal model to be one with a complete system of supply and demand functions for each good. Efforts to develop such a model for Canadian trade have proven singularly unsuccessful.[11] This is not surprising given the extreme econometric difficulties with simultaneity and high multicollinearity among the relevant variables for such a system. Most researchers have resorted to quasi-reduced forms that incorporate elements of both demand and supply. We follow this practice, but we provide two versions of the exports equation - one based more on the supply view, the other based more on the demand view. Both are hybrid models, but they differ in their long-term scale effects and in how short-term cycle effects are introduced.

The supply version is specified as follows:

```
EQ54UFS    log(XNEID) = AT11+AT12*J3A(log(PXNEID/(PFX*PW)))
                        +AT13*DUMCAR+AT16*log(UGPBSS)
                        +AT15*(log(YW/UGPBSS)-J3A(log(YW/UGPBSS))).
```

Note that the scale variable is domestic potential output. This is important both because it represents a key element of the supply interpretation of this equation and because it is an equilibrium variable. The final term in the equation represents the ratio of world income, YW, to domestic potential, relative to a moving average of this ratio. Note that even in the case of differing real growth rates for YW and UGPBSS, the final term becomes constant and influences only the level of exports and not the growth rate. Thus, in the long run exports are dominated by domestic potential output. In the short run, however, the influence of foreign demand through the final term in the equation is very important.

---

[10] If world consumers are rational then it does not make sense that this solution would characterize the steady state unless real relative price changes are costless. If there are any gains to real price stability, fully rational agents might be expected to recognize that they were not getting more goods, only moving the price, and to cease causing this. Although such a market-level rationality goes beyond usual rational-agent arguments, it is interesting that the implication is that the world would operate as if exports were structurally supply determined.

[11] E.g., Francis, 1979.

The price of exports, PXNEID, enters the function relative to the domestic price of the world good it competes with, PFX*PW. This is consistent with a demand view; it is the price ratio relevant to the foreign buyer. A pure supply equation would use the supplier's price ratio, PXNEID relative to the domestic selling price, PD. A negative value of AT12 indicates that an increase in the price of exports reduces the quantity of exports sold, ceteris paribus. A coefficient with absolute value greater than unity means that revenues and volumes move in the same direction in response to a relative price change.

The other term in the equation is the Auto Pact dummy. It has the same justification here as it does in the imports equation. The form of the dummy is identical in both equations.

The alternative model is specified as follows:

```
EQ54UAD    log(XNEID) = AT11+AT12*J3A(log(PXNEID/(PFX*PW)))
                        +AT13*DUMCAR+AT16*log(YW)
                        +AT14*(CAPU-CAPUSS).
```

This differs from EQ54UFS in only two points. The scale variable in EQ54UAD is the foreign income variable, YW, and the domestic cycle effect of the CAPU gap is added to replace the foreign income gap in EQ54UFS. Here, we would look for a negative sign for AT14. This would provide a supply influence in that when there was excess demand for domestic goods exports would be reduced by suppliers as part of their efforts to respond to the cycle.

To this point we have considered the implications of differing scale variables in the export equation. The long-run properties of the model also depend on the parameters, particularly the income elasticities, AT04 and AT16. For a constant real exchange rate in steady state we require the restriction,

$$AT04 = AT16*gx/gd = 1.0, \qquad (5.1)$$

where gx and gd are the growth rates of the scale variables in the exports and imports equations, respectively. If we use EQ54UFS then gx=gd and the restriction is simply that both elasticities be unity. If restriction (5.1) does not hold, and if the relative price terms in the trade equations are constant, then the ratios of exports and imports to domestic output will change continuously. This is not consistent with steady state unless both trade ratios approach zero.[12] Steady state with trade, but without restriction (5.1), requires that the real exchange rate change continuously so that trade grows at the same rate as domestic output.

Net exports of energy

The current version of SAM does not contain a model of domestic energy production or of energy trade. Energy prices are determined by a combination of world market price and domestic policy choice. There is no domestic market influence on the price.

The model does respect the energy-trade identity requiring that what is produced must either be used domestically, put into inventories, or (net) exported.[13] The exact identity is:

EQ50EFS UGPEN = ENC+J1D(INVENT)+XEN-MEN.

Energy inventories are exogenous to the model. Thus, since domestic use is endogenous, and since the identity determines one variable, only one variable, domestic production or net exports, can be set exogenously. The model can be run with either variable exogenous, but if energy output is not set sensibly then all fluctuation in domestic use shows up inversely in energy trade and these movements can have major effects on the current account, the exchange rate, and so on. For these reasons our usual simulation procedure is to specify an exogenous net export series and to determine domestic production from the identity.

---

[12] In a linear aggregate, such as Gross National Expenditure, the aggregate growth rate cannot be constant unless all components grow at the same rate, except in the limit where slower growing components become negligible relative to total expenditure.

[13] We can use more than we produce by importing at the world price.

5.3 Estimates of the Trade Equations

In Tables 5.1 and 5.2 we report the estimates for the two models of trade. The estimates in Table 5.1 are for the version with a domestic-supply constraint on exports. Table 5.2 contains the estimates for the alternative model with a foreign-income constraint on exports. In both tables we report the unconstrained estimates and the estimates with imposed unit elasticity with respect to the scale variables.[14] The estimates are all computed using an iterative Zellner procedure.

The two models are not nested and we have not attempted a comparison using formal non-nested models techniques. Nevertheless, we feel that certain conclusions are justified. In terms of overall fit the two models are virtually identical in both the constrained and unconstrained versions. The same is true for the individual equations. Indeed, the fitted values from the two models are virtually indistinguishable. In terms of the fitted values, the only point of distinction is a small difference in the residual-correlation properties of the export equation. The demand version appears to have slightly lower first-order correlation, but in the context of our systems estimator no formal test can be based on the Durbin-Watson statistic. We have tried various alternative samples and small changes of specification. Although the details of the results

---

[14] Over the historical sample YW has grown less than UGPBSS. If we use the average growth rates to determine restrictions as shown in equation (5.1), we would test AT04=1.0 and AT16=1.1833. The results of this procedure do not differ noteably from those reported in Tables 5.1 and 5.2. The unit restriction is imposed on AT16 because, for simulation over future periods, we generally assume that the domestic and foreign equilibrium growth rates are the same.

Table 5.1

PARAMETER ESTIMATES: TRADE EQUATIONS, SUPPLY-DETERMINED EXPORTS

Equation Variable	Coefficient	Unconstrained		Constrained	
		Point estimate	Asymptotic t-ratio	Point estimate	Asymptotic t-ratio
Imports					
Constant	AT01	-3.381	8.5	-1.640	54.7
Income	AT04	1.164	31.0	1.0	-
Rel. Price	AT02	0.582	4.1	0.375	2.2
CAPU Gap	AT03	1.103	9.3	0.948	6.5
Auto Pact	AT05	0.032	3.9	0.063	12.3
		RSQ=0.996	DW=1.76	RSQ=0.993	DW=1.26
Exports					
Constant	AT11	-0.006	0.0	-1.073	9.0
Income	AT16	0.900	19.1	1.0	-
Rel. Price	AT12	-1.069	6.0	-1.100	5.9
World Income	AT15	0.909	2.2	1.109	2.7
Auto Pact	AT13	0.068	6.4	0.054	6.5
Sample 1960-83		RSQ=0.991	DW=1.58	RSQ=0.989	DW=1.29
		Log likelihood = 97.03		Log likelihood = 89.17	

Table 5.2

PARAMETER ESTIMATES: TRADE EQUATIONS, DEMAND-DETERMINED EXPORTS

Equation Variable	Coefficient	Unconstrained		Constrained	
		Point estimate	Asymptotic t-ratio	Point estimate	Asymptotic t-ratio
Imports					
Constant	AT01	-3.371	8.5	-1.641	56.0
Income	AT04	1.163	31.1	1.0	-
Rel. Price	AT02	0.570	4.0	0.384	2.3
CAPU Gap	AT03	1.110	9.3	0.934	6.2
Auto Pact	AT05	0.032	4.0	0.063	12.4
		RSQ=0.996	DW=1.75	RSQ=0.993	DW=1.28
Exports					
Constant	AT11	0.490	0.8	1.911	15.3
Income	AT16	1.177	16.7	1.0	-
Rel. Price	AT12	-0.671	3.5	-0.781	3.9
CAPU Gap	AT14	-0.482	2.8	-0.631	3.5
Auto Pact	AT13	0.046	4.0	0.070	10.1
Sample 1960-83		RSQ=0.991	DW=1.88	RSQ=0.988	DW=1.70
		Log likelihood = 97.16		Log likelihood = 89.29	

change, we find no alteration to the basic result that the two models are identical in terms of their ability to explain the data.

The restrictions to unit income elasticities are formally rejected with both models. With both versions the restriction on the imports function contributes more to the rejection than does the restriction on the exports equation. One might expect that with increasing trade liberalization, under GATT for example, there would be a danger of upward bias in the scale coefficients. That is, over this sample there is reason to expect rising trade that is not fundamentally linked to the level of output. With the demand version we see roughly equal scale coefficients, greater than unity, consistent with this view. With the supply version, however, the estimated scale coefficient is less than unity. It might be that in the sample the estimator has difficulty separating the permanent and temporary scale effects. But, when we add independent time trends to each equation and re-estimate, the demand version is largely unaffected in terms of the other parameters and permits unit elasticity restrictions on the scale variables, whereas the supply version produces an even lower scale coefficient and still rejects the unit elasticity restrictions at the 95% confidence level (but not at the 99% confidence level). The supply version fits a bit better when independent trends are included, but despite this fact we would interpret the extra evidence as favouring the demand version of the model.

Now consider the influence of relative prices. These effects are significant in both equations in both versions of the model. Moreover, in all cases the sum of absolute price elasticities exceeds unity.[15] The supply version gives higher price responsiveness, primarily from higher estimates of the price sensitivity of exports. On this point we find the results from the supply version more appealing. They are more in keeping with the findings of other research.[16] Moreover, with higher price elasticities we will require smaller real exchange rate responses to foreign shocks, ceteris paribus. It is worth remembering that this will help prevent formal stability problems of the Martin and Masson type from emerging in simulation. Although this is not an empirical argument in favour of the supply version, it is a useful by-product of adopting that approach.

---

[15] A sum of absolute price elasticities exceeding unity is the Marshall-Lerner condition in which a depreciation of the exchange rate improves the trade balance. In our case export prices reflect both domestic and foreign price movements and a depreciation can improve the trade balance even if the sum of absolute price elasticities does not exceed one.

Next consider the disequilibrium influences. The estimates of the CAPU effect on imports are all highly significant and relatively large. In the demand version of the exports equation, domestic conditions come in strongly through the same term. Note that the signs are correct for a stabilizing influence. According to these estimates, when there is domestic excess demand imports rise and exports fall. The net effect is very powerful stabilization of demand cycles through trade buffering. In the supply version the CAPU gap does not appear in the exports equation since it played no useful statistical role, yielding a very small and insignificant coefficient. In the supply version it is foreign demand that captures the significant short-term cyclical movements. Note that the effect labelled 'world income' is the impact of the ratio of world to domestic income, relative to a moving average of this same term. It has no effect in a steady state but a powerful effect in the first few years of a foreign-demand shock. As might be expected, this term plays no useful role in the demand version since foreign demand is already included as the scale variable.

It is worth noting that in response to a positive foreign-income shock the demand version implies that export demand will rise (with the scale variable, YW), but not as much in the short run as in the long run because of the CAPU buffering term. In contrast, the supply version has a larger short-term response than long-run response, ceteris paribus.

The Auto Pact variables are significant and econometrically important in clarifying the identification of the other coefficients.

---

[16] Helliwell et al. (1982) report a sum of 1.41. Other estimates vary widely, but are generally higher than those from the demand version here. See Boothe et al, op. cit. Table C-1.

Figure 5-1

EXPORTS : ACTUAL AND FITTED VALUES
(Billions of 1971 Dollars)

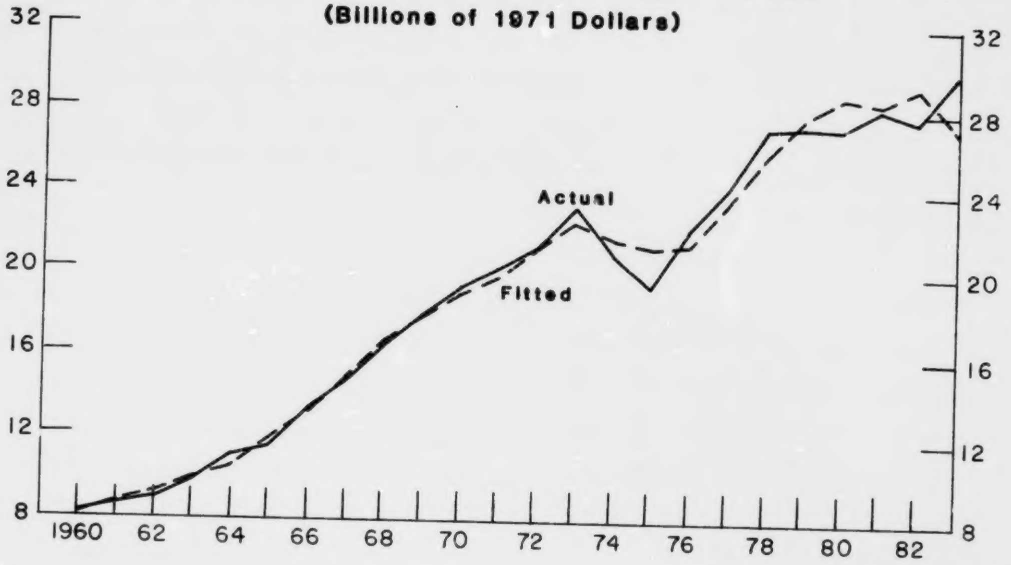
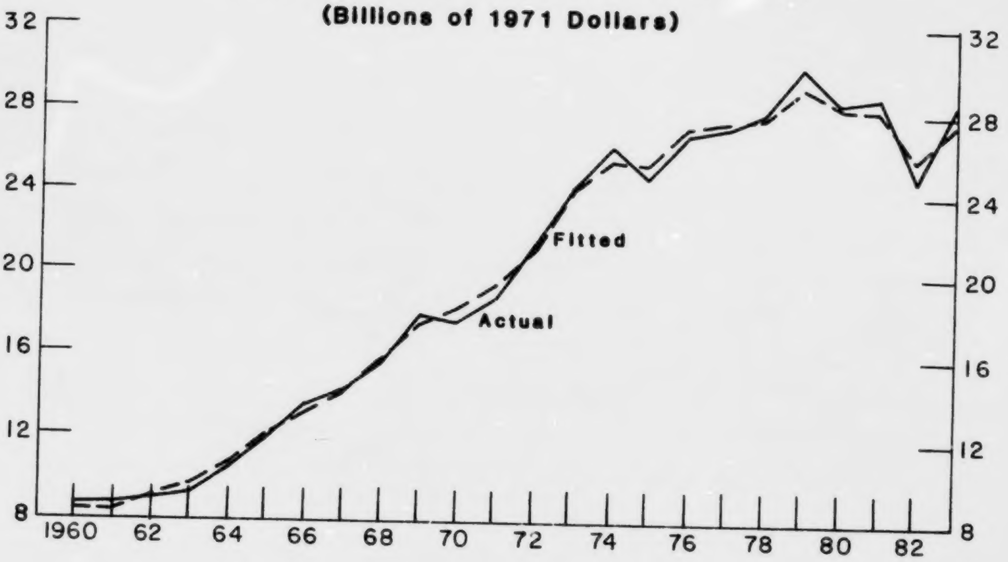


Figure 5-2

IMPORTS : ACTUAL AND FITTED VALUES
(Billions of 1971 Dollars)



We find it impossible to choose between the models on empirical grounds. We present the two versions as equally tenable structures for simulation. As we have emphasized, the long-run properties (particularly the nature of the real exchange rate solution) will be affected by the choice made; the user must consider this fact in designing particular applications of the model. The empirical case for unit income elasticities rests on the argument that there are exogenous trends in trade volume over the sample that are not truly associated with real incomes. We tend to favour this view and we find it convenient to operate with assumptions that permit constant relative prices in steady state. In simulation over future periods this can be achieved with either model. As a standard practice we impose the unit coefficient on AT04 (imports). For simulation over future periods we usually assume that the long-term growth rates of YW and UGPBSS are the same. As such, we restrict AT16 to unity with either version of the exports equation. Any of these particular choices can be altered as required in particular experiments.

The actual and fitted values of exports and imports from the demand version with unit scale elasticities are plotted in Figures 5-1 and 5-2. The equations track the historical evolution of trade reasonably well.

5.4 Foreign Investment and the Stock of Foreign-Owned Domestic Capital: KWT

Foreign investment is determined in SAM as follows:

$$\text{EQ06KAD}\qquad .01*\text{J1P(KWT)} = \text{DNUBSS}+[\underset{\substack{.0126\\(5.0)}}{\text{AS24}}+\underset{\substack{.4557\\(2.1)}}{\text{AS21}}*(\text{RRK}-\text{RRKUS}+\text{AS29})].$$

Sample 1960-77 RSQ = 0.361 DW = 1.38

The model states that the equilibrium growth in foreign-owned capital equals the domestic potential growth, DNUBSS, so that, ceteris paribus, the share of foreign-owned capital stays fixed. There is, however, an unexplained trend in the equation, AS24, which we generally assume to be sample specific and remove for simulations over future periods. The constraint to a unit coefficient on DNUBSS is not rejected at the usual confidence levels,[17] but this is not a strong result given the free constant in the equation.

The other term represents the differential return to physical capital, Canada relative to the United States, with AS29 set to the sample mean of the difference. The U.S. variable, RRKUS, is taken from work by Feldstein.[18] The relative return variable is statistically significant, but the overall explanatory power of the equation is not high.

5.5 Profits Accruing to Foreigners: FOPRO

In SAM we assume that foreigners and domestic owners of producing capital are treated equally, before tax. Both have a claim to the profit stream in proportion to their ownership share. For foreigners we can write:

```
EQ951AI   FOPRO = YBD*J2A(KWT)/(J2A(KCT+KENT+INVCT)
                  + (PEN/P)*J2A(INVENT))
```

where YBD is nominal profits and is multiplied by the average share of total capital owned by foreigners. We measure this using the ratio of averages of end-of-year stocks.

The data for FOPRO are constructed by adding to the recorded payments to foreigners an estimate of profits accruing to foreigners but retained in Canada. We treat such retention as if it were paid and reinvested. Our measure of the foreign-owned stock of capital is based on cumulation from an estimated benchmark stock using data on investment flows inclusive

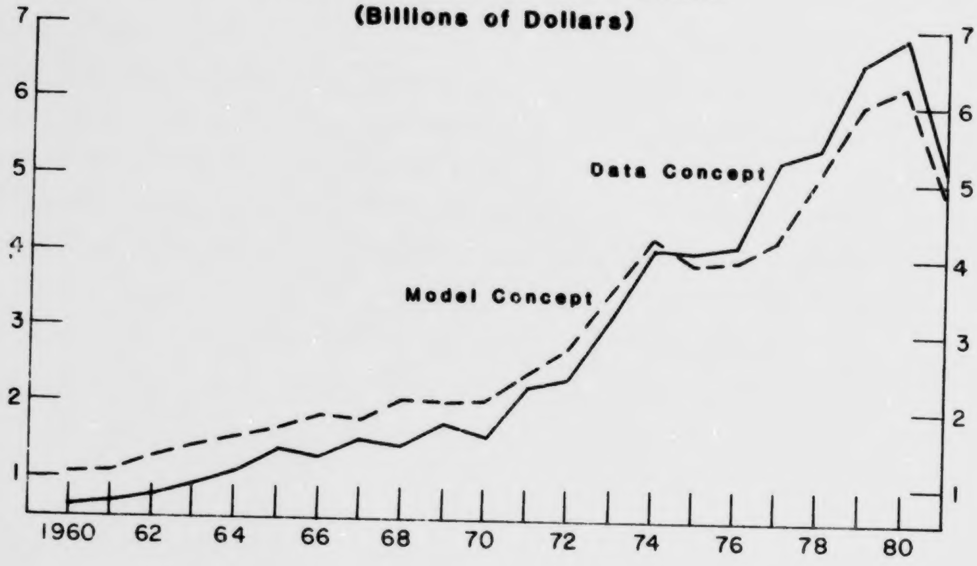
---

[17] The chi-squared value is 1.88, well below the critical value at the 95% confidence level of 3.84.

[18] See Feldstein (1982). The data are the values referred to as the real net return to capital in the article. Feldstein reports values only up to 1977, hence our shorter-than-normal sample for estimation. For data after 1978 and for simulation over future periods we link to the measure of the foreign cost of capital maintained in the SAM data base.

Figure 5-3

PROFITS PAID TO FOREIGNERS
(Billions of Dollars)



of retained earnings accruing to foreigners. Because we have independent measures of all variables in EQ95IAI the identity does not hold exactly in the data sense. In Figure 5-3 we plot the data for FOPRO and the values generated from EQ95IAI. We generate numbers that are too large in the 1960s and generally too small in the 1970s. We follow our usual procedure and define FOPRO data for the simultaneous model from identity EQ95IAI, but retain the direct measure in the data base and compute the historical balancing item for use in the block of recursive transforms of model variables to a national accounts (or more generally, direct measure) basis.

5.6 Relative Trade Prices

In a formal two-good model, with perfect competition, the export price and the domestic-good price must move together. In the abstract model these prices would be the same. In the real-world data, however, the trade bundle is not the same as the domestic-production bundle. As a result, a measure of average export prices will not be the same as a measure of domestic selling prices. Moreover, there is no reason for their relationship to be stable over time. We can define relative price, PRELX, such that:

$$\mathrm{PXNEID} = \mathrm{PRELX} * \mathrm{PD}. \tag{5.2}$$

where PD is the domestic price and PXNEID is the export price in domestic currency.

Similarly, a formal competitive two-good model requires that the import price equal the world price (in foreign currency) multiplied by the exchange rate. In the data, however, this simple we will not hold. We can define a relative price, PRELM, such that:

EQ89UMI $$\mathrm{PMNEID} = \mathrm{PRELM} * \mathrm{PFX} * \mathrm{PW}.$$

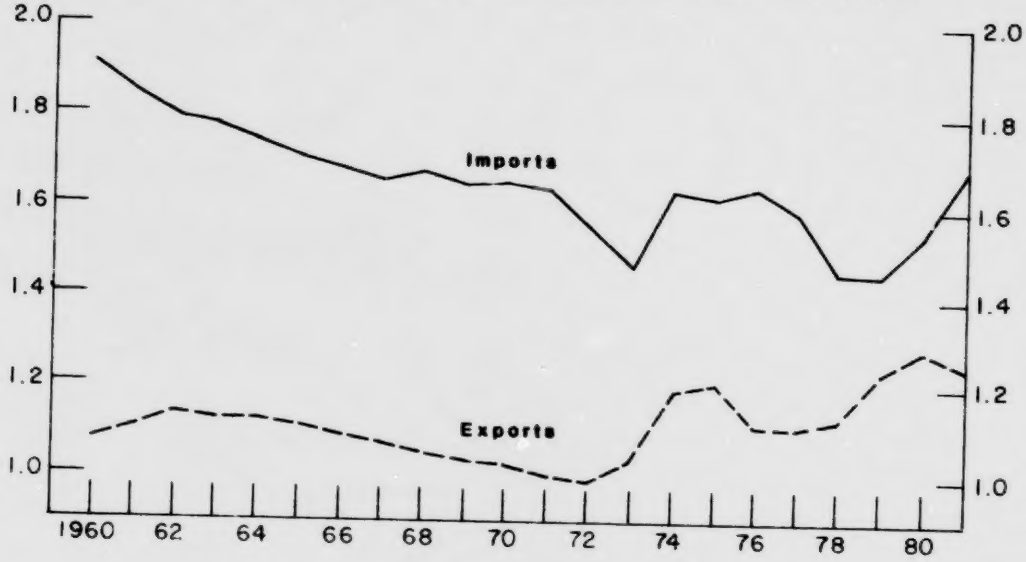
where PW is the world price index and PMNEID is the domestic-currency import price index.

It is useful to think of these equations as decomposing the trade prices into nominal and real components -- any change in a PREL representing a real relative price change. In Figure 5-4 the historical values of PRELX and PRELM are plotted. Clearly, changes in the real components of trade prices have occurred through time.

If the PREL variables were left exogenous, then we would be imposing the simple two-good competitive paradigm on the model's simulation properties. Export prices in domestic currency would move in proportion to changes in the domestic price of the notionally identical good. Similarly, import prices would move in proportion to changes in the domestic equivalent of the world price. After considering the PREL data, however, we concluded that it would be more realistic to permit some relative-trade-price movement in the adjustment process.

Figure 5-4

REAL RELATIVE PRICES OF IMPORTS AND EXPORTS



Theoretically, the idea of modelling export prices has some appeal. In the two-good paradigm of SAM the world good and the export good are conceptually distinct, but the world good and the import good are conceptually the same. Therefore, the 'law of one price' applies conceptually to imports. Canada is a small country and must buy at a world-determined price (unless one appeals to arguments such as imperfect arbitrage or price discrimination that are alien to SAM's depiction of long-run market processes). Conversely, since we sell a conceptually distinct good there can be some price-setting power.

For all these reasons we decided to maintain an exogenous PRELM, to maintain EQ89UMI as a model identity for import prices, and to represent relative trade price movements with a stochastic equation for PXNEID.

The model of relative-export-price movements has two components. First, the relative price of exports depends on the real exchange rate: a real depreciation (that is, a depreciation of the exchange rate with the world price level and the domestic price level unchanged) will cause the price of exports to increase relative to the price of output in the domestic market, a response that tends to blunt the export-volume response to the real depreciation. This relative price effect is implemented by specifying that the price of exports is, in part, a weighted average of the price of domestic output in the domestic market, PD, and the price of world output in domestic currency units, PFX*PW. In effect, it is as if the price of exports were determined partially in world markets. The second determinant of the relative export price is the ratio of commodity prices to the world price index.[19] To measure this effect we use <u>The Economist</u>'s World Commodity Price Index, called PCW2 in SAM. The argument for including this term is that, in fact, many Canadian exports are commodities, and hence special consideration must be given to movements in their relative price, in order that the other parameters be appropriately estimated. Both the commodity price index and the world price index are exogenous in SAM. We specify the equation as follows:

```
EQ90UMP   J1P(PXNEID) = (1-AP76)*J1P(PFX*PW)
                        +AP76*J1P(PD)+AP77*J1P(PCW2/PW).
                        .5954        .1262
                        (3.4)        (2.5)

   Sample      1961-81      RSQ = 0.688      DW = 1.57
```

Both coefficients are significantly different from zero, and AP76 is significantly different from 1.0. About 60% of the export price is associated with the price of the domestically produced good, PD.[20] The actual and fitted values for this equation are displayed in Figure 5-5.

---

[19] In considering why the relative price of exports has moved over the sample we examined three additional types of argument: that the relative price of exports would rise when there was domestic excess demand (as part of the rationing system foreigners would be charged more), that the relative price of exports would rise when the relative price of commodities rose and that the relative price of exports would rise when the current account had generated a cumulative surplus such that the stock of net foreign assets was above the long-run desired level. All three ideas have some empirical support, but we decided to retain only the relative price of commodities as an additional influence in the model of relative export prices.

[20] P, the average revenue of firms, is a weighted combination of the domestic price PD, and the export price, PXNEID. See Chapter 7.

In Figure 5-6 we show the actual current account balance and our fitted value. The series labelled 'fitted' is the series we obtain using fitted values for exports, imports and the price of exports. The current account is one of the 'residual' series that modellers have always found difficult to replicate. Our model does quite well at tracking the historical data.

5.7 Sector Properties

We now consider the trade account response to a 10% depreciation of the exchange rate in which the import price fully reflects the change and hence also rises by 10%. It is assumed that the domestic price level and the world price level are unaffected by the depreciation — the shock changes the real exchange rate. This is a very limited partial experiment. The dynamics of response through prices, output, and feedback through the exchange rate are all suppressed. Not even the asset-accumulation consequences of changes in the current account are included.[21] The only dynamic aspects of the experiments are those that arise from the averages in the trade equations themselves. To allow these to work themselves out we introduce the shock in 1980 and look at the results in 1983.

The volume of net exports of energy is assumed unaffected by the shock. Its Canadian dollar value thus increases by 10%, or $531 million at 1983 volume. Transfers to foreigners, profits accruing to foreigners, and payments of interest to foreigners on Canadian dollar debts are assumed fixed.

The exact results depend on two things: the values of the exogenous variables described above, and the trade-equation dynamics from the gradual adjustment to a relative price change. Recall that the relative price terms appear in the form of a three-period average. Because of this fact, the trade equations, in and of themselves, exhibit J-curve properties. Under all formulations, the trade account initially

---

[21] Simulations that include some dynamic features of the model and the trade equations are presented in Chapter 6.

Figure 5-5

PRICE OF EXPORTS : ACTUAL AND FITTED VALUES

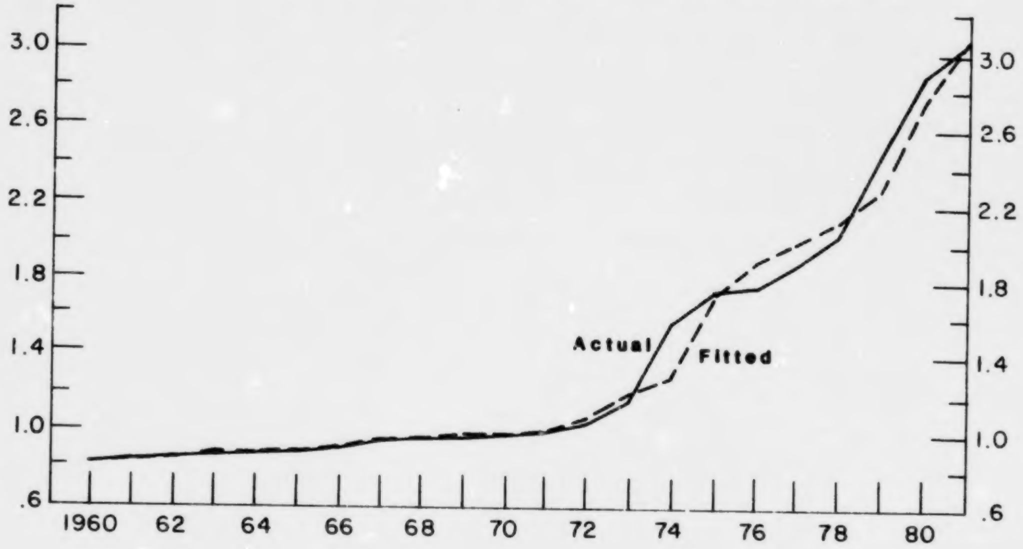
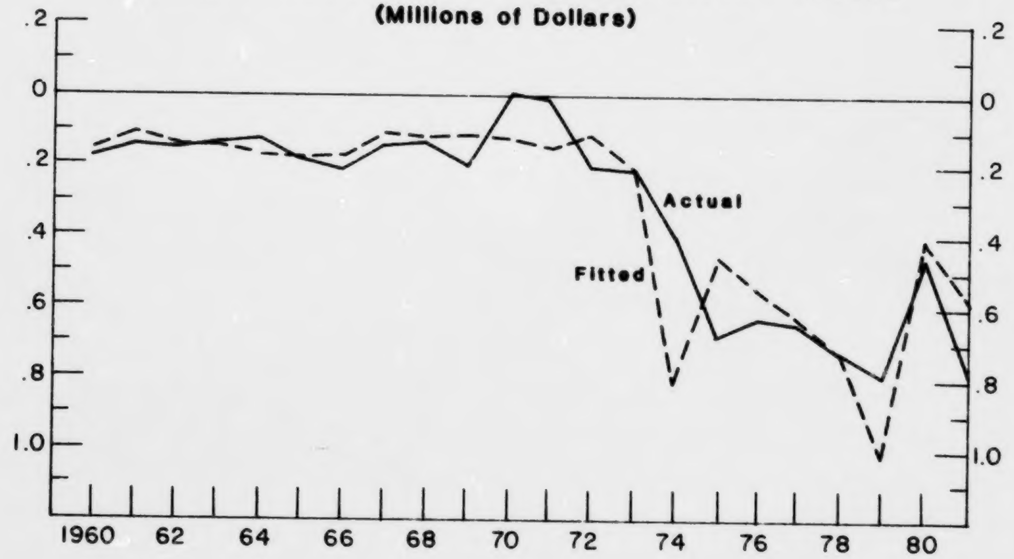


Figure 5-6

CURRENT ACCOUNT : ACTUAL AND FITTED VALUES

(Millions of Dollars)



deteriorates. For the demand versions the deterioration is about $4 billion, whereas for the supply version it is about $3 billion. As the volume response builds, however, the trade account improves. In year 2 the supply version generates a net movement towards surplus. In year 3 the full effects are observed. The demand version yields a move in the surplus direction of $2.5 to $3 billion, about half the $5 to $6 billion generated by the supply version. The versions with unit scale elasticities produce the smaller figures in these ranges. Note that it is the version (i.e., demand vs. supply) and not the parametric restrictions that generate distinctly different answers.

Chapter 6

ASSET MARKETS AND THE DETERMINATION OF RATES OF RETURN, ASSET PRICES, AND THE EXCHANGE RATE

6.1 Introduction

In this chapter we complete the description of the household sector by providing the asset-demand equations that determine the allocation of household wealth among claims on the other three sectors: firms, governments, and foreigners. We also describe the sectoral budget constraints or financing requirements that provide the asset-supply equations, and how the demands and supplies come together in the proximate determination of interest rates and asset prices, including the exchange rate.

It is important to stress the 'proximate' nature of the solutions we discuss. The asset system is part of a simultaneous macro model and cannot be considered an isolated group of equations that determine a subset of variables. There are important links from the real side of the model to the asset system through real interest rates, the level of real wealth (and savings flows), and the price level. The reader is referred to the general discussion of these matters in Chapter 1. For this chapter we largely ignore the overall workings of the model and concentrate on the structure of the asset system and variables proximately determined in asset markets.

The asset-demand system specified here is intended for medium-to long-term analysis. All asset markets are presumed to clear in each period. No adjustment processes are introduced. The money-demand function that holds in the short run comes from the portfolio system. It does not explain well the short-run correlations between money, interest rates and income. In the longer run, however, the real-balance preferences described in Chapter 3 ensure that the model has plausible properties. This system is not intended for short-run analysis or forecasting. For such work, we have developed a version with disequilibrium in all asset markets and in the process have introduced a more standard short-run money-demand function.

We begin the chapter with a review of our accounting conventions and data measures. This is followed, in section 6.2, with an overview of the entire asset system. Then, in section 6.3, we describe the valuation equations that link asset prices and rates of return, and we introduce the concept of expected holding-period yield used in the asset-demand equations. Full details of SAM's asset-demand system, including the econometrics of estimation and the results, are provided in section 6.4. Finally, in section 6.5, we consider the operation of asset markets using partial simulation experiments.

6.1.1 Accounting Conventions and Data Measures

In SAM domestic residents can hold their wealth in four forms: money, other claims on the domestic government sector (bonds), claims on the domestic capital stock (equities), and net claims on foreigners. Foreigners are assumed to hold no domestic money, so currency substitution is not an issue. Foreigners do hold domestic government bonds[1] and claims on the domestic capital stock.[2] All other private sector transactions are consolidated in the net foreign asset position of domestic residents.

In the following review we establish our notation and describe our data. The details of the supply equations are provided in section 6.2.1.

Money: H (average), HT (end-of-period)

There is no explicit banking sector in SAM. Although this precludes study of the money supply process, we do not feel that it is contentious to argue that the central bank can, if it so chooses, control monetary aggregates over the short to medium term. Moreover, in medium- to long-term analysis, a banking sector is probably an unnecessary complication in a small model: most fundamental monetary policy questions can be posed in a non-trivial fashion in the simplified model. We therefore aggregate financial firms with other private sector producers. As a result, the relevant concept of money is high-powered, outside, or base money.[3] The stock of money, like all stocks in SAM, is represented as a year-end value, HT. Where the conceptual model requires an average stock, we use the average of start-of-year and end-of-year values, H = J2A(HT).

---

[1] In the real world most such claims are denominated in a foreign currency, mainly U.S. dollars. For the SAM data we convert all issues in other foreign currencies to U.S. dollar equivalents and treat them as if they were U.S. dollar debt. We also identify in the SAM data the Canadian dollar government debt purchased by foreigners. We do not, however, explicitly model the preferences of foreigners for debt in different currency denominations.

[2] In the data we consider only the 'direct investment' portion of such claims, leaving other portfolio transactions in the 'net foreign asset' category. Conceptually, however, such investment is considered to include both direct and equity market investment.

Government debt: LGT, LGDT, LGFT, FGT, FGBT, FGRT

We consolidate all levels of government into one sector in SAM. Government corporations and public utilities are considered part of the private sector, following standard national accounting conventions. Government debt held by the Bank of Canada and in government accounts, including the Canada and Quebec pension plan accounts, has been eliminated from gross measures of the public debt.

We account for both domestic-pay and foreign-pay government financial instruments. FGT is the end-of-year net stock of government sector foreign currency assets, denominated in U.S. net dollars. It consists of official reserves of foreign currency assets, FGRT, and other foreign currency assets, FGBT. FGBT is negative throughout the sample, indicating cumulated foreign currency borrowing by domestic governments, and is assumed to pay interest at the U.S. bond rate.[4]

It is not possible to obtain a timely direct measure of purchases of domestic-pay government debt by foreigners. We estimate the purchases of such debt by assuming that all foreign currency government debt transactions are with foreigners. We can then measure purchases by foreigners of domestic-pay debt by deducting foreign currency purchases from the data on total purchases of domestic government debt by foreigners. This method will understate domestic currency purchases by foreigners to the extent that domestic residents purchase foreign currency issues of the domestic governments. In any case, this is a small category, and as noted above we do not model the currency preferences of foreigners. The distinction between domestic-pay and foreign-pay debt held by foreigners allows us to model more accurately the interest-flow implications of a change in the exchange rate, and facilitates future development of the model of international financial flows. When our estimate of foreign-held, domestic currency government debt, LGFT, is deducted from total domestic currency government debt, LGT, the amount of government debt held by domestic residents, LGDT, is obtained. It is this measure that is included in the domestic household sector's financial wealth.

---

[3] We have not adjusted the data for changes in required reserves. As noted in Chapter 3, however, we allow for some such effects through shifts of the demand function.

[4] Our measure of the foreign bond rate, RUS, is the constant maturity, 5-year bond yield, B54413, from the Bank of Canada Review. This same rate is used for all foreign currency contracts in the model. To be precise, we do not assume that all outstanding debt pays the current bond rate. Rather, we specify an equation for the average coupon rate that is endogenously updated as new debt is issued and old debt retired.

Equities: QEQT, PEQ, VEQ

SAM records two kinds of domestic capital: energy sector capital and the rest, including the housing stock. For simplicity, we treat all claims to capital as 'equity' claims. Essentially, direct ownership and bond or other fixed-income claims to capital are consolidated with the true equity claims into an amalgam we call equities. The variable QEQT is an index of the number of equities outstanding, end-of-year, based on an index of the market value of equity claims, PEQ, and a measure of the market value of domestically owned capital, VEQ. To derive these measures we use Toronto Stock Exchange price/earnings data to value the earnings stream of the corporate sector. To this we add an estimate of the value of unincorporated business and farm capital, based on a similarly motivated capitalization of earnings flows, as well as the book value of fixed-income claims on firms. Finally, we add an estimate of the value of the housing stock, using an average selling price index to value the constant-dollar stock. From this we deduct an estimate of the capital owned directly by foreigners to get the domestically owned measure, VEQ. The values of PEQ and QEQT are derived from the identity that price times quantity equals value and the flow-financing requirements of firms (see equation EQ63AFS in section 6.2.1), with a normalization rule that sets PEQ to unity in 1971.

Net foreign assets of households: FHT

Our measure of changes in the book value of net foreign assets of the household sector comes from the balance-of-payments identity. This was discussed fully in Chapter 5, but we repeat it here for convenience.

EQ57AAS J1D(FHT) = [XBAL+PI*J1D(KWT) + J1D(LGFT)]/PFX - J1D(FGT).

We determine the data for FHT from this model identity, using a benchmark stock estimate derived from the financial flow accounts.

6.1.2 Valuation of assets

Conceptually, SAM deals with market values of financial instruments. Unfortunately, most of the available data are book values, and hence it is necessary to estimate market values. Here we provide an overview; for details see section 6.3.

For foreign currency assets, valuation has two components: the exchange rate and the foreign currency value of the asset. We estimate the foreign currency market value using a price derived from movements in the foreign interest rate. Given the 'net' nature of these assets it is difficult to provide an historically accurate valuation. In simulation, however, our price measure does move appropriately with the foreign (i.e., U.S.) interest rate.

For domestic government debt we have taken considerable care to derive a reasonable measure of market valuation. For the yield on this debt we combine separate direct measures of current yields to maturity on federal treasury bills, savings bonds, other federal debt, and consolidated provincial and municipal domestic debt. Similarly, we construct a measure of the average 'coupon' rate for the aggregate bond stock. In some cases we have direct measures of the coupon rates of the components; in others we construct our own measures from interest-flows data. The market value and average price of outstanding debt are derived from the yield and coupon-rate data using the present value of the hypothetical bond's after-tax flow payments and redemption value (with an estimated average term to maturity).

The market valuation of assets is an important feature of SAM. Aside from money, which has unit value by definition, all asset prices are determined endogenously, and it is the market value of financial wealth that influences consumption and labour supply decisions. The valuation equations are described in section 6.3. Together with the asset-demand and supply functions, they proximately determine the complete set of asset prices and rates of return.

6.2 An Overview of the Asset System

In SAM we model the complete system of assets that define household sector net wealth. In this we differ from the RDX family of models since we include claims to the real capital stock in the asset system. We have shown (in Chapter 3) how decisions concerning household consumption and labour supply depend on a comprehensive measure of wealth that contains both human wealth and financial wealth, including claims to the capital stock. We have also shown how human wealth represents the present value of labour income and transfer income. Here we complete the household sector accounts and show how financial wealth represents the present value of interest and profit income. The system is complete. All forms of income result in a measure of wealth. All forms of non-human wealth are valued endogenously in the asset system.

6.2.1 Asset supplies

Much of the material on asset-supply functions has been provided in previous chapters. It is useful, however, to bring it together in a comprehensive review.

The version of SAM described in this volume specifies an independent central bank that sets the base money supply. There is no feedback from endogenous variables of the system to the monetary decision. The model can be used with more general reaction functions and with other specifications of monetary targets or instruments. Our choice of an exogenous money supply is simply the default specification of the model.

The supply of government bonds is determined, at least in the short run, by the government financing requirement -- the excess of expenditures over tax revenues must be financed by bond sales. Some such net new issues are purchased by the monetary authority; this provides the mechanism whereby the money supply grows over time. The rest of the deficit must be financed through bond sales to domestic households or to foreigners.[5] Of course, there is no logical necessity for bond sales to provide the effective residual source of finance. The government financing requirement could be satisfied with an exogenous bond supply by making taxes the residual component. While there may be more or less appropriate answers as to what behaviour best describes the financing of government spending in particular periods of history, in the context of simulation analysis there is no right or wrong specification. The user must ultimately determine what questions are to be posed and what assumptions are to be maintained about things like the rules for financing government spending. SAM can handle a variety of assumptions equally well. In particular, SAM can simulate with either taxes or bond sales chosen as the long-run residual source of financing. See Chapter 2 for a more detailed discussion and for our standard simulation rules.

The supply system for domestic currency government debt consists of the government financing constraint and a split of new issues into purchases by foreigners and by domestic residents. The overall net new supply of Canadian dollar debt is given by the financing requirement:

EQ42AGS $J1D(LGT) = GFR - J1D(HT) + PFX*J1D(FGT),$

---

[5] In SAM firms do not hold government debt. Such debt is considered held directly by households. The market value of firms is correspondingly adjusted in the measurement of VEQ.

where GFR is the flow-financing requirement, G-T (consolidated expenditures and taxes, respectively), where HT is the money stock (end-of-period) and where FGT is government sector net foreign assets. LGT is the total stock of Canadian dollar government liabilities held by domestic residents, LGDT, and by foreigners, LGFT. The supply to domestic residents is given by:

EQ43AGS $LGDT = LGT - LGFT.$

We sometimes use a simulation rule that implies that a given portion of new issues is purchased by foreigners.[6] Our default specification, however, leaves LGFT exogenous.

The stock FGT is measured as an asset in SAM, reflecting the part of it that represents official foreign reserves. We call this part FGRT; it is presumed to be non-interest bearing.[7] The rest of FGT represents government borrowing from foreigners in the form of foreign currency bond issues. Such borrowing is recorded as a negative asset, FGBT. The components are linked by the identity:

$$FGT = FGBT + FGRT. \tag{6.1}$$

Interest liabilities arise on FGBT and it is this measure that appears in the calculation of interest payments by government, GTIN, and in the current account, XBAL. However, for the asset-supply equations, in particular the accumulation of foreign assets by households, it is the change in the total, FGT, that matters.

---

[6] The portion of new issues taken by foreigners has remained fairly stable over our sample, although year-by-year there has been considerable fluctuation. Over 1961-81, the average ratio of foreign acquisitions to total new issues is 0.0888. The simple proportionality rule explains about 26% of the variation in J1D(LGFT). The fit is severely reduced by one major outlier in 1978. Over 1976-81, the average ratio is just slightly higher, about 0.0998.

[7] See Chapter 5, section 5.1.1, for our reasons and for further details.

Although the supply of total government sector liabilities is determined by the financing requirement, the split between domestic currency and foreign currency issues is not. In specific simulations we adopt rules appropriate to the experiment. In this volume, however, we treat FGT, FGBT, and FGRT as exogenous variables. Note that it is variation in FGT that provides the mechanism in the model whereby the authorities can 'intervene' in the foreign exchange market. Government financial transactions affect the net foreign asset position of the household sector and through this the exchange rate.

The supply of equities is determined by the investment financing requirements of firms. Firms choose how much capital to put in place. The purchase of this capital must be financed either from profits not distributed to equity-holders, or by selling new equity claims. In the simplest case, where we assume that all profits are distributed, we can write

$$\begin{aligned}\text{EQ63AFS}\quad \text{J1D(QEQT)} = {} & \{\text{PI}*[\text{J1D(KCT+KENT)} - \text{J1D(KWT)}] \\ & + \text{P}*\text{J1D(INVCT)} + \text{PEN}*\text{J1D(INVENT)}\}/\text{PEQ},\end{aligned}$$

where QEQT is the quantity of equity claims at year-end, and PEQ is the average value of such claims over the year. Note that we consolidate the energy and non-energy sector issues and net out direct investment by foreigners to obtain the issues to domestic residents that form part of household sector wealth. Note, further, that inventory accumulation is part of capital formation and must be financed.

The supply of net foreign assets is determined from the balance of payments. Any surplus on the current account results in an accumulation of claims on foreigners. Similarly, any purchases by foreigners of claims to domestic capital generate capital inflows. Finally, any sales of bonds by domestic governments to foreigners, whether in Canadian dollar issues

(J1D(LGFT)) or foreign currency issues (-J1D(FGT)), result in capital inflows. Thus, the model identity that determines the accumulation of net foreign assets by the household sector is the same as that reported above as a data definition:

EQ57AAS J1D(FHT) = [XBAL + PI*J1D(KWT) + J1D(LGFT)]/PFX - J1D(FGT).

6.2:2 Asset demands

With one exception, the asset-demand system in SAM posits that the desired allocation of financial wealth across asset categories depends on the relative real rates of return on the various assets. A typical asset demand has the simple form

$$P_iQ_i/V = A_{io} + \sum_{j\neq i} A_{ij}\,(R_i - R_j), \tag{6.2}$$

where P_i and Q_i are the price and quantity of asset i, R_j is the real after-tax rate of return on any asset j, and V is the total value of the portfolio:

$$V = \sum_i P_iQ_i. \tag{6.3}$$

The price of money is unity and its rate of return is zero in nominal terms or the negative of the inflation rate in real terms. All other assets have a market price and a real return determined in the system. Prices and rates are linked behaviourally through valuation rules that relate the price of each asset to the income stream to which that asset provides a claim -- using present-value equations. These are described in detail in section 6.3.

The asset demands represent stochastic behavioural allocation rules, subject to random errors of optimization. But, given the allocative nature of the portfolio (the shares must sum to unity), the system is singular and subject to various adding-up restrictions. Given that

$$1.0 = \sum_i P_i Q_i / V = \sum_i A_{io} + \sum_i \sum_{j \neq i} A_{ij} (R_i - R_j) + \sum_i \varepsilon_i$$

$$= \sum_i [A_{io} + R_i \sum_{j \neq i} (A_{ij} - A_{ji})] + \sum_i \varepsilon_i$$

must hold for all possible rates and random errors, ε_i, it follows that

$$\sum_i A_{io} = 1.0, \quad \sum_{j \neq i} (A_{ij} - A_{ji}) = 0, \text{ all } i \ . \qquad (6.4)$$

In addition we have the exact singularity result, $\sum_i \varepsilon_i = 0$.

We also impose symmetry restrictions on the system,

$$A_{ji} = A_{ij}, \quad \text{all } i, j. \qquad (6.5)$$

These symmetry restrictions are not implied by the adding up of the system. Neither are they necessary consequences of preferences, unlike the somewhat similar symmetry of substitution effects in goods-demand systems. Indeed, such restrictions are valid in asset-demand systems only for particular preference functions. Nevertheless, given the high collinearity of returns, symmetry restrictions provide extremely useful identifying restrictions in estimation, without significantly limiting the generality of the system for macro analysis.

Given that the real return on money is the negative of the inflation rate, the differential of any of the other rates with that of money is simply the nominal rate of return on that asset:

$$R_i - R_H = R_i - (\text{-INFLATION RATE}) = \text{nominal rate on asset } i.$$

As such, except for the money equation, each demand function in the system above contains a series of real (or nominal) rate differentials plus an own-nominal-rate term. The money-demand function contains the negative of each of the own-nominal-rate terms.

At this point we must mention the one exception to the general functional form of equation (6.2): a small change in the bond equation. It is reasonable to suppose that when real interest rates increase there will be a general substitution away from money towards the other assets. If the interest increase is purely nominal, however, it is less clear that the three competing assets are equivalent. When there is inflation, equity prices and the exchange rate will tend to move with the general price level. Even unanticipated changes in inflation will eventually pass into these asset prices so that there is no long-run loss of real value. Bond prices do not move with domestic prices over time. They return to par regardless of the level of interest rates and inflation. Unanticipated increases in inflation, to the extent that they pass into nominal rates, create capital losses for bond-holders. To the extent that a higher level of inflation is associated with a perception of greater 'riskiness' one would expect bond demand to respond differently to increases in expected real as opposed to nominal rates. To reflect this idea, we add one change to the function for bond demand, making the own-rate term a real rate, and adding the expected rate of inflation as a separate determinant. This allows the pattern of substitutability to depend on whether we are considering real or nominal changes in rates. Of course, under the restriction that the coefficients on the real rate and expected inflation terms are equal, the extension degenerates to equation (6.2).[8]

Given the singularity of the system, only three of the four asset-demand functions are independent. Given the asset supplies, the wealth identity and any three asset-demand functions combine with the three asset-price/rate-of-return link equations to determine three asset prices, three rates of return, and the market value of financial wealth. We specify that all asset markets clear in each period, in the sense that demands and supplies are always equated. Thus, we have markets that are technically 'efficient'. There are no adjustment processes that permit solutions off the demand functions. But such solutions need not have long-run, steady-state properties in every period. Recall, for example, that there are long-run, real-balance preferences that will be satisfied only when the system is in full equilibrium. Similarly, when an asset market 'clears' in a particular period, that means only that there are no rigidities in asset prices or interest rates that prevent a market-clearing solution. Such a solution may provide only a temporary equilibrium. For example, if the rate of return to equity is forced up, relative to world values, an adjustment process will begin whereby international arbitrage through long-term changes in foreign ownership will provide a force moving the 'solution' through time towards a full steady state. In SAM such processes do not occur quickly and so short-term, market-clearing solutions can be quite different from long-term, steady-state solutions.

Exchange rate

An important part of any open-economy model is the approach it takes to the determination of the price of foreign exchange. It is important to distinguish between the nominal exchange rate, PFX, and the real exchange rate (adjusting for international price-level differentials), PFX*PUS/P in SAM. It is the real exchange rate that determines a country's relative competitiveness in world product markets.

---

[8] We also tested for the possibility that unanticipated inflation can explain some of the movements in asset values. The effect did not appear to be quantitatively important and we have not retained it.

In some models, an absolute purchasing-power-parity (PPP) restriction determines the exchange rate in the long run. In such models, small countries are viewed as selling a good that is perfectly substitutable for foreign goods in world markets - unlimited quantities can be sold at the going price. The law of one price then holds such that the exchange rate times the domestic price is fixed at the world value and the real exchange rate is constant (unity without loss of generality). The empirical case against absolute PPP is strong. We do not find this surprising since we view the conditions under which it would be expected to hold as unrealistic. We prefer to consider products as being at least somewhat differentiated and our ability to sell in world markets as being price sensitive on the margin. More important, however, is that the absolute PPP view ignores the asset dimension. There is nothing in this view that says the current account must balance. As such, net foreign assets are free to accumulate or decumulate without limit without influencing the exchange rate. It is our view that the real exchange rate is an important relative price that does respond to market forces -- both real and financial. This does not mean that PPP plays no role. On the contrary, a relative PPP condition -- that the nominal exchange rate will move to offset any ongoing inflation-rate differentials -- is a sensible long-run property for a model of the exchange rate. But such a property is realized only when the real exchange rate has attained an equilibrium level.

Another approach, sometimes called the 'Keynesian' model, concentrates on goods and capital flows and gives the current account special emphasis in the analysis of exchange rate determination. Trade is sensitive to the exchange rate and income (and possibly other variables), and capital flows to the interest rate differential (domestic, relative to the world). Thus, there is a locus of exchange rate and interest rate differentials that, for a given level of income, will be consistent with a balance between the current and capital accounts. An exogenous move to surplus on the current account will generally cause an appreciation of the exchange rate and/or lower domestic interest rates. This is contrary to much of recent experience with exchange rates. For example, a strong U.S. dollar has been associated with a movement towards a current account deficit. More important in our view, is that the 'flows' approach, like the absolute PPP model, ignores the long-run implications of continuously changing asset stocks. If a current account deficit is financed by foreign borrowing and capital inflows, or a surplus is 'financed' by accumulating claims against foreigners, then although the exchange market may temporarily clear, it is doubtful, in our view, that such flows and asset accumulation/decumulation could continue forever.[9] It is here that an 'asset' approach can provide the necessary missing link.

One special case of the asset approach -- the perfect substitutes model -- rejects the 'flows' approach to the capital account and (for a small open economy) provides a theory of the real exchange rate that focuses on the requirements of equilibrium in the domestic product market. This approach shares with the flows model the implication that asset stock accumulation does not matter except inasmuch as overall wealth is concerned. In particular, a country can sell unlimited quantities of bonds to foreigners without being forced to pay a premium on the margin in the form of higher interest rates. We do not find this notion appealing and therefore do not impose perfect substitutability on asset preferences.

For Canada, there is considerable evidence in favour of a perfect-substitutes perspective.[10] Moreover, evidence of portfolio-balance effects has not been strong for other countries. Nevertheless, we expect such evidence to begin to emerge as researchers consider fully the data associated with the movement towards current account deficits in the United States, and movements in the relative real interest rates of various countries over the last few years. We provide some evidence in favour of the portfolio-balance approach from our results below.

It is difficult to design a test for the existence of portfolio-balance effects. Nothing in the portfolio-balance model says that stocks of assets have to stay relatively noise-free or cycle-free over time. Moreover, there is a presumption in the application of 'life cycle' notions to the current account that a country will want to borrow extensively in some stages of its development. In other words the desired share of foreign assets in net wealth may be far from the constant embodied in our simple model. It is therefore difficult to specify an empirical test that constitutes a confirmation or rejection of the approach. If the demand curve is shifting, owing to a life-cycle desire to borrow more, for example, then changes in the net asset position can occur without necessitating exchange rate or interest rate response. Perhaps more important, if the authorities are pursuing targets defined in terms of interest rates or exchange rates then there will be significant simultaneous equations bias if one looks for quantity effects in rate equations -- bias that cannot be handled with single-equation techniques such as two-stage least squares. In the limit, if the authorities fix a rate, no evidence on demand preferences will be identifiable from the data.

---

[9] This is not to say that the current account must be zero in full equilibrium in a portfolio-balance model. With growth there will generally be a non-zero solution. Moreover, we do not mean to imply that Keynesian or PPP models necessarily fail to converge to full equilibrium. As long as the national/domestic income consequences of net foreign asset accumulation are recognized there can be a solution. See Chapter 5, section 5.1.3.

[10] See, for example, Boothe et al., op. cit.

For SAM we have chosen a portfolio-balance approach to assets, including net foreign assets. Among the asset prices in the system is the 'price' of net foreign assets. This price has two components: the price of the asset in foreign currency, and the exchange rate at which the foreign value is converted to Canadian dollars. The foreign currency price of such assets is assumed to be independent of Canadian variables -- we are too small to influence world capital-market equilibria. The variable proximately determined by the asset system is the price of foreign exchange, PFX.

Our version of the asset model is limited. We specify a system in which only the preferences of domestic agents affect the real exchange rate. It is clear that to have a complete picture one must take a world view and specify a system of demand equations for a distribution of assets across currencies for each country and derive equilibrium exchange rates from such a generalized system. For the kinds of counterfactual simulations we normally perform, however, the absence of the world part of the general model may not be a serious deficiency.

It is important to note that in a general simultaneous system no variable is determined by any one equation. We have emphasized the links between PFX and the real side of the model. There are also other asset-system equations that must be considered. Of particular significance is the link between the exchange rate and the expected return to holding foreign assets. The rate of return to a Canadian holding a foreign security is the foreign currency rate of return plus any exchange gains. Similarly, the expected return is directly influenced by expected exchange rate movements. This provides an extremely powerful link between the exchange rate and the rate of return that determines the level of desired holdings through the asset-demand function. When a model of expectations about future spot rates is added, the subsystem becomes complete and proximately determines the actual and expected exchange rates and the expected return on foreign assets. The subsystem is simultaneous; no single structural equation can be thought of as the exchange rate equation. But the whole system rests on the asset view and, in particular, the portfolio-balance view of how preferences impinge on market solutions for the exchange rate.

We do not estimate a complete demand and supply system, but we do use a full-information, maximum-likelihood technique on the complete demand system, including the present-value identities that link asset prices and expected rates of return. We find some supporting evidence for our portfolio-balance model, but although such confirmation is reassuring, we think that the supply simultaneity problem is important. In our view, the nature of international asset preferences remains an interesting empirical question. The limiting case of perfect substitutability is easily imposed on SAM as a simulation rule for all assets or for a subset of assets.

Asset substitutability in SAM: implications for interest rates

We have established that, in SAM, assets are not perfect substitutes. But Canada's net foreign assets are in a form identical to those held by foreigners (e.g., U.S. government bonds) and have identical yields in foreign currency. Hence, there are strong links between (pre-tax) foreign rates and domestic rates. Formally, the expected return to a Canadian holding a foreign bond differs from the foreign yield only by expected exchange rate movements. Ceteris paribus, a rise in foreign interest rates will increase the expected yield on net foreign assets. The degree to which other domestic interest rates will be affected depends on how substitutable domestic bonds are for foreign assets. Our empirical evidence suggests that these assets are relatively good substitutes, but not close enough for the two rates of return to be treated as identical. This is important for the analysis of things like the impact of deficit spending by government in the long run. In a world of perfect substitutes a government can sell unlimited quantitites of bonds in world markets without influencing the domestic interest rate. The extra bonds are taken by domestic residents and foreigners as perfect substitutes for other instruments. In SAM, although there is enough substitutability to permit large shocks and continuous deficit financing without dramatically changing interest differentials, long-run deficit financing implies long-run increases in domestic rates relative to foreign rates. See section 6.5 for simulation evidence on this point.

The relative substitutability of bonds and equities in domestic portfolios determines the stability of relative rates of return on domestic assets in simulation. Our empirical evidence suggests relatively low substitutability. Given the erratic historical movements in profit rates and equity rates of return, however, and the difficulty of measuring adequately ex ante expected returns, it is hard to have confidence in this empirical result, especially as a long-run property. Is it plausible that rates of return on government bonds could rise by large amounts without leading savers to reallocate their portfolios out of equities? Our answer would be no. We think that as a long-run proposition relatively high substitutability makes sense. Moreover, it is the fundamental real rate of return to capital, the marginal productivity of capital in the simplest case, that determines an economy's ability to pay real interest. In the long run, interest payments on government debt must be paid by taxation, and it is hard to find stable solutions in systems where real interest rates are significantly at odds with the real ability and willingness to pay as determined, by and large, by the real growth potential of the economy. For stable growth with full-asset equilibrium the real interest rate must generate savings sufficient to 'finance' the required real investment. In an open economy there will be times when foreigners will be willing to provide any residually required savings. It seems unreasonable to suppose that such borrowing can continue indefinitely. We thus find it both intuitively appealing and convenient to suppose that in the long run the real interest rate on bonds is strongly linked to the real productivity of capital, and that there is high substitutability in asset markets that prevents real bond rates from permanently diverging very far from the real return on capital. Of course, it is a matter of degree. As we have already said, substitution is not so high as to permit governments to engage in permanent deficit financing without real-rate consequences.

6.3 Expected Holding-Period Yields, Asset Prices, and Inflation

6.3.1 Money and expected rates of inflation

Money is an unusual asset because of its role as the unit of account (its price is normalized at unity) and because it earns a zero nominal return. When there is inflation money has a negative real return - the negative of the expected rate of inflation over the holding period.

There is no unique expected rate of inflation valid for all horizons, except possibly in conditions of steady state. For each point in the future there will be a price level and a rate of change of that price, and one can imagine there being a sequence or continuum of point expectations about these values. To define an expected rate of inflation one must either implicitly or explicitly specify something about the presumed horizon. The loss of real purchasing power of any dollar held over any horizon is given by the integral of the inflation sequence over that horizon. Therefore the agent cares about, and forms expectations about, the average rate of inflation over the horizon relevant to the decision being made. In the case of assets that horizon is the holding period. The agent observes the current state of the cycle, including the current rate of inflation. The agent knows, however, that in steady state there will be a rate of inflation given by the money-growth rate less the real growth rate of output.[11] The longer the horizon, the more weight would be given to steady-state conditions by rational agents.

In SAM we provide a very simple way to introduce flexibility with respect to forward-looking expectations. Let DNPS be the long-run expected rate of inflation (the rate expected to prevail when all price-level adjustment and other disequilibria are resolved). Let DNP be the current rate of inflation (or a short-run forecast). It can be shown, under various assumptions about the path from DNP to DNPS, that the effective average expected inflation rate over a decision period can be written as:

$$\mathrm{DNPE} = \alpha\mathrm{DNP} + (1-\alpha)\mathrm{DNPS}, \tag{6.6}$$

where the weights depend on the horizon, the expected speed of adjustment, and the relative discounting of the future in current behaviour. For example, suppose we specify that time preference and the distribution of horizons result in exponentially declining weight being given to the more distant future. Suppose further that we assume an exponential process of adjustment to DNPS. We then have

$$\mathrm{DNPE} = \int_0^\infty \theta e^{-\theta\tau}\{\mathrm{DNPS} + (\mathrm{DNP}-\mathrm{DNPS})e^{-g\tau}\}d\tau$$

$$= \frac{\theta}{\theta+g}\,\mathrm{DNP} + \frac{g}{\theta+g}\,\mathrm{DNPS}.$$

---

[11] Recall that this exact result assumes unit wealth elasticity in the long-run, real-balance-preference function. In our standard simulation rules, agents are not presumed to know the exact long-run behaviour of the monetary authority. Rather, they learn about monetary policy by observing recent behaviour. It is easy to vary this assumption to put more or less foresight into simulations.

Many specific sets of assumptions result in forward-looking expectations written as such linear combinations of current and expected steady-state values. Other specific examples are provided in Chapter 4 in the analysis of expected sales. Although the specific definitions of the weights vary with the assumptions made about horizon and adjustment speed, it is generally true that the longer the horizon and the faster the adjustment to steady state, the more weight will be given to steady-state conditions in computing the average. In the example above, the interpretation is particularly simple. Longer horizons mean lower values of θ ; faster adjustment means higher values of g. Both increase the weight on DNPS.

If the rational-expectations literature and the Lucas critique of model simulation have taught us anything it is that we must always specify expectations formation to suit the particular experiment. We present our procedure not as 'the' answer, but rather as a general approach to the introduction of long-run considerations into forward-looking expectations. In the standard simulation rule for the expected rate of inflation, we specify a 75% weight on the short-run prediction and a 25% weight on the expected steady-state inflation rate. See, for example, EQ03UHE in Appendix B, which provides DNPCE, the average expected rate of inflation for the consumption price index. This is the inflation expectation that influences household decisions. It measures the expected decline in the real value of nominal magnitudes owing to inflation in the units of concern to households -- the consumption bundle.

6.3.2 Bonds: RAC, RC, RGE and PBG

Our model for the average coupon rate, RAC, is a simple recursive updating approximation. It specifies that the new average coupon rate is a weighted average of the old rate and the yield on new issues. The weight on the yield of new bonds includes both the proportion of net new issues in the stock and the proportion of the stock that matures (and is replaced with new issues) in the current year, AS06. To allow for cases where the outstanding stock of government debt is reduced more than required by maturation, a conditional trap is employed to hold the average rate constant on the assumption that debt would be retired in proportion to the existing maturity structure.

$$\text{EQ26AM1}\quad \begin{cases} RAC = J1L(RAC), & (J1D(LGT)/J2A(LGT) + AS06) < 0 \\ RAC = J1L(RAC)*[J1L(LGT)/LGT-AS06] \\ \quad + RG*(J1D(LGT)/LGT+AS06), & (\text{otherwise}). \end{cases}$$

This simple approximation can be expected to provide reasonable results as long as the outstanding stock of debt is fairly evenly distributed by term to maturity. The parameter AS06 represents the inverse of the average term to maturity. It is not possible to obtain measures of the average term to maturity for all components of the consolidated bond stock. We therefore decided to try EQ26AM1, as written, under the assumption that the average term to maturity is constant. To obtain a value for AS06 we used our estimated RAC data and applied ordinary least squares to EQ26AM1. The point estimate of AS06 is 0.204, which corresponds to an average term to maturity of just under five years. The fitted values from the equation are very similar to our estimates of RAC (Figure 6-1), although small differences arise around 1973 and 1978. We see no reason to abandon the simplification of a constant term to maturity. We therefore use EQ26AM1 both as a model identity and as the source of our model data for RAC.

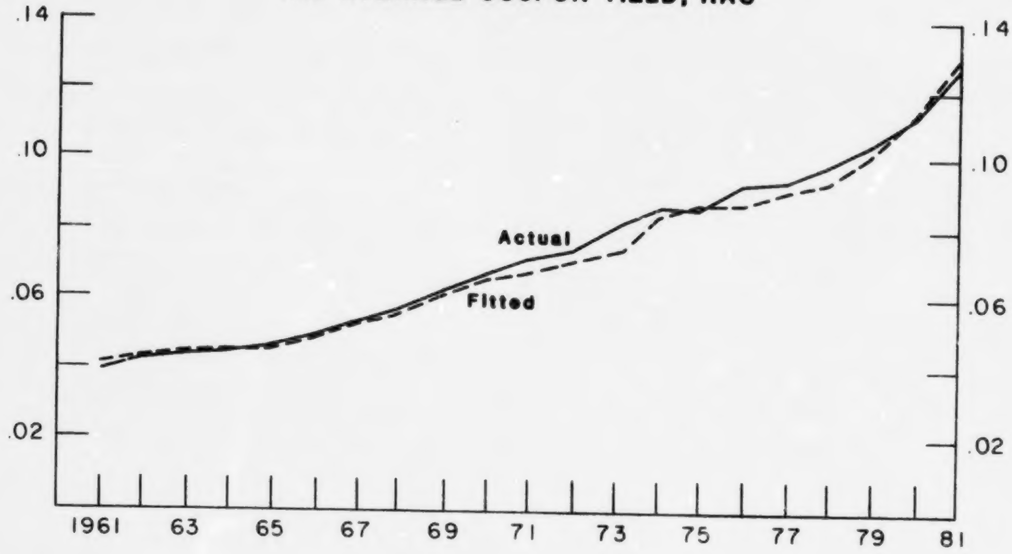
When the current yield to maturity is RG, a bond with coupon rate RAC that matures T periods into the future has a present value, per dollar face value, of:

$$PBG = RAC\int_0^T \exp(-RG\cdot t)dt + \exp(-RG\cdot T)$$

$$= RAC/RG + (1-RAC/RG)\exp(-RG\cdot T). \tag{6.7}$$

Figure 6-1

ACTUAL AND FITTED VALUES OF THE AVERAGE COUPON YIELD, RAC



We make three changes to equation 6.7 to obtain the version used in the model. First, since PBG is defined to be the average price of unmatured issues that were outstanding at the start of the year, we use J1L (RAC) in place of RAC. New issues are counted separately. Second, T is recorded as (1/AS06), consistent with the RAC identity EQ26AM1. Finally, we introduce personal taxation in the simplest possible form. In particular, we assume that interest income and capital gains are treated equally for tax purposes. Therefore, introducing taxation involves simply multiplying each rate in the above integral by (1-RATAX), where RATAX is the average tax rate. Thus PBG is the present value of the after-tax receipts. The model equation is then:

EQ60AMP PBG = J1L(RAC)/RG + (1-J1L(RAC)/RG)*exp(-RG*(1-RATAX)/AS06).

See section 6.4 for elaboration of these points.

The valuation equation EQ60AMP applies to the term to maturity. If the holding period is the same as the term to maturity, then the valuation equation would apply directly and the expected holding-period yield and the yield to maturity would be identical. We specify, however, that the holding period is shorter than the average term to maturity for government debt. We could write another present-value calculation, using RAC for the interest component of the contract and specifying some explicit model of expected capital gains to complete the model, but we choose a simpler approach. We use EQ60AMP to provide the basic link between prices and interest rates, and specify another equation to link the yield to maturity to the expected holding-period yield. This involves adding to the yield to maturity a measure of the extra return expected owing to capital gains over the holding period. Such expectations arise from expectations of changes in market yields. Recall that although asset markets clear in every period, the temporary equilibria need not be stable or expected to be stable over a decision horizon. In particular, when shocks enter a system, asset prices and interest rates may tend to overshoot, in the short run, the values that will prevail as the whole system responds to the shock. This would be taken into account by rational agents, as would other perceived influences on rate movements, such as supply responses of authorities to support policy targets.

We summarize these ideas by specifying that the expected holding-period yield is

$$RGE = RG + AE66*LOG(PBGN/PBG), \qquad (6.8)$$

where PBGN is, notionally, the price that would prevail if asset markets were in a steady-state configuration with the current stock of bonds and the current historically determined RAC. We are not able to solve the model analytically for PBGN. Instead, we specify a proxy

$$PBGN = AE50*PBG + (1-AE50). \qquad (6.9)$$

Because of overshooting, PBGN will tend to lie between the current bond price, PBG, and the very long-run price, 1.0.[12] Hence, the linear combination in equation (6.9) is usually a reasonable proxy. For the model equation, EQ70AME, we substitute equation (6.9) into equation (6.8) and renormalize to write the dependent variable as RG. The bond-demand function is normalized on RGE and we determine the bond price from the present-value equation, EQ60AMP.

To obtain a value for AE66, the rate at which current disequilibrium is expected to be dissipated, we tried various market yields as proxies for RGE and various fixed values for AE50. To allow for level differences between the proxy bond and the model's composite bond we added a free constant to the equation for estimation. We obtained results for AE66 ranging from about 0.08 to about 0.3. A typical result was the following, where RGE is the one-to-three year federal bond yield, and AE50 is set at 0.3.

$$RGE = \underset{(7.5)}{-.0098} + RG + \underset{(2.5)}{.134}*\log(PBGN/PBG)$$

For the model we fix AE66 and AE50 at 0.12 and 0.3, respectively, and generate the RGE data from the identity (with no constant term).

For monetary shocks the effect of the distinction between yields to maturity and expected holding-period yields is to diminish movements in bond prices and interest rates. Consider a monetary expansion from an initial full equilibrium. The expansion in nominal assets will tend to push up bond prices and reduce yields. Because the initial effects tend to be relatively large, however, expectations of a return to real-rate equilibrium (especially for a one-time level increase of money) will generate expected future capital losses in bonds and hence decrease the level of bond demand. This tends to blunt the price increase and interest rate decline.

---

[12] In the very long run, assuming no new shocks, RAC will converge on the market yield, RG, as the stock of debt is rolled over. As this occurs, the average price of outstanding issues will converge to 1.0. Note that the movement in the average price as rollover occurs does not create capital gains to be captured in the term log(PBGN/PBG). To a holder of an 'old' bond there may be capital gains as time passes, but these are represented in RG, the yield. Only new changes in market rates generate capital-value changes that we must consider in equation 6.8.

6.3.3 Equities: REQE

Owners of equity have a claim to the stream of profits earned by firms. Let the current profit rate (per unit of equity) be Y and suppose that the normal profit rate is YN. Suppose, further, that profits are expected to return to normal levels at the exponential rate g. The present value of the profit stream, discounting at rate ρ , is then

$$PV = \int_0^\infty e^{-\rho s}[YN+(Y-YN)e^{-gs}]ds$$

$$= YN/\rho + (Y-YN)/(g+\rho)$$

$$= Y/\rho + (g/(g+\rho))\ (YN/\rho-Y/\rho)$$

or, inverting,

$$\rho = Y/PV + (g/(g+\rho))*(YN-Y)/PV. \qquad (6.10)$$

In terms of SAM notation, the expected yield, ρ, is REQE, and the present value, PV, is PEQ. By multiplying by QEQ on the top and bottom of the right side of equation 6.10, we convert the Ys in the numerator to profit flows and both denominators to the value of the total claims to those flows.

We could use the equation in this form, but we prefer to transform the expected real capital gain from the return-to-equity basis of equation 6.10 to a return-to-physical-capital basis. The return to physical capital is more stable and more closely represents the longer-term resource-allocation signal that we want to reflect in the model's asset decisions. It is not our purpose to model speculative swings in stock market prices. The model equation is:

```
EQ96AMP    REQE = (YBD-FOPRO)/(PEQ*J2A(QEQT))
                + AE40*(RRKSS-RRK)+J3A(.01*J1P(PI))
```

The first term is exactly as derived above, except that we use 'disposable' profits (after corporate tax) and we net out profits accruing to foreigners (since the base is domestically owned equity). The second term represents expected *real* capital gains. As noted, we use RRKSS and RRK, steady-state and current rates of return to physical capital, in place of the equity-based measure in equation 6.10. The above analysis is in real terms, essentially assuming a fixed price level. We add expected *nominal* capital gains on equity claims at the expected rate of inflation for the capital-goods price. It is important to repeat that we are not trying to model the stock market. This term represents the expected increase in the nominal value of the capital stock.

To obtain a value for AE40 we use the dividend yield from the Toronto Stock Exchange as a proxy for the real component of REQE. We add a free constant to the equation and estimate without the inflation expectation. The results are not well determined and are not robust to sample variation. For the 1961-81 period, the point estimate of AE40 is 0.05, and we use that value; but we have no strong case against a zero restriction, such that the expected equity return is simply the current profit yield, the future being entirely discounted.

6.3.4 Net foreign assets: RACUS, RFE, PFXE, PBFH

To account for movements in net interest payments to or from foreigners we specify an equation for the average coupon rate on net foreign assets. The equation has the same basic structure as the RAC equation reported in section 6.3.2:

$$\text{RACUS} = \alpha \text{J1L(RACUS)} + (1-\alpha)\text{RUS}. \tag{6.11}$$

The current average coupon rate is given by a linear combination of the former average value and the yield on newly issued U.S. bonds; the weight, α, depends on the relative importance in domestic portfolios of gross new issues (or purchases). Because the net stock, FHT, can cross zero, the simple weighting function used for RAC cannot be adopted for RACUS. Instead we specify α using an inverse tangent transform that provides a very close approximation to the appropriate linear function over the 'normal' range and is well behaved at extrema. See EQ27AMI for the exact form. A similar equation, EQ28AMI, is specified for RACUSG, the average coupon rate for the government sector's net foreign assets (debts).

The foreign currency value of net foreign assets is represented in an equation identical in form to that described for the domestic bond price. We write:

```
EQ00AMP   PBFH = J1L(RACUS)/RUS + (1-J1L(RACUS)/RUS)*
                 exp(-RUS*(1-RATAXUS)/AS07).
```

As is the case in our PBG equation, this represents the after-tax present value of a bond with coupon rate RACUS, that matures in (1/AS07) years, when the pre-tax yield to maturity is RUS.

The net foreign asset position of the domestic private sector is modelled as if it were entirely in the form of five-year U.S. government bonds, with yield to maturity RUS. We assume a one-year decision horizon and, in principle, should distinguish between RUS and the expected yield over the one-year holding period. To avoid dealing with the U.S. term structure we assume that the expected one-year yield is RUS.[13] The expected holding-period yield in Canadian terms is then:

EQ62AMI $RFE = RUS + \log(PFXE/PFX)$,

where PFXE is the exchange rate expected to prevail one period into the future. For our data (for the flexible-rate period since 1971) we use the one-year forward rate as a proxy for PFXE and use EQ62AMI to define RFE.[14] In the simulation code EQ62AMI is normalized on PFX and RFE comes from the demand function for net foreign assets.

We treat the specification of the one-year-ahead expected exchange rate as the provision of a simulation rule. Like other expectations rules, this one must be chosen to reflect the nature of the experiment and, in particular, the desired degree of rationality or speed of response to shocks. Two elements must be contained in the expectations-formation rule, however, at least as long-run properties. First, there must be a term that reflects long-run inflation differentials so that relative purchasing power parity holds over the long run for the nominal exchange rate. Otherwise, no real steady state can persist (except in the case of no inflation differential) because nominal movements would continuously disturb the real equilibrium and necessitate perpetual disequilibrium adjustment. Second, there must be some mechanism that permits expectations to adjust to be consistent with the level equilibrium for the real exchange rate. This can be done in various ways. Ideally, we would like to be able to compute, even if only approximately, the equilibrium real exchange rate so that we could specify an adjustment towards that equilibrium as part of the dynamics. Unfortunately, this does not seem feasible. Our solution is to use a standard adaptive-expectations term that guarantees consistent expectations in steady state.

---

[13] The yield on one-year U.S. treasury bills is highly correlated with RUS. For 1955-83 the correlation is 0.984. Nevertheless, there are some movements in the U.S. term structure that are worthy of recognition. We intend to relax our current assumption the next time SAM is estimated. It would simplify the model to assume that all foreign assets are one-year bonds so that holding-period and maturity concepts were identical. This would allow us to eliminate the measure of foreign-market value (PBFH) and to avoid a term-structure equation. But many of Canada's portfolio and government financial transactions with foreigners are in longer-term instruments; e.g., virtually all provincial borrowing (in FGBT) is in long-term instruments. We therefore prefer to retain the formal distinction between maturity and holding period for foreign assets.

[14] For the fixed-exchange-rate period we specify PFXE to be the mean of the actual values of PFX. For the period before 1962 we use the lagged exchange rate as a proxy for PFXE.

Our standard rule is as follows:

```
EQ08AHE    J1P(PFXE) = AE61*J1P(PFX*PUS/PC)
                       + AE62*100*J2A(log(PFX/J1L(PFXE)))
                       + AE63*100*(DNPS-DNPUS)
                       - AE64*100*log(SALES/UGPCSS).
```

How we define the inflation differential, DNPS-DNPUS, depends on how rapidly we want the effects of purchasing power parity to influence expectations.[15] The adaptive-expectations term, with coefficient AE62, appears as a two-period average because of a formal transformation from continuous to discrete time following the Bergstrom-Wymer procedure.[16] Note that it is written in terms of the current and previous forecast errors (J1L(PFXE) is the expectation, formed last year, about this year's PFX). In steady state this term is zero, but if an adjustment process is changing the real exchange rate, nominal exchange rate expectations will gradually adjust via the error terms.

The remaining two terms in EQ08AHE are sometimes used to enhance the dynamics. The first term adds another link between movements in the real exchange rate and the expected spot rate. Note that in steady state this term is zero; it affects only the dynamics, not the final solution. The last term, with coefficient AE64, is a measure of excess demand in the domestic-product market. This term can be included to capture a rational expectation that the equilibrium real exchange rate is affected by the requirement of domestic real equilibrium. This term is not necessary to

---

15. In the equation reported in Appendix B, DNPS-DNPUS is measured as the actual inflation differential.

16. See Chapter 2, section 2.3.3.

establish a link between other real variables and the real exchange rate. Such a link exists in the structure of the behavioural equations and the interaction of sectoral budget constraints, regardless of whether or not PFXE is specified to be sensitive to the output gap. Such a term can be used, however, to increase the rationality of expectations and to speed up the response to shocks.

Our standard parameters are: AE61,0; AE62, 0.2; AE63, 1.0; AE64,0 (0.2 if used). When considering the parameters we estimated equations like EQ08AHE using various techniques and various sample periods, with the one-year forward rate as a proxy for PFXE. We obtained values for AE62 between 0.2 and 0.4 that were reasonably well determined. The others were more erratic, but generally had the correct signs. The unit restriction on AE63 was generally not rejected. We do not consider the estimation of the PFXE equation a serious exercise. There is high simultaneity between PFX and PFXE, and single-equation techniques, such as two-stage least squares, seem an inadequate response. The PFXE equation is best thought of as a simulation rule to be provided by the user.

6.4 Estimation of the Demand System

6.4.1 The complete system for estimation

The system to be estimated consists of the asset-demand functions and the system of identities linking asset prices and yields. Consider, first, the notation and conventions for measuring asset values.

Valuation of new and outstanding assets

Asset prices and all yields and rates are treated conceptually and measured as averages over the year. Asset stocks, however, are recorded as year-end values. This facilitates the link between stocks and flows since the flow financing requirements represent the total flows for the year and generate stock changes from the beginning to the end of each year. A continuous-time demand function in the form

$$P \cdot Q = (A0 + A1 \cdot R)V,$$

when integrated over an annual interval creates the same type of non-linearity problems addressed in Chapter 2 (section 2.3.3). For example, the integral of P•Q is not generally the product of the integrals. Our specification of these products varies with the type of financial instrument and the type of price.

For equities, new issues and old issues provide the same kind of claim to the profit stream and hence command the same price. So we simply write the integral of the product as the product of the integrals, appealing to the approximation results presented in Armstrong (1985c). Thus the value share of equities appears in the code as

$$\text{Equity share} \equiv PEQ*J2A(QEQT)/V.$$

PEQ is the average price for the year and J2A(QEQT) is the average quantity, proxied by the average of beginning-of-year and end-of-year values.

Bonds represent a very different kind of financial instrument. New issues generally appear at, or close to, par (a price of 1.0 in SAM) and bear the current yield to maturity, RG, as a contract interest rate. Because the yield tends to change over time, there will generally be a wide variety of prices extant for bonds, depending on conditions at issue and time remaining to maturity. We feel that it is important to distinguish clearly between new issues that have a price or value of unity and old bonds whose value depends on current market conditions. It is conceptually convenient to think of all new issues in any year as bearing the same contract yield, RG, and a unit price.[17] The price of outstanding issues is then applicable to the stock outstanding at the start of the year, and we write

$$\text{Bond share} \equiv [PBG*J1L(LGDT) + .5*J1D(LGDT)]/V$$

where the .5*J1D(LGDT) represents the average value of new bonds held (i.e., new issues are assumed held for half the year, on average).

The U.S. dollar value of household net foreign assets is written the same way using the average value of assets outstanding at the start of the year, PBFH. Conversion to Canadian dollars, however, requires multiplication by PFX for all assets, regardless of the issue date.

$$\text{Foreign asset share} \equiv PFX*[PBFH*J1L(FHT) + .5*J1D(FHT)]/V.$$

Personal taxation

In the real world the taxation of income and changes in capital value is a complex affair. For previous working versions of SAM we developed explicit measures of expected after-tax yields, applying as best we could the current and past institutions of taxation. The resulting constructs were complicated and introduced inflation-induced non-neutralities into interest rate determination, both in terms of the levels of all rates and the term structure. Although such detail is extremely interesting in some cases, particularly when it is assumed that institutions remain fixed, we have opted for simplicity in the basic model. Besides, in counterfactual simulation over the medium to long term, who can say whether it is more or less reasonable to assume that particular institutional rules will remain unchanged? When inflation became a significant issue, indexation was introduced in the tax system. Indeed, our research generally indicates that, over time, institutions tend to adapt; assumptions that particular institutions, like specific tax rules, will stay the same when the environment changes are often wrong.[18] We therefore decided to keep the basic version of SAM simple and to retain the complex treatment of taxation for those cases where such issues are of paramount importance. We assume that one average tax rate applies to all returns from assets, whether in the form of interest or dividend payments or in the form of capital gains. Under this assumption it is easy to provide clear and complete consistency between asset valuation and asset after-tax yields and, more important, retain a very simple form for the personal tax function.

---

[17] If we did not use this approach we would have to rewrite flow constraints such as the government financing requirement to account for changes in interest rates and bond prices during the year. Although in times of rapidly changing interest rates our approach misses something, the distinction between new issues and the cumulated past is much more important.

The asset-demand system

In the singular, four-asset system only three demand conditions can be specified independently. For our parameterization we begin with the share equations for equities, bonds, and net foreign assets:

$$\begin{aligned}\text{PEQ*J2A(QEQT)/V} &= \text{AS30 + AS36*QTIME + AS31*(1-RATAX)}\\ &\quad \text{*(REQE-RGE-AE89) + AS32*(1-RATAX)}\\ &\quad \text{*(REQE-RFE-AE88-AE89) + AS33}\\ &\quad \text{*(1-RATAX)*REQE}\end{aligned} \tag{6.12}$$

$$\begin{aligned}\text{(PBG*J1L(LGDT) + .5*J1D(LGDT))/V} &= \text{AS40 + AS46*QTIME}\\ &\quad \text{+ AS41*(1-RATAX)*(RGE-RFE-AE88) + AS42}\\ &\quad \text{*(1-RATAX)*(RGE-REQE+AE89) + AS43}\\ &\quad \text{*(1-RATAX)*(RGE-DNPCE) + AS44*DNPCE}\end{aligned} \tag{6.13}$$

$$\begin{aligned}\text{PFX*(PBFH*J1L(FHT) + .5*J1D(FHT))/V} &= \text{AS50 + AS56*QTIME}\\ &\quad \text{+ AS51*(1-RATAX)*(RFE-RGE+AE88)}\\ &\quad \text{+ AS52*(1-RATAX)*(RFE-REQE+AE88+AE89)}\\ &\quad \text{+ AS53*(1-RATAX)*RFE}\end{aligned} \tag{6.14}$$

---

[18] See Selody and Lynch (1983). The change in energy royalties after the OPEC crises provides another good example.

The fourth share equation can be written in terms of the parameters in equations (6.12-6.14) using the adding-up restrictions. Recall that we also impose symmetry restrictions: AS42 = AS31, AS52 = AS32, and AS51 = AS41. Under these restrictions the residual money-demand equation is simply

$$J2A(HT)/V = 1.0-AS30-AS40-AS50 - (AS36+AS46+AS56) *QTIME - (1-RATAX)*(AS33*REQE + AS43*(RGE-DNPCE) + AS53*RFE) - AS44*DNPCE. \quad (6.15)$$

We do not use any of these equations directly in the simulation version of the model. We invert the demand function for net foreign assets (equation 6.14) and normalize on RFE (EQ65AHD). Similarly, we renormalize the bond-demand function (equation 6.13) on one of the RGE terms (EQ64AHD). The three asset-demand functions (6.12-6.14) are added together to obtain EQ59AHD, which is normalized on PEQ. This exploits the symmetry restrictions, because in the addition all the rate-differential terms (involving pairs of RGE, RFE, and REQE) cancel. Of course, the summation gives us 'everything else except money' and so what we use in simulation is equation (6.15) rewritten as EQ59AHD.

Two features have been introduced into the asset-demand system for estimation. First, we add to the basic model the possibility of exogenous trends in the desired asset shares. For simulation over future periods we shut off these trends. Second, we introduce parameters AE88 and AE89, specified as the sample means of the differentials (RGE-RFE) and (REQE-RGE), respectively, so that all rate differentials have means of zero in estimation. These parameters are fixed from the data and not estimated with the system of behavioural parameters.

If inflation rises and nominal interest rates follow, there will be substitution into net foreign assets and equities from money and/or bonds. If AS44 is positive, there will also be substitution from money into bonds (i.e., there will be substitution from money into all other assets). If AS44 is negative, however, there will be substitution out of bonds when inflation and nominal interest rates rise. In extreme cases this effect can dominate the nominal-rate effects on money demand and lead to substitution into money in the short run. Recall, however, that in the long run this cannot happen; the long-run money-demand function requires substitution out of money when nominal rates rise. The short- and long-run conditions are reconciled mainly through price-level adjustments.

Finally, note that although only three demand equations are independent, for the system to be closed we also need an equation for total nominal financial wealth. We use the direct definition of V as the sum of the four components:

```
EQ67AMI   V = J2A(HT) + PEQ*J2A(QEQT) + PBG*J1L(LGDT) + .5*J1D(LGDT)
              + PFX*[PBFH*J1L(FHT) + .5*J1D(FHT)].
```

6.4.2 Estimation procedures

For estimation we must choose a method of introducing the stochastic specification. There is some power in the argument that since the choice variables are asset shares, the random errors should be associated with the share variables. Instead, we specify that the errors are associated with the dependent variable in the normalized form used in simulation -- the share for equities, but the rates for bonds and net foreign assets. This choice affects the design of the estimator. We have not investigated how much impact the choice has on our results.

Owing to the cross-equation parameter restrictions, some systems estimator is essential for the asset-demand system. Moreover, the existence of strong simultaneities among asset prices and yields suggests that there are great advantages to a maximum-likelihood procedure that can appropriately incorporate into the estimator the identities linking asset prices and yields. These identities are: the condition linking RFE and PFX, the expected holding-period yield equation for equities that links

PEQ and REQE, the present-value equation linking PBG and RG, and the expected holding-period equation linking RGE to RG and PBG:

$$
\begin{aligned}
\mathrm{RFE} &= \mathrm{RUS} + \log(\mathrm{PFXE}/\mathrm{PFX})\\
\mathrm{REQE} &= (\mathrm{YBD}-\mathrm{FOPRO})/(\mathrm{PEQ}*\mathrm{J2A}(\mathrm{QEQT}))+\mathrm{AE40}*(\mathrm{RRKSS}-\mathrm{RRK})+\mathrm{J3A}(.01*\mathrm{J1P}(\mathrm{PI}))\\
\mathrm{RGE} &= \mathrm{RG} + \mathrm{AE66}*\log(\mathrm{PBGN}/\mathrm{PBG})\\
\mathrm{PBG} &= \mathrm{J1L}(\mathrm{RAC})/\mathrm{RG} + (1-\mathrm{J1L}(\mathrm{RAC})/\mathrm{RG})\\
&\quad *\exp(-\mathrm{RG}*(1-\mathrm{RATAX})/\mathrm{AS06}).
\end{aligned}
$$

Parameters AE40, AE66, and AS06 are predetermined and are not part of this estimation problem. The feedbacks through PFXE and RAC are ignored in the estimation.

We also make the heroic assumption that asset supplies can be considered exogenous to the estimation. We feel that this simplification is one of the major reasons for the common difficulties researchers have encountered in identifying asset-demand systems. We suffer some of the same difficulties, but one must always stop somewhere in generalizing empirical problems.[19]

The estimator we use is similar in design to that employed for the supply side of the model (Chapter 4). It uses the same restriction on the covariance matrix, namely independence of the disturbances, for essentially the same reason. We found very high correlation of the estimated errors and there is reason to believe that in such cases there will be efficiency gains through the imposition of covariance restrictions.[20] The estimation was carried out using a purpose-designed application of the Goldfeldt and Quandt GQOPT program, using the Davidon-Fletcher-Powell and Nelder-Meade algorithms in a sequential combination we

---

19. Early in the development of SAM we experimented with estimation of a similar demand system but with some supply-side endogeneity. The results were not encouraging and we abandoned the attempt. It remains a problem we would like to tackle, since we are convinced that it will prove important in empirical work on asset markets.

20. See Armstrong (1985a) for a description of the maximum-likelihood estimator with covariance restrictions and the case for efficiency gains from independence restrictions.

have designed for such problems. The t-statistics were generated using the GRADX routine in the same program.

6.4.3 Estimation Results

The estimation of asset-demand systems has always proven a difficult econometric task. For example, rates tend to be highly collinear, making precise identification of partial effects and individual parameters extremely difficult. Like most other researchers we have had problems estimating the asset system. However, we have results that seem worthy of consideration. We remind the reader that because of our long-run perspective we have not introduced any adjustment processes in the asset-demand system. There are no lagged dependent variables in the estimated system. Despite this, residual autocorrelation is not a severe problem.

It proved impossible to estimate the parameters jointly when AS43 and AS44 were free to vary in the system. Some apparent near singularity led to all of the bond-equation parameters exploding together towards plus or minus infinity, without other parameters being much affected. To avoid this problem we had to determine AS43 and AS44 using a different approach. We began by fixing AS40 and AS46 at values determined from OLS estimation of the bond-share equation. Using these restrictions, we then estimated the simultaneous system, with AS44 also imposed at zero. That gave us a preliminary value for AS43. We then imposed this value on the system, freeing the other parameters. With the rest of the system fixed, we tried a small grid of values for AS43 and AS44 and settled on AS43 = 0.19 and AS44 = 0.0. With these final values for AS43 and AS44, the rest of the parameters were estimated simultaneously using the FIML procedure.

The second problem is a similar difficulty with the 'own rate' in the foreign-asset equation. Even with AS43 constrained, AS53 interacted with the parameters of the bond equation in a similar, albeit less extreme, fashion. To avoid this problem we imposed AS53 at zero. Note that the own rate still enters the foreign-asset equation -- through the two cross terms. Only the differential relative to the money return is eliminated.

The final difficulty is that use of untransformed data for the market value of equity makes the wealth measure extremely volatile, and this volatility is passed into the share measures. The value of capital represents about 87% of financial wealth. Researchers have always found it difficult to explain fluctuations in capital valuation with any model, regardless of complexity, let alone a simple one with no attention to institutions or historical detail. For estimation we therefore use smoothed data for the market value of capital. This is not a change to the model for simulation, but rather a filter applied to the data in estimation.

The remaining parameters are simultaneously determined using the FIML procedure described above. The 'own rate' in the equity equation is very small, 0.006, and statistically insignificant. Its asymptotic t-ratio is only 0.02. We constrain it to zero[21] in the results that follow. The other parameters are not affected by this restriction.

Point estimates are reported in Table 6.1. Only AS31 (and AS42 by symmetry) is statistically insignificantly different from zero. Indeed, this is the only parameter that is troublesome in terms of its relative size. The low positive value for AS31 indicates low substitutability for bonds and equities. This is clearly what our data require. A restriction of AS32 to the region of high substitutability is clearly rejected. As we argued earlier, however, we do not feel comfortable with a long-run property whereby asset-holders would respond weakly to permanent changes in the rate differential between bonds and equities. As a result, for the standard simulation model, we impose a higher coefficient. In cases where one wants to retain the flavour of the estimated result, one can leave AS31 at a low value for the first few periods of simulation and then increase it. The rationale for this is that our estimates may provide a good measure of impact effects, but a poor measure of the longer-term tradeoffs that would be observed.

The results are significant and reasonable. Note, in particular, that the relatively large and significant AS41, AS51 pair represents

---

[21] Under the restriction the log of the likelihood function drops by about 0.002 on a base of 186. The restriction is not rejected.

Table 6.1

PARAMETER ESTIMATES: ASSET DEMANDS

Coefficient	Point estimate	Asymptotic t-ratio
AS30	0.8721	443.8
AS31	0.0812	0.8
AS32	0.2900	2.3
AS33	0	constrained
AS36	0.0023	6.3
AS40	0.0923	53.2
AS41	1.2422	3.6
AS42	0.0812	0.8
AS43	0.19	constrained
AS44	0	constrained
AS46	-0.0015	4.5
AS50	0.0117	6.4
AS51	1.2422	3.6
AS52	0.2900	2.3
AS53	0	constrained
AS56	-0.0009	2.5

relatively high substitutability between bonds and foreign assets. It also indicates a significant asset effect on the exchange rate, through the return on foreign assets. That is, our dependent variable in this equation is RFE, and the coefficient on the foreign asset ratio is AS51/(AS51+AS52). This coefficient is statistically significant. The model has been estimated by a FIML procedure, taking into account the identity linking RFE and PFX. Given the error specification, the FIML results are independent of normalization. Thus, one can think of the foreign-asset-demand equation as proximately determining PFX, and the results as establishing a significant link between the net foreign asset share and PFX.

The estimates indicate a moderate, but significant, substitution effect for equities and net foreign assets. We find both the sign and the size plausible here. For example, it would be surprising to observe a higher substitutability of equities and foreign assets than of government bonds and foreign assets.[22]

In Table 6.2 we report the individual equation statistics. They are reported for the variables as normalized for the stochastic specification in estimation: the equity value share, and the after-tax expected rates for bonds and net foreign assets. The rate equations both fit quite well, and the equity share is at least respectable.[23] The other statistic is the first-order autoregression coefficient for the residuals. All equations have small positive residual correlation. We report these results for completeness, although in a FIML context the importance of individual equation results is unclear. The system is highly simultaneous and is treated as such through the likelihood function.

Table 6.2

INDIVIDUAL EQUATION STATISTICS
(Sample, 1963-81)

Equation	Dependent variable	RSQ	AR1
Equities	PEQ*J2A(QEQT)/V	0.572	0.352
Bonds	RGE*(1-RATAX)	0.943	0.381
Net Foreign Assets	RFE*(1-RATAX)	0.944	0.218

To sum up, we have been moderately successful in obtaining reasonable empirical results for our simple asset system. Not all parameters can be simultaneously determined, but we are able to obtain much of the system through estimation. The use of a systems estimator, in particular a FIML procedure that can account for asset-price/rate-of-return simultaneities, is important.[24]

---

[22] Given that we find significant substitutability between equities and foreign assets, and between bonds and foreign assets, the case for imposing higher substitutability on bond/equity preferences appears stronger.

[23] Recall, however, that this is with the smoothed data for PEQ. If actual historical values were used, we would have a much worse fit.

6.5 Properties of the Asset System

We now report the results of three shocks to the part of the model that comprises the asset system. This includes the four asset-demand equations, the equations linking yields to asset prices, the equations linking expected yields to market-determined yields, and the asset-supply equations. For these experiments, the supply of equity consists of the endogenous response of foreign investment to rate-of-return differentials. The real profit stream of firms, the price of investment goods, and the physical stock of capital are held exogenous. The supply of net foreign assets consists of the endogenous response of trade to changes in the exchange rate, and the endogenous response of the current account to changes in interest payments on the foreign debt. Trade in energy goods, domestic income, and all foreign variables are held exogenous in real terms. Similarly, most components of government expenditures and taxes are held exogenous in real terms. The only exceptions are interest payments on the debt and personal taxes. We use our standard simulation rule to link taxes and interest payments -- real interest payments on the debt are paid from personal taxes (see Chapter 2, sections 2.4.4. and 2.4.5). Feedback from tax-rate changes to real decisions is ignored here. With these assumptions, the supply of government bonds (government financing requirement) simplifies to movements in interest payments net of taxes.

6.5.1 A money-level shock

The first partial simulation demonstrates the subsystem's response to a swap of money for government bonds. We increase the stock of money by a once-and-for-all 10%. Since the real side of the model is excluded from the partial experiment, there is no point in using the model's estimated price dynamics. Instead, the price level is assumed to move by 2% per year until it too increases by 10%. Ultimately, therefore, there is no real shock. Initially, the real value of government debt is lower in the shock than in the control, because the financing requirements of government are met more through money and less through bonds; but eventually, because of the simulation rule for government real interest payments, the real value of government debt returns to its control value. This shock is designed to illustrate two points. First, a purely nominal shock does not permanently alter relative interest rates in this model. Second, in the subsystem considered here the exchange rate moves with the general price level; the dynamics of interaction between the exchange rate, trade and net-foreign-asset acquisition are complicated but controlled.

---

[24] The system results are much more robust and sensible than can be obtained using single-equation techniques.

The responses of the three asset prices are displayed in Figure 6-2. The price of bonds, PBG, is initially above the control level by 1.6%, reflecting the reduction in the real value of government debt. The return of the bond price to control takes about 15 years, and involves some overshooting. Both the exchange rate and the price of equity tend to lead the increase in the general price level, implying that, on impact at least, the increase in money results in a real depreciation of the Canadian dollar, and a real 'boom' in the equity market. In the first year of the shock the depreciation is 3.8%. This implies a real depreciation of about 1.8%. By year 6 the exchange rate and the equity price surpass 90% of their ultimate equilibrium changes, and by year 12 they are at their long-run values. In the case of the exchange rate there is some overshooting. Of course, these partial simulation dynamics depend on the five-year rule for price adjustment. If prices are faster (slower) to adjust, asset-price adjustment will be faster (slower) also.

The asset rates of return reflect the movements in the real prices of assets (Figure 6-3). All three rates of return are down initially, with the return on bonds showing the smallest drop (35 basis points) and the return on foreign assets showing the largest drop (56 basis points).[25] All rates overshoot their eventual equilibrium in the adjustment process that ensues. Generally, the distortion in the structure of relative interest rates that results from the shock is not severe. There is, however, a significant secondary cycle in the rate of return on foreign assets, RFE. This secondary cycle is associated with a cycle in net foreign assets, FHT, (Figure 6-4), that itself is associated with the trade response to the initial distortion in the real exchange rate. The return to full stock equilibrium for foreign assets is a slow process, according to our model and estimated parameters.

Figure 6-5 displays the profile of profits to foreigners, FOPRO, interest payments on the government debt, GTIN, and interest on foreign debt, INT. All three measures are stabilizing at levels 10% higher than their control values. The slight drift in GTIN over most of the simulation period represents the slow return of the coupon rate on government bonds to equilibrium as the stock of debt rolls over.

6.5.2 A money-growth shock

In this partial simulation we increase the money supply by a perpetual 10% per year. That is, money growth is 10% higher per annum than in the control. The simulation rule for price inflation is 2%, 7%, 10.5%, 14.3%, 10%, ..., which is calculated such that by year 5 of the shock the proportional change in the price level equals the proportional change in the money stock. The pattern of inflation response is roughly what we would find in a full-model simulation, but we ignore the level

---

[25] These results must be tempered by two comments. First, the relative size of the response of the return on net foreign assets is largely determined by the extent to which exchange rate expectations respond to current exchange rate movements. Our rough estimates (see section 6.3.4) indicate much less than proportionate response. Hence, the actual depreciation in the first few periods of the shock is not matched by changes in expected future spot rates, creating an expected future appreciation and correspondingly lower expected return on net foreign assets. We are not confident about the size of this response. Second, because this version of SAM does not have nominal interest rate terms in the estimated portfolio money-demand equation, the short-run response of interest rates (especially the bond rate) is curtailed compared to what would be obtained with a standard short-run money-demand function. We remind the reader that we do not use this version for short-run analysis.

Figure 6-2

MONEY-LEVEL SHOCK:PRICES
Shock Minus Control %

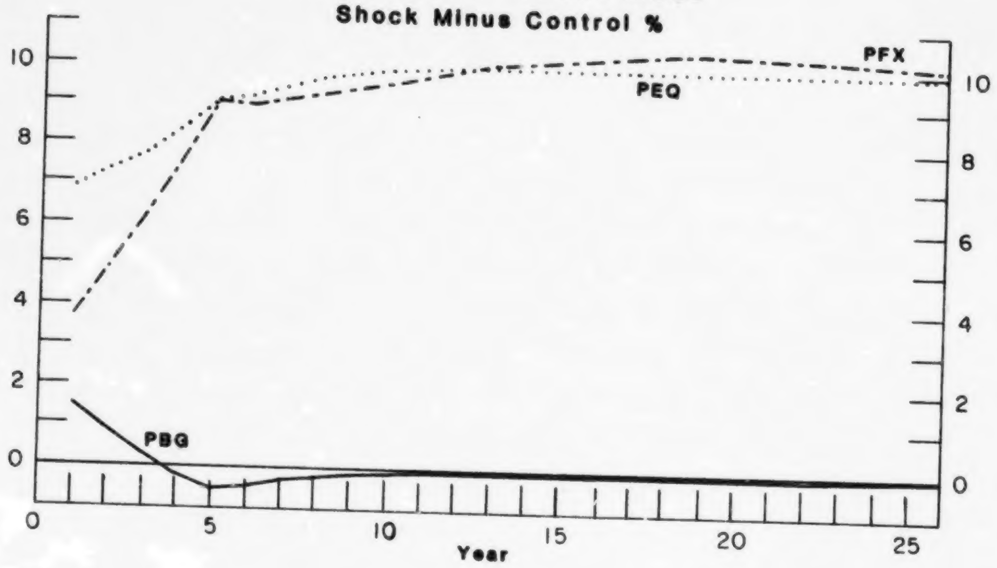


Figure 6-3

MONEY-LEVEL SHOCK:INTEREST RATES
Level Shock Minus Control

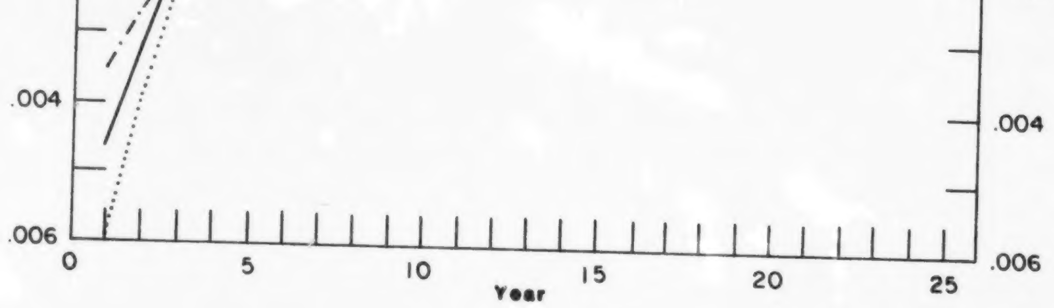


Figure 6-4

MONEY-LEVEL SHOCK: BALANCE OF PAYMENTS
Level Shock Minus Control
(Millions of Dollars)

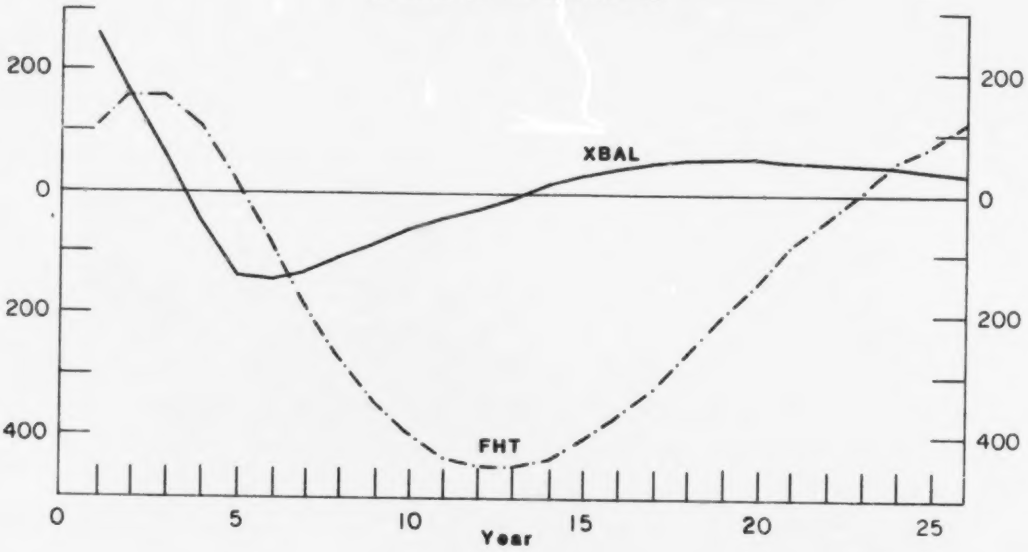
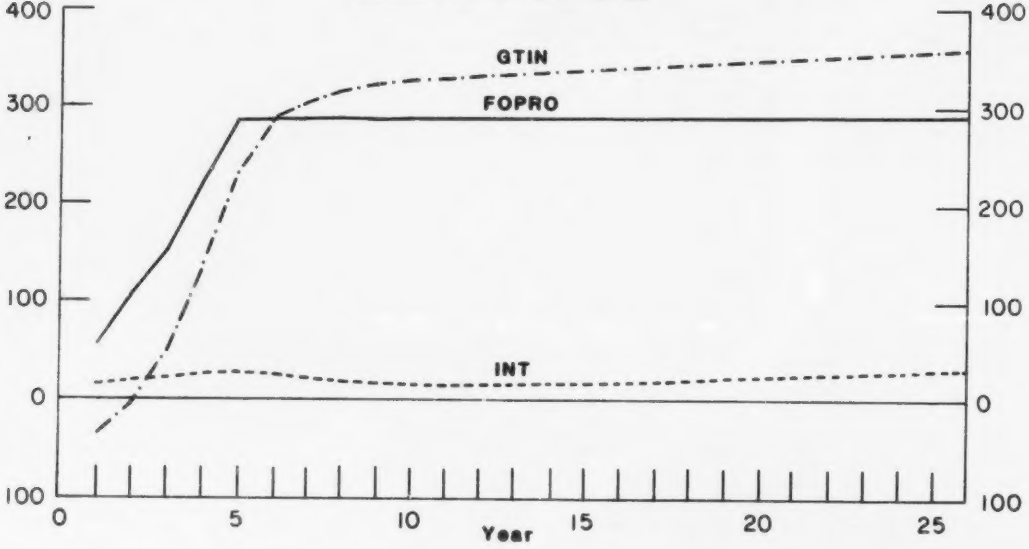


Figure 6-5

MONEY-LEVEL SHOCK: INTEREST PAYMENTS
Level Shock Minus Control
(Millions of Dollars)



shift of prices that would be associated with interest rate changes in the full model.

In Figure 6-6 we illustrate the level shock-minus-control values for the rate of change of the price level, the rate of change of the equity price index, and the rate of change of the exchange rate. The price of equity tends to lead the aggregate price level, while the exchange rate tends to lag it. For the results reported here we specify exchange rate expectations such that there is no confusion about the origin of the shock (it is known that it is a nominal shock), but it is of unknown magnitude. The lag in the exchange rate is thus due to error learning about the size of the shock. If we specified expectations in which agents were confused about the origin of the shock, then the initial delay in exchange rate adjustment would be greater, and the eventual overshoot of the exchange rate would be more severe. Because the shock is purely nominal, any initial real appreciation of the exchange rate will be counterbalanced by a later real depreciation, and the real value of the exchange rate will eventually return to its pre-shock level.

The response of interest rates to the money-growth shock is shown in Figure 6-7. The imposed fast response of prices to the shock, in particular expectations of inflation, keeps the nominal rates from ever turning down, even in the first year.[26] In the initial few years of the shock, when prices are in disequilibrium and exchange-market participants are in the early stages of error learning, the government bond rate, the net foreign asset rate, and the return on equity are slowly diverging. After price adjustment is complete, the various rates drop to reflect the drop in measured inflation, then they begin to converge on their eventual equilibrium as the error-learning process continues and stock equilibrium is restored. The extent of stock disequilibrium can be seen in Figure 6-8, where the stock of net foreign assets and its corresponding flow (the current account balance) are displayed. Once the level adjustment of the real exchange rate is complete, and stock equilibrium is restored, all

---

26. If expected inflation were not forward looking, then interest rates would fall slightly in the first year even in the model without nominal interest rate terms in the portfolio money-demand equation. The absence of such interest rate terms, however, is the principal reason for the unusual impact effects.

Figure 6-6

MONEY-GROWTH SHOCK: RATES OF CHANGE
Level Shock Minus Control

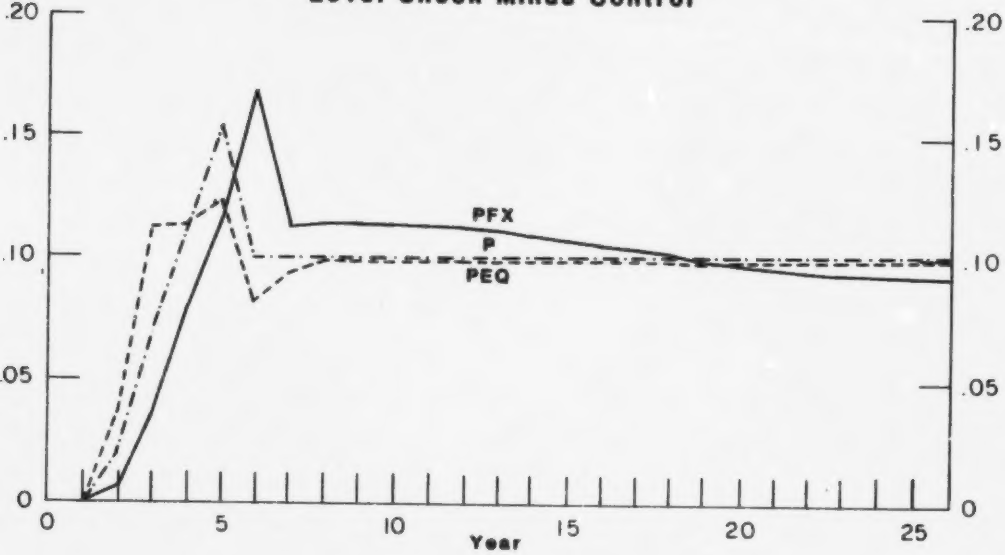
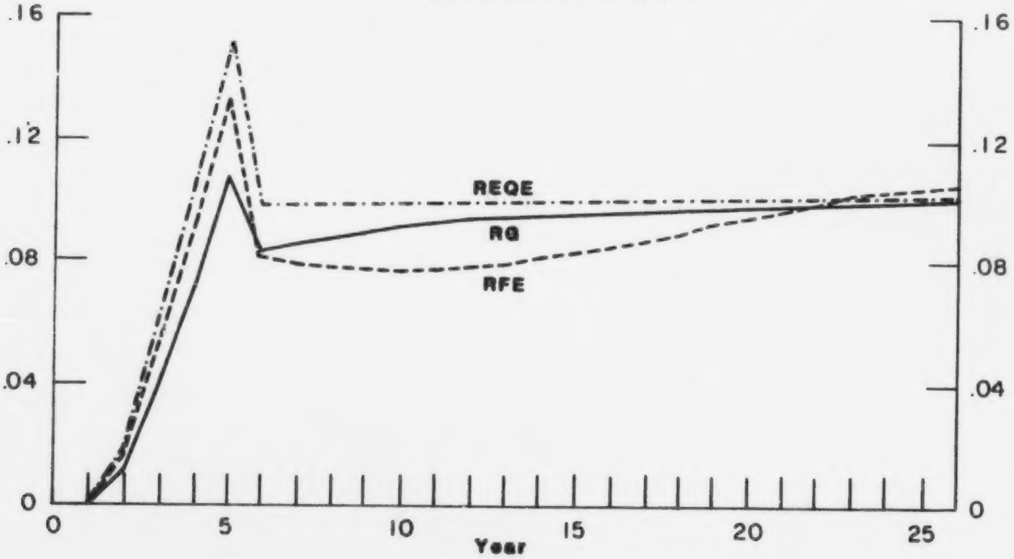


Figure 6-7

MONEY-GROWTH SHOCK: INTEREST RATES
Level Shock Minus Control



three rates of return will converge on a level 10 percentage points higher.

6.5.3 A government expenditure shock

In the third and final partial simulation we consider a deficit perpetually higher by 2% of the initially outstanding bond stock. The deficit is not monetized, but financed through increased issues of government debt. The resulting higher interest payments on the government debt are financed through increased taxes. Nevertheless, the perpetual nature of the deficit implies that the real bond stock is growing at a rate 2% greater in the shock than in the control.

The fact that the proportion of government bonds in total financial wealth is continually increasing in this simulation means that relative asset prices must be continually changing. This follows from the feature of the model that bonds, foreign-pay assets, and equities are not perfect substitutes, although they are close substitutes. The interest rate consequences of the perpetual real deficit are shown in Figure 6-9. In an effort to persuade households to hold an increasing proportion of their wealth in the form of debt, government must offer an ever-higher yield, RG. This implies an ever-increasing distortion of the rate structure as (but less than proportionately) the yield on government debt pulls up the yield on other financial instruments. In Figure 6-10 the interest rate consequences of the real deficit are clear; real interest payments on the government debt are continually rising. Note, however, that in this partial simulation no response of real domestic saving to the higher interest rates is allowed.

The fact that interest rates are slowly drifting up, and not increasing explosively, is due to the simulation rule that interest payments on the government debt are financed by taxes. The fact that interest rates are slowly drifting up, and not stabilizing at some higher level, is due to the partial nature of the simulation (no savings response) and the assumption that the real growth rate of the economy is zero.

Figure 6-8

MONEY-GROWTH SHOCK:BALANCE OF PAYMENTS
Level Shock Minus Control
(Millions of Dollars)

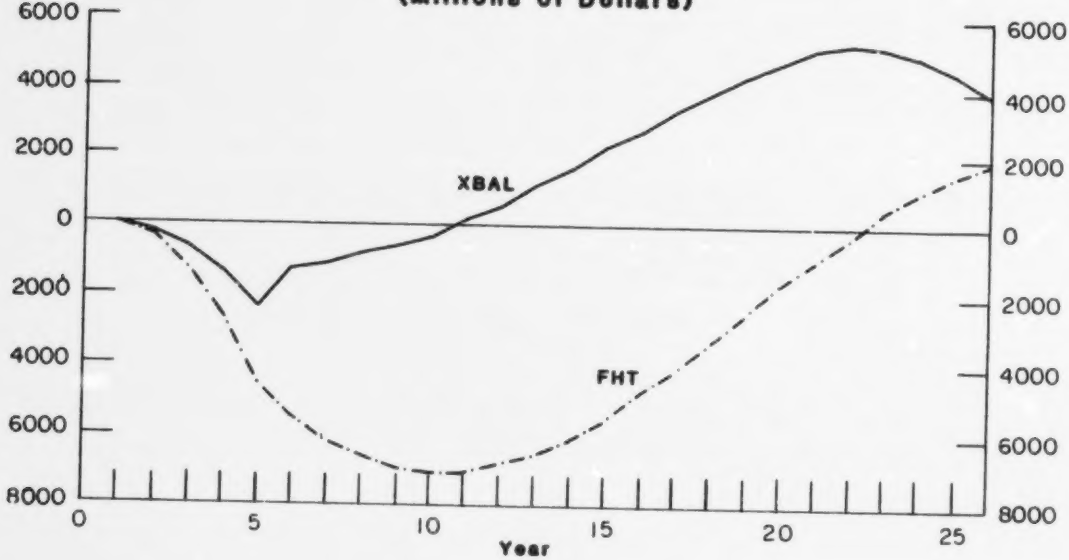


Figure 6-9

GOVERNMENT EXPENDITURE SHOCK:INTEREST RATES
Level Shock Minus Control

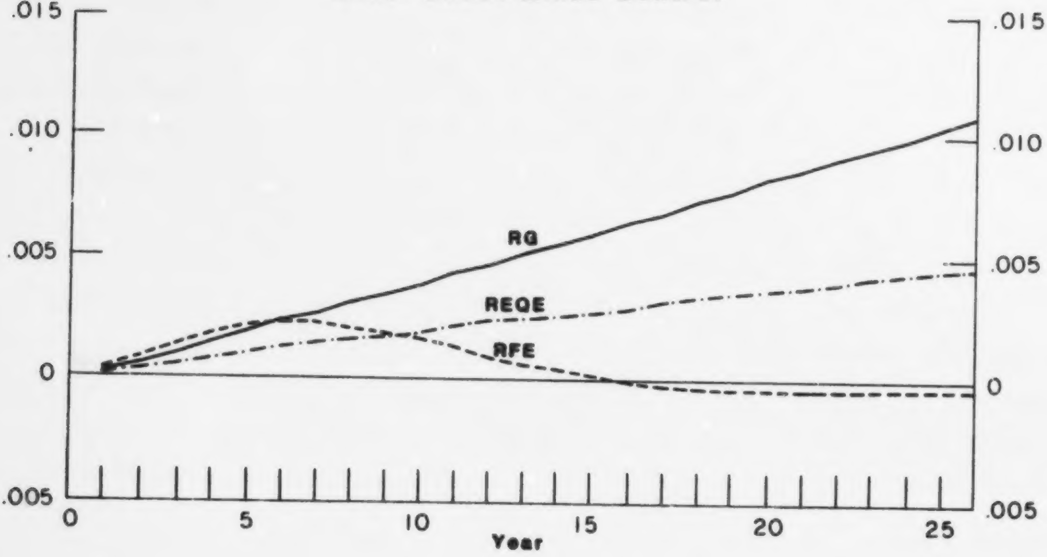


Figure 6-10

GOVERNMENT EXPENDITURE SHOCK:INTEREST PAYMENTS
Level Shock Minus Control
(Millions of Dollars)

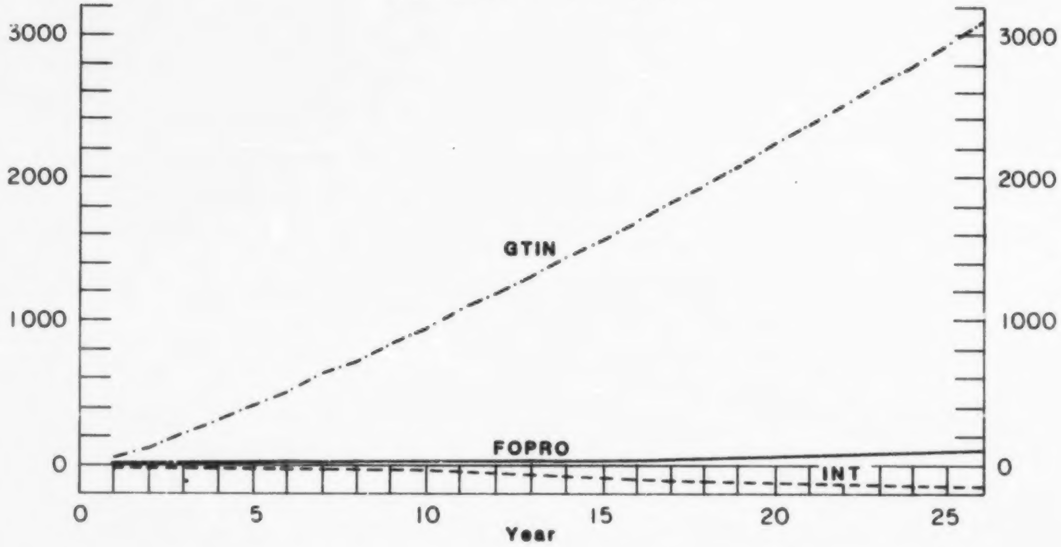


Figure 6-11

GOVERNMENT EXPENDITURE SHOCK: ASSET PRICES
Shock Minus Control %

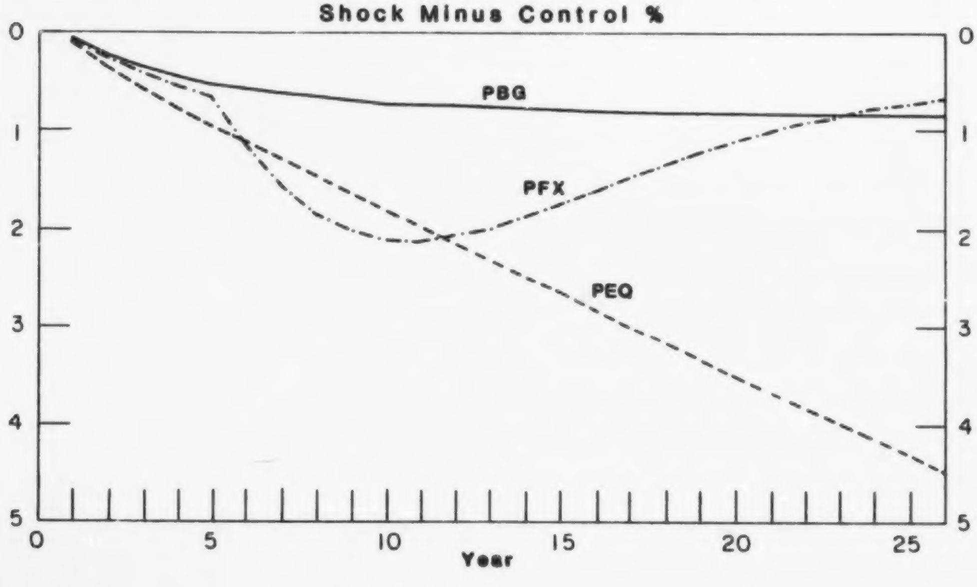
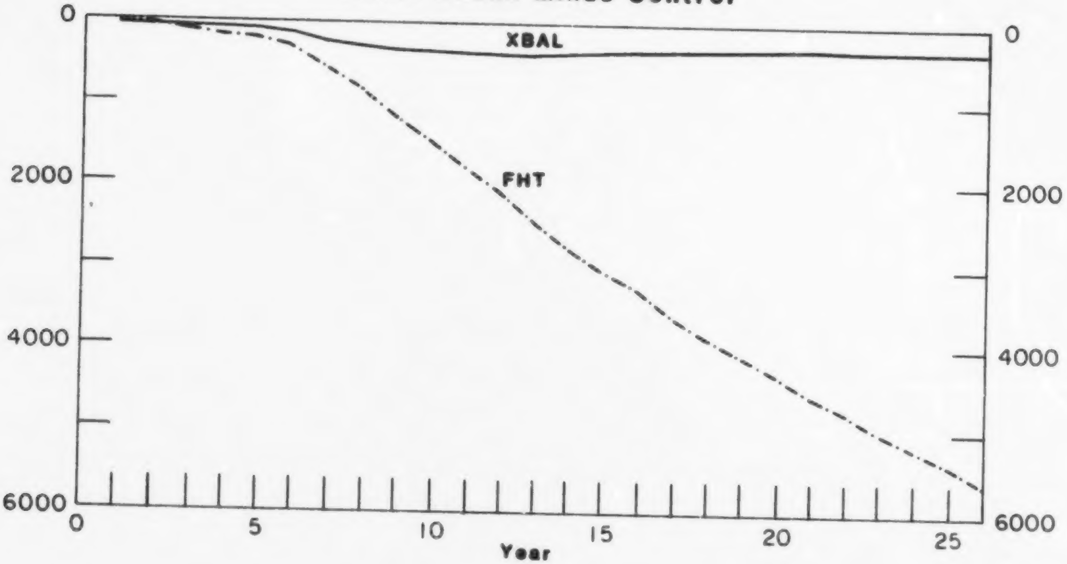


Figure 6-12

GOVERNMENT EXPENDITURE SHOCK: BALANCE OF PAYMENTS
Level Shock Minus Control



In Figure 6-11 we show the evolution of asset prices. It is the price of equity, PEQ, and the exchange rate, PFX, that continually change in response to the shock. Essentially, these are the only prices that are free to move. The price of bonds stabilizes because eventually the bond stock rolls over. The price of equity drops as the rate of return on equity rises in response to the rising relative bond interest rates. The exchange rate eventually undergoes a perpetual real depreciation as households borrow more from foreigners in an effort to accommodate the increasing proportion of government bonds in their portfolios. The increasing level of foreign indebtedness by the household sector is evident in Figure 6-12, where a current account XBAL in constant deficit implies an ever-increasing stock of foreign indebtedness, -FHT. Essentially, the perpetual deficit forces the private sector to borrow in foreign markets and lend the proceeds to domestic governments.

Chapter 7

MARKET DYNAMICS: THE DETERMINATION OF PRICES AND WAGES

7.1 Introduction

In this chapter we describe the determination of the domestic price of the non-energy good, PD, and the price of labour, W. Up to this point we have documented all elements of supply and demand in the product and labour markets; all that remains is to establish how these elements come together to determine prices and wages.

We also describe the accounting identities and other equations that link the rest of the prices in the model to PD, W and exogenous variables. There are eight such prices in SAM: PMNEID, the price of non-energy imported goods; PI, the investment deflator; PG, the government expenditure deflator; PXNEID the price of non-energy exports; PEN, the domestic energy price; PC, the price of the consumption bundle (a combination of domestic and imported goods); P, the average selling price for non-energy firms (a combination of domestic, PD, and foreign, PXNEID, sales prices); and WG, the government sector wage. These prices share the characteristic that, although endogenous to the model, they are not determined directly by the forces of supply and demand in the primary markets of SAM.

The reader will recall from Chapter 6, that in SAM the demands and supplies of assets are equated in each period. Asset prices and yields are considered free to vary to clear the asset markets. The markets for labour and the domestic non-energy good are treated differently.

Flow demand in the product market is satisfied in every period, but the price does not move to clear the market in the same sense as in asset markets. There is some price movement in response to excess demand, but the primary mechanisms for adapting to fluctuations in flow demand are the use of inventories as a buffer stock, variation in the degree of capacity utilization, variation in trade, and (to a lesser extent) variation in factor use. Chapter 4 contains a detailed discussion of this process. Here, the key point is that although the output market 'clears' in every period, firms deliberately move off the long-run supply curve. The effect is to limit the fluctuations of the product price in the market. This is not to say that firms are treated as price setters. Price determination is a market process, influenced by the decisions of both demanders and suppliers. Prices are not set by the direct decisions of any particular agent or group.

We treat the labour market in a similar fashion. A market process determines the evolution of wages. This process is influenced by the decisions of both demanders and suppliers; no single agent 'sets' wages. Wages do not vary to equate demand and supply in every period, and SAM provides no explicit explanation of why. Any of the standard explanations of rigidites, such as contracts, could be used, but such explanations would be outside the formal model. In any case, SAM's markets allow involuntary unemployment and excess employment. Demand for labour is always satisfied; suppliers adapt and move off the long-run supply curve.

The chapter is organized as follows. In section 7.2 we provide an overview of the market processes that provide SAM's wage and price dynamics. We then examine, in section 7.3, the detailed specifications of the equations and the estimates. Finally, in section 7.4, we document the identities and link equations involving prices and wages.

7.2 An Overview of Price and Wage Determination

7.2.1 Determination of the domestic price of the non-energy good:PD

One major difference between SAM and most other models is SAM's explicit use of level equilibrium conditions as forces in adjustment processes. Price determination provides a good example. Consider a simplified version of SAM's equation for PD, the domestic price of the domestic non-energy good:

$$\begin{aligned} \mathrm{J1D(log(PD))} &= \alpha\mathrm{log(PS/PC)} + \beta\mathrm{log(SALES/UGPCSS)} \\ &\quad + \gamma\mathrm{log(INVCD/INVC)} + \mathrm{DNPX.} \end{aligned} \qquad (7.1)$$

The rate of change of PD is postulated to depend on the extent of price level disequilibrium and on the state of excess demand. Excess demand is measured in the flow dimension by a sales gap and in the stock dimension by an inventory gap. There is also an underlying trend, DNPX, that contains expectations and other influences that persist in a steady state when all gap terms are zero.

SAM contains mechanisms that guarantee that any product market excess flow demand will eventually be eliminated, and that firms will eventually adjust inventory stocks to the desired levels. As this adjustment occurs, the sales and inventory gap measures go to zero, and equation (7.1) simplifies to a statement that domestic prices grow at the trend rate, DNPX, as long as price level equilibrium has been established. Recall (from Chapter 3) that the nominal level equilibrium condition is that real-balance preferences be satisfied. This in turn requires that PC, the consumption price index, attain its equilibrium value, PS. In the equation for PD, we specify that if prices are too low (high), in the sense of PC being below (above) PS, there will be an increase (decrease) in the rate of change of PD. Since PC is an index based on PD (and the price of imports),[1] the influence of the level disequilibrium passes through PD to PC and will continue to do so until level equilibrium is generated. Thus, equation (7.1) describes the process in SAM that ensures nominal levels attain their equilibrium values. This technical analysis of the dynamics reflects a fundamental economic assumption used in SAM - that markets possess basic equilibrating forces that tend to bring the overall system to full equilibrium. This is the essence of what we mean by a 'market process'. No agent explicitly solves for the market-clearing solution and imposes that solution on the system. The solution is simply generated by the dynamics of the marketplace.

Although the nominal levels equilibrium condition provides the dominant long-run property of equation (7.1), the state of excess demand in the product market plays an important role in the short and medium runs. SALES represents the current state of aggregate demand in the flow dimension. It consists of demands from various sources: the domestic household sector (consumption net of non-energy imports), the domestic corporate sector (gross investment including normal inventory accumulation), the government sector (non-wage expenditures) and foreigners (non-energy exports):

---

[1] The exact equation linking these prices is given in section 7.4.

EQ78UMD
$$\begin{aligned}\mathrm{SALES} = {} & \mathrm{CON} + \mathrm{IEN} + \mathrm{IC} + \mathrm{GEXPNW}/\mathrm{PG} \\ & + \mathrm{XNEID} - \mathrm{MNEID} + \mathrm{DNUCSS}*\mathrm{INVCD}.\end{aligned}$$

'Normal' inventory growth is included, rather than actual changes in inventories, because of the use of inventories to buffer shocks. If demand falls unexpectedly, then inventory stocks will rise. It is not appropriate to count this as 'sales' when deriving a measure of flow excess demand. It is, however, appropriate to count the growth in desired inventories, because such growth is part of the demand that must be satisfied from potential output. Hence, we add a term that represents the steady-state increase in inventory stocks -- the level of desired inventories, INVCD, times the steady-state real growth rate of the non-energy sector, DNUCSS.

To measure excess demand, the measure SALES is compared with the model's endogenous measure of potential output from the non-energy sector, UGPCSS. If there is excess demand according to this measure, then the domestic price of the non-energy good will rise faster than it otherwise would, ceteris paribus. One advantage of a fully articulated model such as SAM is that it is possible to derive measures of excess demand that are firmly based on consistent representations of demand and supply.

A market can generate flow equilibrium, at least temporarily, wherein flow demand is equal to flow supply, but where stock equilibrium has not been attained. We add a stock measure to the representation of the way excess demand may be transmitted to prices. Specifically, we add the gap between desired and actual inventory stocks.[2]

---

[2] An earlier attempt to reconsider stock effects in price-dynamics equations is found in Rose, Selody, and Masson (1982). Many other models now contain similar structures.

We often refer to the trend term, DNPX, as the expected rate of inflation, because in steady state expectations must be consistent with the actual equilibrium rate of inflation, and away from steady state expectations play a major role in market dynamics. The price dynamics equation, however, does not represent anyone's conscious choice. It is a market process, a result of demand and supply influences, and it is affected by the decisions of many agents. As such, DNPX is best thought of as an underlying trend that reflects the fundamental determinants of the equilibrium inflation rate. In considering equation (7.1) the reader might be misled into thinking that since there are other mechanisms to drive real excess demand to zero, PD can rise at any arbitrary DNPX. In other words, one might think that actual inflation is whatever is expected, in a causal sense. This is not so. The level equilibrium condition, that PC must go to PS, will eventually force the actual inflation rate to conform to the money-growth setting, and expectations must adapt. If, for example, expectations were wrong such that DNPX was greater than the equilibrium inflation rate, given by the exogenous money growth rate less the real growth rate, and if PC were initially at PS, then the higher-than-equilibrium inflation rate for P would force PC to rise faster than PS. As a result, a level equilibrium gap would open and would serve to moderate the actual inflation rate. Inflation expectations would gradually adjust and DNPX would move to the 'correct' equilibrium value. This would be true regardless of whether expectations were specified to be forward looking (and to consider money growth) or simply backward-looking functions of recent inflation. However, if expectations, or whatever is behind DNPX, are specified to be totally exogenous and not responsive to actual price changes, then unless the monetary authority accepts the exogenous DNPX and sets money growth accordingly, the model has no consistent solution.

Money influences prices through three mechanisms in equation (7.1). First, to the extent that a monetary shock disturbs flow or stock equilibrium in the product market, prices will respond (the sales and inventory gap terms). Second, since SAM's expectations are forward looking, money-growth shocks will influence inflation expectations and actual inflation (through DNPX). Finally, there is the money market signal -- the extent of disequilibrium in the level of real balances, measured here as the extent of price level disequilibrium. There are many ways one can think about this last link. Our preferred explanation is that it represents the market process of establishing overall equilibrium. One can also consider it a forward-looking, price-level expectations mechanism; a complement to the growth rate consideration in DNPX. In a similar vein, one could note that an excess supply of real balances (such that PS exceeds PC) signals a latent excess demand for goods. Indeed, both consumption and investment demand are responsive to the monetary disequilibrium.

In models requiring real disequilibrium before prices (or wages) change, measures of excess demand provide the core of the model. SAM can run in this mode if we cut off the forward-looking expectations and level equilibrium terms. But the use of an explicit equilibrium condition in an equation changes the role of excess-demand measures in dynamics. After the first few periods of a shock, excess-demand measures tend to become a source of ongoing cycles, rather than a force towards full equilibrium.[3] As a result, we find it useful to view the estimated coefficients on the excess-demand variables as valid for short-run analysis, but not necessarily for longer-term dynamics (when the shock has become clear and agents can adjust their behaviour). In simulation, we sometimes shut the product-market terms off after the first few years of a shock, or at least reduce their relative importance. Such adjustments are not automatic; they are left to the user in simulation.

Despite its emphasis on equilibrium, SAM is not configured to generate extreme rational expectations results, wherein fully anticipated monetary shocks pass directly into prices without disturbing real solutions. For such a property one would require a set of parameters that give high weight to the price level gap and inflation expectations that respond quickly to changes in the money growth rate. Moreover, one would require that there be no sources of non-neutrality elsewhere in the model, for example in the process of wage determination. None of these things is a feature of the standard version of SAM. But SAM is easily configured to emulate such a world through changes in the parameters.

---

[3] See Rose, Selody, and St-Gelais (1985) for an exploration of these issues.

7.2.2 Determination of the money wage:W

Many of the points made for price dynamics also apply to wage dynamics. A simplified form of our model is:

$$\mathrm{J1D}(\log(\mathrm{W})) = \alpha \log(\mathrm{WS}/\mathrm{W}) + \beta(\mathrm{RNAT}-\mathrm{RNU}) + \mathrm{DNPX} + \mathrm{DNPRX}. \qquad (7.2)$$

The rate of change of the wage is specified to depend on the level equilibrium condition for wages, the state of excess demand for labour, and trend terms that capture inflation expectations and productivity growth.

In Chapter 4 the conditions of equilibrium in the product market are documented. In particular, we show that there is a unique real efficiency wage, WRESS, consistent with full equilibrium in the product and factor markets and the zero-excess-profits condition. From this real efficiency wage and the equilibrium price level there follows an equilibrium nominal wage, WS. For full equilibrium in the model the actual money wage, W, must adjust to WS. The first term in equation (7.2) postulates a market force taking the wage towards its zero-excess-profits, equilibrium value.

The second term represents the influence of excess demand in the labour market. RNAT is the exogenous level of unemployment that will prevail in full equilibrium. In general, there is no necessary connection between the equilibrium level of unemployment and endogenous variables in a macro model. In particular, factors that change the level of supply and/or demand for labour and change the equilibrium level of employment need not change the equilibrium level of unemployment.[4] RNU is the endogenously determined rate of unemployment. Thus, RNAT-RNU measures the state of excess demand in the labour market.[5]

There are two components of trend wage growth. One, DNPRX, is associated with real wage growth. The trend in real wages is associated with productivity gains. In full equilibrium, the real wage must rise at the rate of labour-embodied technical progress. The other, DNPX, is a pure inflation effect. For simplicity of exposition in this introductory discussion we have used the same trend inflation in equations (7.1) and (7.2). This reflects the fact that, in steady state, the nominal efficiency wage must rise at the expected (and actual) rate of price inflation. In the full model, described in section 7.3, we take into account some short-run factors, such as the difference between expectations of firms (who care about the producer price) and expectations of consumers (who care about the price of the consumption bundle). These details are important only for short-run dynamics, since the two inflation rates will be equal in full equilibrium.

SAM does not require the unemployment gap to generate equilibrating movements of the wage, as is the case in many other models. In fact, we view the role of the unemployment gap as that of providing the impact and short-run effects of shocks, and not the long-run adjustment mechanism.

---

[4] This is not to say that there cannot be any such link. However, standard explanations of variables like RNAT begin with search costs, the role of welfare and unemployment benefit rates, minimum wages, demographics and so on. They emphasize microeconomic phenomena and not macro variables. While there are possibilities in SAM for experiments in which the natural rate is made endogenous in simulation, we are content to leave RNAT exogenous in the standard model. But see the discussion of the labour market in section 1.2.2 for a description of short-run trade-offs and the concept of an endogenous non-inflationary rate of unemployment.

[5] The unemployment gap appears linearly in equation (7.2). Readers of early SAM papers (for example, Rose, Selody and Masson, 1982) may recall that we once used a non-linear form, whereby the effect of the gap increased with its size. Once we had introduced the level equilibrium conditions into the dynamics, however, we found it unnecessary to go beyond the simple linear form. Moreover, because of the narrow range of experience in our sample, it is very difficult to say anything, empirically, about the functional form. Although there is more support for non-linearity in the effect of unemployment on wage changes than for non-linearity in the effect of excess demand for goods on price changes, neither non-linearity significantly improves the historical explanatory power of the model. For models that rely only on quantity 'gaps' to move wages or prices, the appeal of non-linearity for counterfactual simulations remains. For us, however, the level equilibrium conditions provide the same extra effect as the state of disequilibrium worsens and the quantity gaps are less important.

The level-equilibrium term in the wage equation provides stabilizing adjustment in the full model. The unemployment gap is not necessarily stabilizing, however. Sometimes, for full equilibrium, the real wage must move in a direction counter to that indicated by the unemployment gap. For a full equilibrium one must have equality of demand and supply in both the product and labour markets. While a lower wage in the face of an excess supply of labour will lead to a higher quantity of labour demanded through a price substitution effect, there is also an income effect, and this can dominate. Indeed, unemployment can in principle reflect deficient demand resulting from a wage that is too low for full equilibrium. One advantage of SAM is that the requirements of full equilibrium can be computed and used to condition dynamics. In simulation, we often allow the unemployment effect to operate fully only for the first few years, reducing its importance thereafter. Technically, the model will handle shocks regardless. But the convergence to full equilibrium may be longer and more cyclic than users will find reasonable if cycle-amplifying influences are left to operate over long periods of simulation time with estimated coefficients.

It is useful to emphasize that SAM does not postulate that a real gap must open before there is wage response to a nominal shock. Money has a direct influence through WS. In some models monetary policy can only work through a real disequilibrium channel (an unemployment gap) or indirectly through inflation expectations. This is not the case in SAM. Indeed, with a particular parameterization, SAM can emulate the extreme rational expectations world where fully anticipated nominal shocks pass directly into wages without the necessity of a real disequilibrium. This is not, however, the parameterization one obtains in estimation.

7.3 The Price and Wage Equations: Structure and Estimates

7.3.1 The domestic price of the non-energy good:PD

The equation we estimate is:

```
EQ82UMP  .01*J1P(PD) = DNPX + AP50*J2A(LOG(PS/PC))
                       + AP51*J2A(LOG(SALES/UGPCSS))
                       + AP52*J2A(LOG(INVCD/J2A(INVCT)))
                       + AP53*J1D(LOG(1+RINDT)),
```

This equation has two features not described in the previous section. The first is an explicit transformation from continuous to discrete time. The basic equation is presumed to hold in continuous time and is integrated twice to obtain a form that can be estimated using discrete-time data. This results in a transformation of the gap variables to averages.[6] The second feature is the introduction of a variable that records the impact of indirect tax changes on domestic prices in the short run.[7] The term will operate only in a period where there is a change in the tax rate. The incidence of the tax, in the initial period, is represented by the parameter AP53. Thereafter, adjustment occurs and the final incidence of the tax depends mainly on what happens to PS as a result of the tax change. If PS were unaffected, the influence of indirect taxes on market prices would be transient; the short-run influence would be gradually dissipated in the adjustment to level equilibrium. There is no tax term in the PS equation, but there could be tax effects through a general equilibrium influence of taxes on real interest rates or wealth.

In a full-equilibrium steady state, with a constant indirect tax rate and a constant inflation rate, all terms in EQ82UMP are zero except for

---

[6] The analysis of this transformation is identical to that in section 6.3, and will not be repeated here. The reader is referred to that section for details.

[7] The functional form is derived as follows. The market price and the price at factor cost are linked by PM = PFC(1+R), where R is the indirect tax rate. In continuous time, using D as the differentiation operator, Dlog(PM) = Dlog(PFC) + Dlog(1+R). We use the tax term with a discrete-time difference in EQ82UMP.

DNPX, the trend inflation rate. Note in particular that no real output gap is necessary for prices to rise at the equilibrium inflation rate once inflation expectations have adjusted and other inertias have been dissipated. Similarly, the price-level gap, log (PS/PC), is zero in steady state. Both PS and PC rise at the equilibrium inflation rate and the level gap remains zero.

In steady state the actual inflation rate will be the money-growth rate less the real growth rate, and this will be expected. Recall from section 6.3.1 that our formulation of expected inflation involves a forward-looking weighted average of the expected path of future inflation. For the domestic price, PD, the specific inflation expected is:

```
EQ01UFE  DNPDE = AE12*J3A(.01*J1P(PS))
                 + (1-AE12)*J1L(J2A(.01*J1P(PD)),
```

with 75% of the weight given to recent actual rates of change of PD. In steady state, when interest rates are constant and real wealth is growing at the exogenous real growth rate, the rate of change of PS will stabilize at the money growth rate less that same real growth rate. Assuming for the moment that DNPX converges to that equilibrium inflation rate, we can see from EQ82UMP that actual inflation will conform. Finally, we can see from EQ01UFE that expectations will also converge on the steady-state value.

For simulations over future periods we normally do not distinguish between trend and expected inflation rates. There is no distinction of principle, however, between actual trends and expected trends. There can be systematic expectations errors when inflation rates are changing dramatically. In addition, there can be other specific influences on actual price changes that are not captured by the gap varaibles in EQ82UMP. A good example is a price-controls program such as that experienced in Canada under the Anti-Inflation Board (AIB).[8] We allow for this effect and for other systematic differences between trend and expected inflation in the historical sample:

$$DNPX = AP54 + D69*(AP55+DNPDE) + AP56*D7678. \qquad (7.3)$$

The D7678 term represents the AIB period. The other terms reflect the fact that we make the trend (and expected) inflation rate, prior to 1969, a constant to be estimated, AP54. The DNPDE formula becomes operational at the beginning of 1969. We do not, however, force equality of DNPX and DNPDE, even after 1969. We introduce the coefficient AP55 to allow for a possible difference. If AP55=AP54 then DNPX and DNPDE are equal, except for the AIB period.

The results of estimation by ordinary least squares are reported in Table 7.1. We provide both the free estimates and those obtained when the initial incidence of indirect tax changes is constrained to unity (i.e., full proportional pass-through). Note that according to the unconstrained point estimates there is a greater than one-for-one immediate pass-through of indirect tax changes. But the restriction to proportional pass-through is not rejected,[9] and we retain the results with the restriction for SAM. The other coefficients are not much affected in any case.

The constant shift, AP55, is not sufficient to cancel AP54. Indeed, in a formal test the restriction is clearly rejected.[10] Thus, over the sample there is a significant trend inflation over and above our measure of expected inflation. In the later part of the sample, this extra trend is about 1.5% per annum. We remove this residual constant rapidly over 1982-83 to reflect the considerable decline in the actual rates of change

---

[8] Many researchers have found that, regardless of the model, the AIB period requires special factors. We, too, find it necessary to control for this period with a special-factors dummy, so that the other parameters of the model can be appropriately quantified. It is arbitrary, however, whether we consider this effect part of 'expectations' or part of the independent trend (as we do in the text).

[9] The chi-squared is only 0.4, far below the critical value of 3.8 for the likelihood ratio test of the restriction at the 95% confidence level.

[10] The chi-squared is over 10, sufficient to reject the restriction at the 99% confidence level.

Table 7.1

PARAMETER ESTIMATES: DOMESTIC GOOD, RATE OF PRICE CHANGE

		Unconstrained		AP53 = 1.0	
Variable	Parameter	Point estimate	t-ratio	Point estimate	t-ratio
Price Gap	AP50	0.2147	2.1	0.1930	2.1
Sales Gap	AP51	0.2794	2.5	0.2618	2.5
Inven. Gap	AP52	0.3785	1.4	0.3531	1.4
Indir. Tax	AP53	1.375	1.9	1.0	constrained
Constant	AP54	0.0208	3.9	0.0217	4.4
69 Shift	AP55	-0.0054	0.7	-0.0066	0.1
AIB	AP56	-0.0575	5.4	-0.0554	5.3
Sample 1961-81		RSQ = 0.883 DW = 1.87		RSQ = 0.880 DW = 1.74	

of prices. For simulation over future periods our default specification is that the trend inflation rate is the expected rate. This is easily changed as required in particular experiments.

Now consider the demand-pressure variables in the model. The flow excess-demand measure, the SALES gap, is statistically significant and economically important. According to our estimates, a flow excess demand of 1% will cause price increases to be just under 0.3 percentage points higher than would otherwise be the case. The inventory stock effect on prices is larger, in proportional terms, according to our point estimates, but less well determined.

The measure of price level disequilibrium also produces an important and statistically significant effect. According to our point estimates, a 1% price-level gap alters the current rate of price change by about 0.2 percentage points in the 'correct' direction (to establish the level equilibrium condition).

Finally, we find a very powerful special effect during the period 1976-78. Our result cannot tell us why this occurred -- whether it is an AIB effect or an effect of the announcement of the new monetary regime, or both. But the effect is large, sufficient to reduce the overall 'trend', DNPX, to about 3.6 percentage points below DNPDE, the 'expected' rate of inflation (defined to exclude any announcement effects).

Intending to test for a disequilibrium effect of wages on prices, we added a wage-gap term to equation EQ82UMP. This term measured the wage, W, relative to the model's computed equilibrium value, WS (i.e., log(W/WS)). For the wage mark-up interpretation we would expect a positive coefficient -- relatively high wages passing into prices. We found a significant negative effect. This raises an interesting possibility. The wage gap is an alternative measure of the extent of nominal disequilibrium. WS is directly related to PS. It may be that in the historical sample the gap, based on our measure of WS relative to W, provides a good proxy for the extent of nominal level disequilibrium, a substitute for the direct measure using PS and PC. In Table 7.2 we report the results obtained when we invert the wage gap (i.e., to log(WS/W)) and use it as a proxy for the price-level disequilibrium. We provide only the results for constrained proportional pass-through of indirect tax changes. Again, this restriction is not close to being rejected.

The results are very similar to those reported in Table 7.1. But this equation fits a bit better and shows a much stronger excess flow

Table 7.2

PARAMETER ESTIMATES: DOMESTIC GOOD, RATE OF PRICE CHANGE, WAGE GAP AS PROXY FOR PRICE GAP

Variable	Parameter	Point estimate	t-ratio
Price Gap	AP50	0.2241	3.8
Sales Gap	AP51	0.4619	4.4
Inven. Gap	AP52	0.2463	1.2
Constant	AP54	0.0222	5.5
69 Shift	AP55	-0.0080	1.5
AIB	AP56	-0.0503	6.5
Indir. Tax	AP53	1.0	constrained

Sample 1961-81 RSQ = 0.920 DW = 2.29

demand influence on prices. There is a corresponding reduction in the point estimate of the inventory effect. It is worth repeating that the wage effect has the wrong sign for the mark-up interpretation. The results show that when wages are high, relative to the computed equilibrium, there will be downward pressure on prices, as there is on wages. This is true even if we also include the direct price-gap measure in the regression. Evidently, the cycle aspect of mark-up arguments is captured by the other terms, and the longer-term aspects of such arguments by our direct links to money through the trend rate of change of prices and the level-equilibrium effect.

7.3.2 The wage equation

The equation we estimate is:

```
EQ83LMP  .01*J1P(W) = AP60*J2A(LOG(WS/W))
                      + AP61*J2A(RNAT-RNU) + DNPRX
                      + AP62*D7678
                      + AP63*[AP64*DNPE+(1-AP64)*DNPCE]
                      + (1-AP63)*J1L[AP64*J1D(LOG(P))
                            + (1-AP64)*J1D(LOG(PC))]
```

```
with     DNPRX = DNPRL + AP65 + AP66*QTIME.
```

The term DNPRX represents the trend in real wages, notionally productivity growth. DNPRL is the trend productivity growth identified from the estimates of the technology. The presence of AP65 and AP66 is an indication that there are trends in wages that cannot be explained using the productivity growth estimates from the technology. The first two terms in EQ83LMP are the wage and unemployment gaps. Except for averaging, they are as described in section 7.2. The next term is the AIB dummy. It is introduced to extend our test of whether an explanation of the AIB period requires special factors.

The inflation term has two layers. The 'outer' combination, with weights AP63 and (1-AP63), combines a forward-looking, expectations-based adjustment of nominal wages with a purely backward-looking adjustment based on recent rates of price change. The closer AP63 is to 1.0, the more weight is given to the expectations terms. The 'inner' combination allows expectations held by both sides of the market to play a role in nominal wage determination. Firms care about the overall producer price, P. Households care about the consumption bundle price, PC. Both form expectations about the rate of price change that concerns them, and we allow both expectations to have an impact on the actual rate of wage change, constraining the combined effect to have a unit coefficient. In steady state, both prices will inflate at the same rate. In response to shocks in the short run, however, there can be relative price changes that give the dual perspective some importance. This is particularly true when the shock influences the relative price of imports, such that the perspectives of households and firms differ. Note, however, that such influences are transitory, unless the equilibrium wage is affected by the same shock. The constraint that the weight on the two 'perspectives' be the same for expectations as for the lagged actual rates of price change

Table 7.3

PARAMETER ESTIMATES: WAGE EQUATION

Variable	Parameter	Point estimate	t-ratio
Wage Gap	AP60	0.1004	2.7
Unemp. Gap	AP61	2.1878	7.6
AIB	AP62	-0.0040	.6
Forward Weight	AP63	1.0	imposed
DNPE Weight	AP64	0.4803	1.2
Constant	AP65	0.0122	5.6
Trend	AP66	-0.0016	4.9

Sample 1961-81 RSQ = 0.955 DW = 1.85

was imposed as an identifying restriction owing to high collinearity among the price change measures. But, since our estimates lead us to impose AP63 at 1.0, this last issue becomes irrelevant, as the whole second term is omitted under the restriction.

The results of an ordinary-least-squares estimation are shown in Table 7.3. We test and do not reject, at the 95% confidence level, the restriction to 1.0 for AP63. The free point estimate was greater than unity, a result difficult to interpret in the context of our test. We can say that there is a decided empirical preference for the forward-looking expectations (including the influence of money growth) over the measure of recent rates of price change.

The disequilibrium wage gap comes in with the appropriate sign and is statistically significant. The modulus of the coefficient is not large, but is large enough to be important. A 10% wage disequilibrium would, according to our point estimate, lead to a change of 1 percentage point in the rate of wage change, ceteris paribus. The unemployment gap has a large and highly significant effect. It is measured as a proportion (that is RNAT and RNU are decimal fractions, not percentages). So an unemployment gap of, say, 3 percentage points, would imply a measured gap of 0.03, and this would, according to our point estimate, lead to a 6-percentage-point change in the rate of wage change in the first year, ceteris paribus. Although this effect is very strong, there has not been an unemployment gap as large as 3 percentage points in our 1961-81 sample.

Our point estimate of the AIB dummy is very small and insignificantly different from zero. In this we differ from other researchers. We find that the standard model explains wages adequately throughout the AIB period without special dummy factors. This does not mean that the AIB had no effect on wages -- just that what effect there was came through actual and expected inflation, according to our results. It is worth noting that the AIB period coincides with a time when wages were somewhat above their equilibrium levels. As a result, the wage-gap term in our regression accounts for most of the influence picked up by other researchers using AIB dummies. Indeed, what is left is not statistically significant.

Our point estimate of the relative weight of producer and consumer rate-of-price-change expectations indicates about an equal weight on the two. This parameter is not well determined, however, and we cannot claim to have a very precise estimate. The 95% confidence interval spans the whole range of interest.

The technology-identified productivity trend, DNPRL, cannot, on its own, adequately explain the historical trend in real wages. In the first part of the sample especially, wages grow faster than can be explained by inflation and our estimate of productivity growth. By the end of the sample, however, the unexplained part is very small. Strictly speaking, failure of a single technical progress estimate to explain both the real output trend and the real wage trend would be a rejection of the supply model in SAM. We cannot report any such formal test, since we have not imposed the restriction in a joint estimation of the wage dynamics and the production system. We intend to extend the scope of simultaneous estimation to include this test in future work.

In Figures 7-1 and 7-2 we plot the actual and fitted values for the rates of change of the domestic price and the money wage. The fitted values for the rate of change of the domestic price underestimate the data in the final two years of the sample. Otherwise, they are quite accurate. In Figure 7-2 we also show two trend lines. The line marked 'model trend' indicates values obtained using the productivity growth estimates from the technology (plus the expected inflation). The 'adjusted trend' values also include the effect of AP65 + AP66*QTIME. The figure clearly illustrates our point that we cannot come close to explaining real wage growth in the 1960s using estimated productivity growth. Either excess demand effects were enormous or the model as whole is missing something. We do not feel that pure cycle arguments can explain the difference so we have added the exogenous extra trend. It is easy to see that this extra influence is virtually zero by the end of the sample; we remove it completely beginning in 1982. The figure also shows clearly that the model's disequilibrium terms explain a great deal of the historical fluctuation in wage changes.

Figure 7-1

RATE OF CHANGE OF THE DOMESTIC PRICE

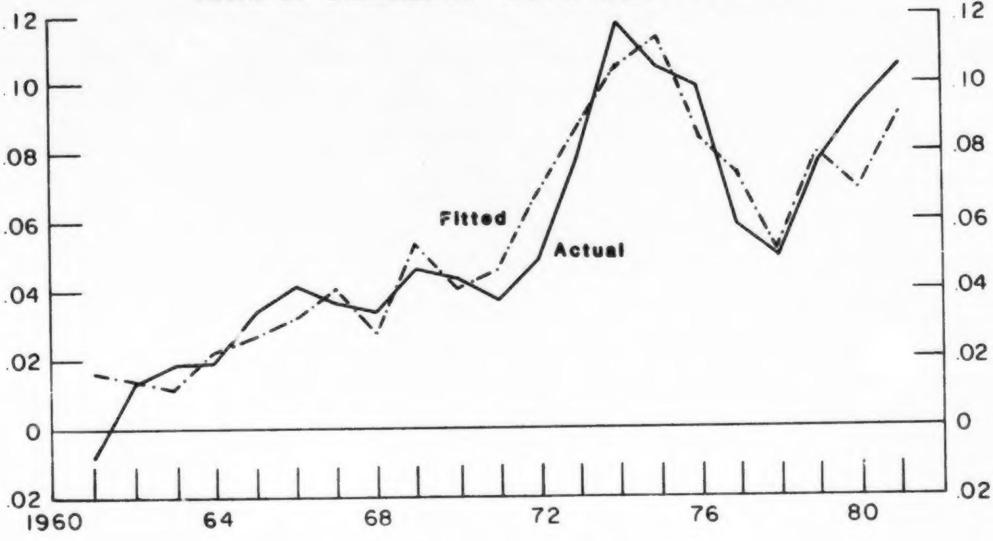
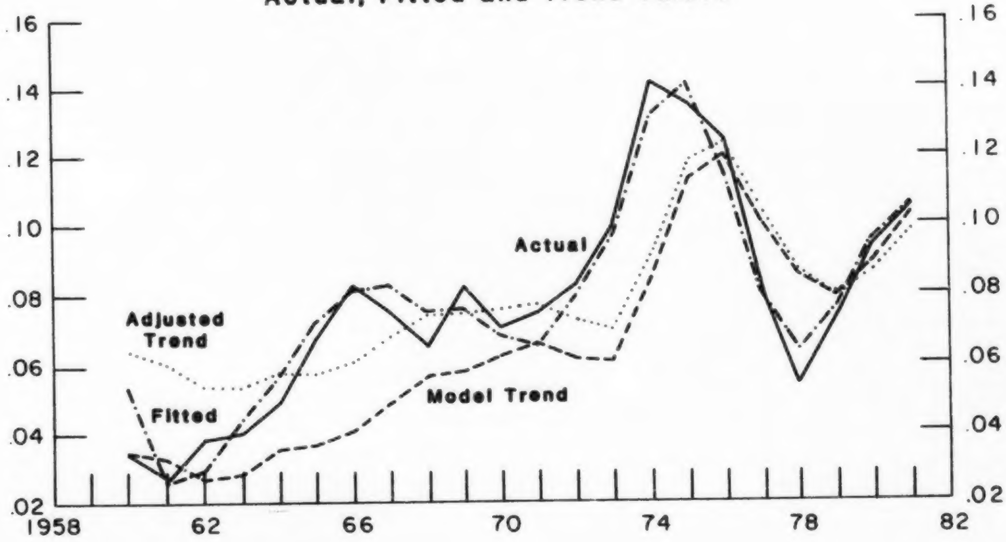


Figure 7-2

RATE OF CHANGE OF WAGES
Actual, Fitted and Trend Values



7.4 Identities and Market-Link Equations

SAM identifies two domestically produced goods (energy and 'the rest') and one imported good. The equations describing energy and import prices are discussed in Chapters 4 and 5, respectively. Essentially, both prices are set in world markets, in real terms, and Canada is presumed to be too small to influence world prices. The first part of this chapter describes the model of the determination of the domestic price of the non-energy good. There are no other markets and so no model-based theory of additional relative price effects is logically possible.[11]

The choice of an accounting framework does not remove the difficulty encountered when one tries to apply that framework to the historical data. In the data there are many relative price movements, for example, that are quantitatively important. Essentially, there are two ways one can proceed. Method one is to adapt the basic data to the framework of the model; for example, by defining appropriate averages and using those averages consistently in the model. Most often this is what we have done in SAM. A good example is our construction of coupon rate, yield and price measures for government debt. But such an approach is not always practicable or appropriate.[12] If one wishes to retain data that do not conform to the abstraction or accounting conventions of the model, such as more than one measure of the price of a notionally distinct good, then to maintain the integrity of the model in simulation one must employ method two. In method two, such measures are linked to appropriate model variables using identities in such a way that any movements in simulation fully respect the abstraction and accounting conventions of the model. Thus, for example, we permit no endogenous movements of relative prices that do not come from the formal markets of the model.[13]

---

[11] It is a common practice in larger macroeconometric models to suppose one production technology and one aggregate good that has multiple end-uses with distinct prices. Thus, for example, the same output can be used in capital formation and in consumption with different prices. In a larger model where extra detail is important or for short-run analysis where users wish to exploit average historical relationships among prices (for example, the timing of the response to shocks), it may be quite reasonable to consider such an approach to modelling. However, for a small model like SAM that emphasizes longer-run properties and tries to exploit the strengths of a fully consistent theory, it makes no sense to permit endogenous changes in such things as relative prices that cannot be given a formal basis in terms of the markets and explicit accounts of the model.

[12] For example, if a variable appears in two distinct identities it may be impossible to find a measurement convention that satisfies all the requirements of the model's abstraction. In other cases, where a suitable data transformation is possible, it is unclear whether using such a transformation prior to using the data in the estimation of behavioural parameters is appropriate.

There is no separate capital-goods industry in SAM and so there can be no formal model of changes in the relative price of such goods.[14] Thus, although we retain the investment deflator in the data for historical measurement, in simulation the capital-goods price is tied to the average price of the domestic good, P, net of indirect taxes:

EQ86UMI $PI = PRELIP*P/(1+RINDT)$

The historical relative price measure, PRELIP, is defined by the identity and is exogenous in simulation.[15] We link PI to a price net of indirect taxes because in Canada most capital-goods purchases are exempt from such taxes.

In an identical manner we link the government non-wage expenditure deflator to the endogenous domestic price:

EQ87UMI $PG = PRELG*PD$,

with PRELG defined by the identity historically and exogenous in simulation.

---

[13] The one exception to this statement is the price of exports. Although exports are notionally the same good sold in the domestic market we allow some endogenous movement of the export price relative to the domestic selling price. The reasons for this and the pertinent equation are provided in Chapter 5.

[14] We refer here only to the purchase price of a unit of capital (the investment deflator); the user cost of capital is a key endogenous variable in SAM.

[15] For simulation over future periods we usually fix the exogenous relative price variables at their last historical values. If the historical data exhibit substantial fluctuation or cycling, however, we use a sample mean, a recent average, or even an autoregressive approach to a constant value.

Just as there is only one domestic non-energy good in SAM, there is only one labour market. So we link the government sector wage,[16] WG, to the endogenous private sector market wage, W:

$$WG = WREL*W. \tag{7.4}$$

This equation does not appear separately in the model, but is embedded in the government expenditure and personal income equations.

The consumption price index, PC, is an important price in SAM. Since it is the price of household consumption, it is used to convert nominals to reals (or vice versa) in all decisions reflecting household preferences. For example, all real asset values are computed using this deflator. A fixed-weight combination of the domestic price, PD, and the import price, PMNEID, marked up by tariffs and indirect taxes, tracks the data quite well. We specify:

```
EQ85UMI  LOG(PC) = AP02*LOG(PD)
                 + (1-AP02)*LOG(PMNEID*(1+RTAR)*(1+RINDT)/AP03),
                     .6786                                1.1883
                     (10.8)

     Sample 1960-81          RSQ = 0.995
```

with AP03 set at 1.1883 to normalize PMNEID*(1+RTAR)*(1+RINDT) to unity in the base period, 1971. Although this equation is treated as an identity in the model, we determine a value for AP02 using ordinary least squares.[17] The point estimate reported above is for AP02. The estimated weight on the domestic price is just under 68%. Although the fixed-weight formulation fits the historical data quite well, it is worth keeping in mind that the weights in such a price index depend, in principle, on the quantity weights in the consumption bundle, and these depend, in turn, on the relative price of domestic and imported goods. In cases where large relative price changes are a feature of the simulation, it might be more realistic to allow for a shift in the weights.[18]

---

[16] The government sector wage differs from the private sector wage in the data because we have separate measures of government wage expenditures and employment. To preserve the identity that expenditures equal employment times a wage, we are forced to permit a second wage measure. This was judged preferable to overriding the government expenditures data or the employment data.

Equation EQ85UMI determines the actual value of the consumption price index and the relative price, P/PC. This relative price is important in SAM. For example, household decisions are influenced by human wealth, which depends on the real wage from the household perspective (i.e., valued using PC). The real wage restriction from the profit-maximization (see Chapter 4) is expressed in terms of firms' average revenue, P. The human wealth valuation thus depends on the endogenous relative price, P/PC. For such subjective valuation equations we do not use the actual relative price -- it tends to be erratic. Instead, we use a smoothed relative price, PRELC, that adjusts gradually towards actual P/PC, and eventually equals that ratio:

EQ69UMI PRELC = J1L(PRELC) + AP20*J1L(P/PC-PRELC).

Parameter AP20 is set at 0.7. This value is roughly what one obtains from estimating EQ69UMI with PRELC defined as a trend through the P/PC data. It implies relatively rapid adjustment in wealth valuation to relative price changes.

---

17. The national accounts consumption deflator is used as a measure of PC for this regression.

18. Using the actual share of domestic good consumption in total consumption as a time series to replace AP02 does not improve the historical fit. However, in simulation one could use changes in the model's equilibrium share of domestic consumption to modify the weight.

The second example of a price index equation that links a composite price to its components is an identity for the average revenue of non-energy firms. Since we retain the various component deflators, such as PI and PG, we must define the link between the average price, P, and the component prices so that the identity that P is nominal output divided by real output is preserved:

```
EQ94UMI   P = (PC*CON+PI*(IC+IEN) + GEXPNW
              + PXNEID*XNEID - PMNEID*MNEID)/
              (UGPC-J1D(INVCT)).
```

Note that since the export price is influenced by world prices (see Chapter 5), the identity provides for a direct influence of world prices on the average price of domestic output. In fact, all component prices are linked to the world price and/or the endogenous domestic price. In particular, P will move with PD (the stochastic domestic price), since PD influences most other component prices directly (e.g., PC via EQ85UMI).

APPENDIX A - Mnemonic Definitions

APPENDIX A-1 Endogenous Variables

APPENDIX A-2 Exogenous Variables

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

BASETR : AVERAGE EXPECTED, REAL, AFTER-TAX, PER CAPITA GOVERNMENT TRANSFERS TO PERSONS

CAPU : CAPACITY UTILIZATION RATE FOR CAPITAL

CCAB : CAPITAL CONSUMPTION ALLOWANCES, PRIVATE SECTOR

CCRDIF : REAL COST OF CAPITAL, DIFFERENTIAL BETWEEN THE DOMESTIC ECONOMY AND THE REST OF THE WORLD

CCRESS : LONG-RUN EXPECTED USER COST OF CAPITAL, REAL EFFICIENCY UNITS

CCRISS : LONG-RUN EXPECTED USER COST OF INVENTORIES

CON : REAL HOUSEHOLD EXPENDITURES ON DOMESTIC GOODS AND IMPORTS

DNPCE : AVERAGE EXPECTED INFLATION, CONSUMPTION PRICE

DNPDE : AVERAGE EXPECTED INFLATION, DOMESTIC PRICE OF NON-ENERGY GOOD

DNPE : AVERAGE EXPECTED INFLATION, AVERAGE UNIT REVENUE OF NON-ENERGY FIRMS

ENC : ANNUAL ENERGY USE BY FIRMS

ENCD : PERCEIVED EQUILIBRIUM ANNUAL ENERGY USE

FHT : NET FOREIGN ASSETS OF HOUSEHOLDS (LIABILITIES IF NEGATIVE)

FOPRO : PROFITS ACCRUING TO FOREIGNERS

GEXPNW : CURRENT-DOLLAR NON-WAGE GOVERNMENT EXPENDITURES

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

GEXPW : CURRENT-DOLLAR GOVERNMENT WAGE EXPENDITURES

GFR : GOVERNMENT FINANCING REQUIREMENT

GNE : REAL (CONSTANT DOLLAR) GROSS NATIONAL EXPENDITURE (NATIONAL ACCOUNTS DEFINITION)

GNE$: CURRENT DOLLAR GROSS NATIONAL EXPENDITURE (NATIONAL ACCOUNTS DEFINITION)

GTIN : ANNUAL INTEREST PAYMENTS ON GOVERNMENT DEBT

IC : ANNUAL GROSS INVESTMENT IN NON-ENERGY SECTOR

INVCD : DESIRED STOCK OF NON-ENERGY INVENTORIES, ANNUAL AVERAGE

INVCT : STOCK OF NON-ENERGY INVENTORIES, END OF YEAR

KCD : PERCEIVED EQUILIBRIUM NON-ENERGY CAPITAL STOCK, ANNUAL AVERAGE

KCT : ACTUAL STOCK OF NON-ENERGY CAPITAL, END OF YEAR

KENT : ACTUAL STOCK OF ENERGY SECTOR CAPITAL, END OF YEAR

KWT : FOREIGN-OWNED CAPITAL STOCK, END OF YEAR (DIRECT INVESTMENT)

LC : EMPLOYMENT IN THE NON-ENERGY PRIVATE SECTOR, ANNUAL AVERAGE

LCD : PERCEIVED EQUILIBRIUM EMPLOYMENT IN THE NON-ENERGY PRIVATE SECTOR

LG : EMPLOYMENT IN THE GOVERNMENT SECTOR

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

LGDT : STOCK OF GOVERNMENT DEBT HELD BY DOMESTIC RESIDENTS, END OF YEAR

LGT : TOTAL DOMESTIC-CURRENCY GOVERNMENT DEBT, END OF YEAR

LS : LABOUR FORCE (PARTICIPATION RATE)

LSS : PERCEIVED EQUILIBRIUM LABOUR FORCE (PARTICIPATION RATE)

MNEID : IMPORTS OF GOODS AND SERVICES EXCLUDING ENERGY AND INVESTMENT-INCOME PAYMENTS

P : PRICE INDEX FOR NON-ENERGY BUSINESS OUTPUT (AVERAGE REVENUE INCLUSIVE OF INDIRECT TAXES)

PBFH : AVERAGE U.S. DOLLAR VALUE OF FOREIGN-CURRENCY ASSETS/LIABILITIES OF HOUSEHOLDS (PAR=1.0)

PBG : AVERAGE PRICE OF OUTSTANDING GOVERNMENT BONDS (PAR=1.0)

PC : PRICE INDEX FOR CONSUMPTION

PD : PRICE OF DOMESTIC OUTPUT IN DOMESTIC MARKET

PEN : ENERGY PRICE INDEX

PENR : REAL ENERGY PRICE AT FACTOR COST

PENRSS : REAL ENERGY PRICE USED BY FIRMS FOR LONG-RUN PLANS

PEQ : EQUITY PRICE INDEX (SUM OF 3 ASSET-DEMAND FUNCTIONS; BONDS, FOREIGN ASSETS, EQUITIES)

PFX : CANADIAN DOLLAR PRICE OF U.S. DOLLAR

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

PFXE : ONE-YEAR-AHEAD, EXPECTED EXCHANGE RATE

PG : GOVERNMENT EXPENDITURE PRICE INDEX

PGNE : GROSS NATIONAL EXPENDITURE DEFLATOR

PI : INVESTMENT PRICE INDEX

PMNEID : NON-ENERGY IMPORT PRICE INDEX

PRELC : LONG-RUN RELATIVE PRICE, FIRMS' AVERAGE REVENUE/ CONSUMPTION PRICE

PS : LONG-RUN EXPECTED CONSUMPTION PRICE INDEX (INVERTED LONG-RUN, REAL-BALANCE PREFERENCES)

PXNEID : NON-ENERGY EXPORT PRICE INDEX

QEQT : STOCK OF DOMESTICALLY HELD CLAIMS TO CAPITAL, END OF YEAR

RAC : AVERAGE COUPON RATE ON OUTSTANDING DOMESTIC-CURRENCY GOVERNMENT BONDS

RACUS : AVERAGE COUPON RATE ON OUTSTANDING FOREIGN-CURRENCY ASSETS/LIABILITIES OF HOUSEHOLDS

RACUSG : AVERAGE COUPON RATE ON OUTSTANDING FOREIGN-CURRENCY ASSETS/LIABILITIES OF DOMESTIC GOVERNMENTS

RATAX : AVERAGE PERSONAL INCOME TAX RATE

RATAXN : AVERAGE EXPECTED VALUE OF AVERAGE PERSONAL INCOME TAX RATE

RDRH : HOUSEHOLD REAL DISCOUNT RATE

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

REQE : EXPECTED RATE OF RETURN ON CORPORATE LIABILITIES

RFE : EXPECTED DOMESTIC HOLDING-PERIOD YIELD ON FOREIGN ASSETS (INVERTED DEMAND FUNCTION)

RG : BEFORE-TAX YIELD TO MATURITY ON GOVERNMENT BONDS

RGE : EXPECTED HOLDING-PERIOD YIELD ON GOVERNMENT BONDS (INVERTED DEMAND FUNCTION)

RMTAX : MARGINAL TAX RATE ON PERSONAL INCOME

RNU : RATE OF UNEMPLOYMENT

ROY : ROYALTIES AND REMITTANCES FROM CROWN CORPORATIONS

ROYH : ENERGY INDIRECT TAXES CALCULATED TO EQUATE RETURNS TO CAPITAL IN ENERGY AND NON-ENERGY SECTORS

RRFINE : EXPECTED REAL RATE OF RETURN, AFTER TAX, ON HOUSEHOLD'S PORTFOLIO

RRGTIN : PORTION OF PERSONAL TAX RATE ASSOCIATED WITH PAYMENT OF REAL INTEREST ON GOVERNMENT DEBT

RRK : GROSS PROFIT RATE (RESIDUAL FACTOR RATE OF RETURN)

RRKSS : MINIMUM-COST, FULL-EMPLOYMENT, ZERO-EXCESS-PROFIT, RETURN TO CAPITAL (NORMAL GROSS PROFIT RATE)

SALES : FINAL SALES WITH NORMAL INVENTORY GROWTH

SALN : AVERAGE EXPECTED FINAL SALES, NON-ENERGY PRIVATE SECTOR

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

TAXIC : INDIRECT TAXES ON DOMESTIC NON-ENERGY GOODS

TAXIP : TARIFFS AND INDIRECT TAXES ON IMPORTS

TAXP : DIRECT TAX REVENUE FROM PERSONS

TCC : PROFITS-TAX REVENUE FROM CORPORATIONS

TOBQ : RATIO OF MARKET VALUE OF CAPITAL TO ITS REPLACEMENT COST

TOBQSS : LONG-RUN RATIO OF THE MARKET VALUE OF CAPITAL TO ITS REPLACEMENT COST

TRANSP : CURRENT-DOLLAR GOVERNMENT TRANSFERS TO PERSONS

TTEN : ENERGY TERMS OF TRADE

UGPB : TOTAL REAL PRIVATE SECTOR OUTPUT IN UGPC UNITS

UGPBSS : ZERO-EXCESS-PROFIT, FULL-EMPLOYMENT, REAL PRIVATE SECTOR OUTPUT (PERCEIVED POTENTIAL OUTPUT)

UGPC : REAL NON-ENERGY OUTPUT

UGPCSS : ZERO-EXCESS-PROFIT, FULL-EMPLOYMENT, REAL NON-ENERGY OUTPUT (PERCEIVED POTENTIAL OUTPUT)

UGPEN : ENERGY OUTPUT

UIB : ANNUAL UNEMPLOYMENT INSURANCE BENEFIT RATE

V : FINANCIAL WEALTH, ANNUAL AVERAGE MARKET VALUE

APPENDIX A-1 ENDOGENOUS-VARIABLE MNEMONICS

VHPV : HUMAN WEALTH VALUED AT AVERAGE EXPECTED REAL WAGE AND EXPECTED UNEMPLOYMENT

VHPVN : HUMAN WEALTH VALUED AT LONG-RUN EXPECTED REAL WAGE AND EXPECTED UNEMPLOYMENT

VHPVT : HUMAN WEALTH, GOVERNMENT-TRANSFER COMPONENT

VHPVU : HUMAN WEALTH - ADJUSTMENT FOR CYCLICAL UNEMPLOYMENT

VHPVW : HUMAN WEALTH - WAGE AND NORMAL UNEMPLOYMENT COMPONENT

VNSS : HUMAN WEALTH VALUED AT LONG-RUN EXPECTED REAL WAGE AND FULL EMPLOYMENT

W : PRIVATE SECTOR ANNUAL WAGE

WRESS : LONG-RUN EXPECTED ANNUAL WAGE, REAL EFFICIENCY UNITS (LONG-RUN, ZERO-EXCESS-PROFIT CONDITION)

WS : LONG-RUN EXPECTED NOMINAL WAGE

XBAL : CURRENT ACCOUNT BALANCE (ADJUSTED FOR UNREPATRIATED PROFITS)

XNEID : EXPORTS OF GOODS AND SERVICES EXCLUDING ENERGY AND INVESTMENT-INCOME RECEIPTS

YB : BUSINESS PROFITS (MODEL CONCEPT)

YBD : BUSINESS PROFITS AFTER PROFITS TAX

YPERS : PERSONAL INCOME

APPENDIX A-2 EXOGENOUS-VARIABLE MNEMONICS

CAPUSS : LONG RUN DESIRED CAPACITY UTILIZATION RATE FOR CAPITAL

CCRW : WORLD REAL USER COST OF CAPITAL

CCTAX1 : COMBINED EFFECTS ON CAPITAL COST OF FISCAL INCENTIVES (EG.INV. TAX CREDIT)

CCTAX2 : COMBINED EFFECTS ON INVENTORY USER COSTS OF FISCAL INCENTIVES (EG. INTEREST DEDUCTIBILITY)

DELK : RATE OF DEPRECIATION OF THE CAPITAL STOCK

DNHE : EXPECTED RATE OF GROWTH OF THE MONETARY BASE

DNPOP : TREND RATE OF GROWTH OF THE POPULATION

DNPRL : TREND RATE OF TECHNICAL PROGRESS (I.E. PRODUCTIVITY GROWTH)

DNUBSS : TREND RATE OF GROWTH OF REAL PRIVATE SECTOR OUTPUT

DNUCSS : TREND RATE OF GROWTH OF REAL PRIVATE SECTOR NON-ENERGY OUTPUT

DUMCAR : DUMMY VARIABLE REPRESENTING THE SHIFT IN TRADE LEVELS DUE TO THE AUTO PACT

FGBT : NET STOCK OF FOREIGN SECURITIES HELD BY DOMESTIC GOVERNMENTS (LIABILITY IF NEGATIVE)

FGRT : STOCK OF FOREIGN RESERVES (END OF YEAR)

GNWT : TREND RATIO OF GOVERNMENT EXPENDITURES ON GOODS AND SERVICES TO NOMINAL OUTPUT

APPENDIX A-2 EXOGENOUS-VARIABLE MNEMONICS

GTRAN : TREND GROWTH RATE OF PER CAPITA GOVERNMENT TRANSFERS TO PERSONS

GWAGE : TREND GROWTH RATE OF THE REAL WAGE

HT : MONETARY BASE (END OF YEAR)

IEN : GROSS REAL INVESTMENT OF THE ENERGY SECTOR

INVENT : STOCK OF ENERGY INVENTORIES (END OF YEAR)

LEN : EMPLOYMENT IN THE ENERGY SECTOR

LGFT : STOCK OF DOMESTIC CURRENCY GOVERNMENT SECURITIES HELD BY FOREIGNERS

LGST : TREND RATIO OF GOVERMENT EMPLOYMENT TO LABOUR FORCE

MEN : REAL IMPORTS OF PRIMARY ENERGY

NK : RATIO OF POPULATION 14 AND UNDER TO POPULATION 15 AND OVER

NPOP : POPULATION 15 AND OVER INCLUDING ARMED FORCES

PCW2 : WORLD COMMODITY PRICE INDEX

PENW : WORLD PRICE INDEX FOR CANADA'S ENERGY TRADE

PRELEN : RELATIVE PRICE, DOMESTIC ENERGY CONSUMPTION/ENERGY TRADE (EXCLUDING POLICY EFFECTS..SEE REN)

PRELG : GOVERNMENT NON-WAGE EXPENDITURES/OUTPUT

APPENDIX A-2 EXOGENOUS-VARIABLE MNEMONICS

PRELIP : RELATIVE PRICE, INVESTMENT/OUTPUT

PRELM : RELATIVE PRICE, IMPORTS/WORLD OUTPUT

PRODL : INDEX OF LEVEL OF PRODUCTIVITY GROWTH

PUS : U.S. IMPLICIT G.N.P. DEFLATOR

PW : WORLD PRICE INDEX

PZ : IMPLICIT PRICE DEFLATOR FOR TERMS THAT CONVERT GROSS PRIVATE DOMESTIC PRODUCT AND GROSS NATIONAL PRODUCT

QTIME : ANNUAL TIME INDEX (1971=0)

QTXRFM : TAX REFORM DUMMY (1972)

RATAXUS : U.S. AVERAGE PERSONAL INCOME TAX RATE

RBTAX : WEIGHTED AVERAGE MARGINAL CORPORATE INCOME TAX RATE

REN : POLICY-DETERMINED RATIO OF DOMESTIC TO WORLD PRICE OF ENERGY

RINDT : RATE OF INDIRECT TAX

RNAT : NATURAL RATE OF UNEMPLOYMENT

ROYEX : DEVIATION OF ACTUAL ROYALTIES FROM LEVEL THAT EQUATES RATES OF RETURN IN ENERGY AND NON-ENERGY SECTORS (UP TO 1973 OPEC)

APPENDIX A-2 EXOGENOUS-VARIABLE MNEMONICS

RRKUS : RETURN TO CAPITAL IN U.S.

RTAR : AVERAGE TARIFF RATE ON NON-ENERGY GOODS AND SERVICES

RTAXB : BASE AVERAGE PERSONAL INCOME TAX RATE (EXCLUDING TAXES ASSOCIATED WITH PAYMENTS OF INTEREST ON THE PUBLIC DEBT)

RTR : PROPORTIONAL DEVIATION OF ACTUAL ROYALTIES FROM LEVEL THAT EQUATES RATES OF RETURN IN ENERGY AND NON-ENERGY SECTORS (AFTER 1973 OPEC)

RUIB : RATIO OF UNEMPLOYMENT INSURANCE BENEFITS TO LONG RUN EXPECTED WAGE RATE

RUS : U.S. INTEREST RATE (FIVE YEAR FEDERAL BONDS)

STIME : SIMULATION TIME INDEX (FIRST PERIOD OF SIMULATION = 1)

TRANSF : GOVERNMENT TRANSFERS TO FOREIGNERS

TST : TARGET RATIO OF GOVERNMENT TRANSFERS TO NOMINAL OUTPUT

WREADJ : HISTORICAL ADJUSTMENT OF LONG RUN EXPECTED REAL WAGE (TO MEAN CONSISTENCY WITH DATA)

WREL : RELATIVE WAGE, GOVERNMENT/PRIVATE SECTOR

XEN : REAL EXPORTS OF PRIMARY ENERGY

YW : REAL WORLD OUTPUT

ZXO1 : BALANCING ITEM, MODEL G.N.E. TO NATIONAL ACCOUNTS DATA

APPENDIX B - Model Equations

```
APPENDIX B   MODEL EQUATIONS
----------   ---------------

EQ00AMP       PBFH: AVERAGE U.S. DOLLAR VALUE OF FOREIGN-CURRENCY ASSETS/LIABILITIES OF HOUSEHOLDS (PAR=1.0)
    PBFH  =  J1L(RACUS)/RUS  +  ( 1 - J1L(RACUS)/RUS ) * EXP(-RUS*(1-RATAXUS)/AS07 )


EQ01UFE       DNPDE: AVERAGE EXPECTED INFLATION, DOMESTIC PRICE OF NON-ENERGY GOOD
    DNPDE  =  AE12 * J3A(.01*J1P(PS))  +  ( 1 - AE12) * J1L(J2A(.01*J1P(PD)))


EQ02UFE        DNPE: AVERAGE EXPECTED INFLATION, AVERAGE UNIT REVENUE OF NON-ENERGY FIRMS
    DNPE  =  AE11 * J3A(.01*J1P(PS))  +  ( 1 - AE11) * J1L(J2A(.01*J1P(P)))


EQ03UHE       DNPCE: AVERAGE EXPECTED INFLATION, CONSUMPTION PRICE
    DNPCE  =  AE10 * J3A(.01*J1P(PS))  +  ( 1 - AE10) * J1L(J2A(.01*J1P(PC)))


EQ04KFI        KENT: ACTUAL STOCK OF ENERGY SECTOR CAPITAL, END OF YEAR
    KENT  =  ( 2./( 2. + DELK ) ) * IEN  +  ( ( 2. - DELK )/( 2. + DELK ) ) * J1L(KENT)


EQ05KMP       CCRDIF: REAL COST OF CAPITAL, DIFFERENTIAL BETWEEN THE DOMESTIC ECONOMY AND THE REST OF THE WORLD
    CCRDIF  =  J1L(CCRDIF)  +  AH98 * J1L(RDRH - CCRW - CCRDIF + DELK + AH96)


EQ06KAD        KWT: FOREIGN-OWNED CAPITAL STOCK, END OF YEAR (DIRECT INVESTMENT)
    .01 * J1P(KWT)  =  DNUBSS  +  AS21 * ( RRK - RRKUS + AS29 )  +  AS24 * ( STIME.LT..5 )
```

```
EQ07IHE        RATAXN: AVERAGE EXPECTED VALUE OF AVERAGE TAX RATE ON PERSONAL INCOME
     RATAXN  =  AG50 * J1L(RATAX)  +  ( 1 - AG50) * J1L(RATAXN)


EQ08AHE        PFXE: ONE-YEAR-AHEAD, EXPECTED EXCHANGE RATE
     J1P(PFXE)  =  AE61 * J1P(PFX*PUS/PC)  +  ( AE63 + AE67*( STIME.GT.7.5 ) ) * J1P(P/PUS)
                   +  100. * (  AE62*LOG(PFX/J1L(PFXE)) - AE64*LOG(SALES/UGPCSS)  )


EQ09EGR        TTEN: ENERGY TERMS OF TRADE
     TTEN  =  REN * PRELEN * ( J1L((TTEN/REN)/PRELEN) + AA04*J1L((PFX*PW*(1+RINDT))/P-(TTEN/REN)/PRELEN) )


EQ10EMP        PENRSS: REAL ENERGY PRICE USED BY FIRMS FOR LONG-RUN PLANS
     PENRSS  =   TTEN * PENW / PW


EQ11KML        CCRISS: LONG-RUN EXPECTED USER COST OF INVENTORIES
     CCRISS  =  ( CCRW + CCRDIF - DELK) * CCTAX2


EQ12IHI        BASETR: AVERAGE EXPECTED, REAL, AFTER-TAX, PER CAPITA GOVERNMENT TRANSFERS TO PERSONS
     BASETR  =  (  AH11 * (TRANSP-TAXIP) / PC + ( 1 - AH11) * UGPBSS * TST  ) / NPOP
```

```
APPENDIX B   MODEL EQUATIONS
----------   ---------------

EQ13IHI         VHPVT: HUMAN WEALTH, GOVERNMENT-TRANSFER COMPONENT
     VHPVT  =  NPOP * BASETR * ( ( EXP(AH35*(AH30*GTRAN+(1-AH30)*AE24-RDRH)) - 1)/( AH30*GTRAN + (1-AH30)*AE24 - RDRH
               ) + ( EXP(AH35*(AH30*GTRAN+(1-AH30)*AE24-RDRH))*(EXP((AH36-AH35)*(AH31*GTRAN+(1-AH31)*AE24-RDRH))-1)
               )/( AH31*GTRAN + (1-AH31)*AE24 - RDRH ) + ( EXP(AH35*(AH30*GTRAN+(1-AH30)*AE24-RDRH)+(AH36-AH35)*(AH31
               *GTRAN+(1-AH31)*AE24-RDRH))*(EXP((AH37-AH36)*(AH32*GTRAN+(1-AH32)*AE24-RDRH))-1))/( AH32*GTRAN + (1-AH32
               )*AE24 - RDRH ) + ( EXP(AH35*(AH30*GTRAN+(1-AH30)*AE24-RDRH)+(AH36-AH35)*(AH31*GTRAN+(1-AH31)*AE24-RDRH
               )+(AH37-AH36)*(AH32*GTRAN+(1-AH32)*AE24-RDRH))*(EXP((AH38-AH37)*(AH33*GTRAN+(1-AH33)*AE24-RDRH))-1))
               /( AH33*GTRAN + (1-AH33)*AE24 - RDRH ) - ( EXP(AH35*(AH30*GTRAN+(1-AH30)*AE24-RDRH)+(AH36-AH35)*(AH31
               *GTRAN+(1-AH31)*AE24-RDRH)+(AH37-AH36)*(AH32*GTRAN+(1-AH32)*AE24-RDRH)+(AH38-AH37)*(AH33*GTRAN+(1-AH33
               )*AE24-RDRH))/(AH34*GTRAN+(1-AH34)*AE24-RDRH) ) )


EQ14KML         CCRESS: LONG-RUN EXPECTED USER COST OF CAPITAL, REAL EFFICIENCY UNITS
     CCRESS  =  ( (CCRW + CCRDIF) * CCTAX1 * PRELIP ) / CAPUSS


EQ15LML          WRESS: LONG-RUN EXPECTED ANNUAL WAGE, REAL EFFICIENCY UNITS (LONG-RUN, ZERO-EXCESS-PROFIT CONDITION)
     WRESS  =  ( 1 - ( CCRISS*(AF01-(AF75*(QTIME.GT.11.5))-(AF02*CCRISS)+AF03*(QTIME.GT.-5.1)*(QTIME.LT.-.9)) ) ) *
               ( ( AF10*((1-AF05)**(1/AF07)))/( (1-(((AF04*(1+((1-AF04)/AF04)*(((PENRSS*AF04)/(CCRESS*(1-AF04)))**(AF06
               /(AF06-1)))))**((AF07*(1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07/(1-AF07)))*(((AF08*AF05)/(1-AF05))**(1
               /(1-AF07)))*((CCRESS/(1-(CCRISS*(AF01-(AF75*(QTIME.GT.11.5))-(AF02*CCRISS)+AF03*(QTIME.GT.-5.1)*(QTIME
               .LT.-.9)))))**-(AF07/(1-AF07)))*(1-AF05)*((AF04/(AF04*(1+((1-AF04)/AF04)*(((PENRSS*AF04)/(CCRESS*(1-AF04
               )))**(AF06/(AF06-1)))))+(1-AF04)/((1-AF04)*(1+(AF04/(1-AF04))*(((CCRESS*(1-AF04))/(PENRSS*AF04))**(AF06
               /(AF06-1))))))**(AF07/AF06))*((AF10*(1-AF05))**(AF07/(1-AF07)))))**((1-AF07)/AF07) ) )  -  WREADJ
```

```
EQ16IHL        VNSS: HUMAN WEALTH VALUED AT LONG-RUN EXPECTED REAL WAGE AND FULL EMPLOYMENT

     VNSS  =  VHPVT  +  AH00 * NPOP * ( ( PRODL*WRESS*(1-RATAXN)*PRELC)/( 1 + RINDT ) ) * ( ( EXP(AH35*(AH25*GWAGE+
              (1-AH25)*AE24-RDRH)) - 1)/( AH25*GWAGE + (1-AH25)*AE24 - RDRH ) + ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24
              -RDRH))*(EXP((AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-RDRH))-1))/( AH26*GWAGE + (1-AH26)*AE24 - RDRH ) +
              ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-RDRH)+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-RDRH))*(EXP((AH37-AH36
              )*(AH27*GWAGE+(1-AH27)*AE24-RDRH))-1))/( AH27*GWAGE + (1-AH27)*AE24 - RDRH ) + ( EXP(AH35*(AH25*GWAGE
              +(1-AH25)*AE24-RDRH)+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-RDRH)+(AH37-AH36)*(AH27*GWAGE+(1-AH27)*AE24
              -RDRH))*(EXP((AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24-RDRH))-1))/( AH28*GWAGE + (1-AH28)*AE24 - RDRH ) -
              ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-RDRH)+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-RDRH)+(AH37-AH36)*(AH27
              *GWAGE+(1-AH27)*AE24-RDRH)+(AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24-RDRH))/(AE24-RDRH) ) ) * ( 1 - RNAT
              + RNAT*( RUIB/(1-(QTXRFM*RATAXN)) ) )


EQ17UML        PS: LONG-RUN EXPECTED CONSUMPTION PRICE INDEX (INVERTED LONG-RUN, REAL-BALANCE PREFERENCES)

     LOG(PS)  =  LOG(J2A(HT))  -  AS00  +  AS01 * ( RDRH + DNHE - DNUBSS )  -  LOG(VNSS+V/PC)


EQ18LML        WS: LONG-RUN EXPECTED NOMINAL WAGE

     WS   =   PS * PRELC * PRODL * WRESS / ( 1 + RINDT )


EQ19LHL        LSS: PERCEIVED EQUILIBRIUM LABOUR FORCE (PARTICIPATION RATE)

     LSS / NPOP  =  AH00  -  ( ( AH01*AH02*((1+NK)**AH03)*((V/PS+VNSS)/NPOP))/( (1-RMTAX+AH02*(1-RATAXN)*
                    ((1+NK)**AH03))*(WS/PS)*(1-RNAT+RUIB*RNAT*((1-RMTAX)**(QTXRFM-1))) ) )
```

```
APPENDIX B   MODEL EQUATIONS
----------   ---------------

EQ20LFL          LCD: PERCEIVED EQUILIBRIUM EMPLOYMENT IN THE NON-ENERGY PRIVATE SECTOR
     LCD  =  ( 1 - RNAT - LGST ) * LSS  -  LEN


EQ21UFL          UGPCSS: ZERO-EXCESS-PROFIT, FULL-EMPLOYMENT, REAL NON-ENERGY OUTPUT (PERCEIVED POTENTIAL OUTPUT)
     UGPCSS  =  LCD * AF10 * PRODL * ( ( (1-AF05)*(1+((AF04*(1+((1-AF04)/AF04)*(((PENRSS*AF04)/(CCRESS*(1-AF04)))**
                (AF06/(AF06-1)))))**((AF07*(1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07/(1-AF07)))*(((AF08*AF05)/(1-AF05
                ))**(1/(1-AF07)))*((WRESS/CCRESS)**(AF07/(1-AF07)))))**( 1/AF07 ) )


EQ22KFL          KCD: PERCEIVED EQUILIBRIUM NON-ENERGY CAPITAL STOCK, YEAR AVERAGE
     KCD  =  ( UGPCSS/( AF10*CAPUSS ) ) * ( ( AF08*AF05)**( - 1./AF07 ) ) * ( ( AF04*(1+((1-AF04)/AF04)*(((PENRSS*AF04
             )/(CCRESS*(1-AF04)))**(AF06/(AF06-1)))))**( - 1./AF06 ) ) * ( ( (((AF04*(1+((1-AF04)/AF04)*(((PENRSS*AF04
             )/(CCRESS*(1-AF04)))**(AF06/(AF06-1)))))**((AF07*(1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07/(1-AF07)))*(
             ((AF08*AF05)/(1-AF05))**(1/(1-AF07)))*((WRESS/CCRESS)**(AF07/(1-AF07))))/(1+((AF04*(1+((1-AF04)/AF04)*
             (((PENRSS*AF04)/(CCRESS*(1-AF04)))**(AF06/(AF06-1)))))**((AF07*(1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07
             /(1-AF07)))*(((AF08*AF05)/(1-AF05))**(1/(1-AF07)))*((WRESS/CCRESS)**(AF07/(1-AF07)))))**( 1/AF07 ) )


EQ23EFL          ENCD: PERCEIVED EQUILIBRIUM ANNUAL ENERGY USE
     ENCD  =  ( UGPCSS/AF10) * ( ( AF08*AF05)**( - 1./AF07 ) ) * ( ( (1-AF04)*(1+(AF04/(1-AF04))*(((CCRESS*(1-AF04))
              /(PENRSS*AF04))**(AF06/(AF06-1)))))**( - 1./AF06 ) ) * ( ( (((AF04*(1+((1-AF04)/AF04)*(((PENRSS*AF04)/
              (CCRESS*(1-AF04)))**(AF06/(AF06-1)))))**((AF07*(1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07/(1-AF07)))*((
              (AF08*AF05)/(1-AF05))**(1/(1-AF07)))*((WRESS/CCRESS)**(AF07/(1-AF07))))/(1+((AF04*(1+((1-AF04)/AF04)*(
              ((PENRSS*AF04)/(CCRESS*(1-AF04)))**(AF06/(AF06-1)))))**((AF07*(1-AF06))/(AF06*(1-AF07))))*(AF04**(AF07
              /(1-AF07)))*(((AF08*AF05)/(1-AF05))**(1/(1-AF07)))*((WRESS/CCRESS)**(AF07/(1-AF07)))))**( 1/AF07 ) )
```

```
EQ24UFE        SALN: AVERAGE EXPECTED FINAL SALES, DOMESTIC NON-ENERGY PRIVATE SECTOR
     SALN  =  AF40 * SALES  +  ( 1 - AF40) * UGPCSS


EQ25UFL         INVCD: DESIRED STOCK OF INVENTORIES, NON-ENERGY SECTOR, YEAR AVERAGE
     INVCD  =  ( AF01 - AF75*(QTIME.GT.11.5) - AF02*CCRISS + AF03*(QTIME.GT. -5.1)*(QTIME.LT. -.9) ) * SALN


EQ26AMI        RAC: AVERAGE COUPON RATE ON OUTSTANDING DOMESTIC-CURRENCY GOVERNMENT BONDS
     RAC  =  J1L(RAC) * ( (J:D(LGT)/J2A(LGT)).LT. -AS06 )  +  (  J1L(RAC) * (J1L(LGT)/LGT - AS06)
          + RG * (J1D(LGT)/LGT + AS06)  ) * ( (J1D(LGT)/J2A(LGT)).GE. -AS06 )


EQ27AMI        RACUS: AVERAGE COUPON RATE ON OUTSTANDING FOREIGN-CURRENCY ASSETS/LIABILITIES OF HOUSEHOLDS
     RACUS  =  J1L(RACUS) * ( .5 - ( ATAN(AS08*(J1D(FHT)/FHT+AS07-.5)+AS09*((J1D(FHT)/FHT+AS07-.5)**3.))/3.141593 ))
            +  RUS * ( .5 + ATAN(AS08*(J1D(FHT)/FHT+AS07-.5)+AS09*((J1D(FHT)/FHT+AS07-.5)**3.))/3.141593 )


EQ28AMI        RACUSG: AVERAGE COUPON RATE ON OUTSTANDING FOREIGN-CURRENCY ASSETS/LIABILITIES OF DOMESTIC GOVERNMENTS
     RACUSG  =  J1L(RACUSG) * ( .5 - ( ATAN(AS08*(J1D(FGBT)/FGBT+AS07-.5)+AS09*((J1D(FGBT)/FGBT+AS07-.5)**3.))/3.141593
              ) )  +  RUS * ( .5 + ATAN(AS08*(J1D(FGBT)/FGBT+AS07-.5)+AS09*((J1D(FGBT)/FGBT+AS07-.5)**3.))/3.141593 )


EQ29IGI         GTIN: ANNUAL INTEREST PAYMENTS ON GOVERNMENT DEBT
     GTIN  =  RAC * J2A(LGT)   -   RACUSG * PFX * J2A(FGBT)
```

APPENDIX B   MODEL EQUATIONS

```
EQ30IGR         UIB: ANNUAL UNEMPLOYMENT INSURANCE BENEFIT RATE

     UIB  =  RUIB * WS


EQ31UGD        GEXPNW: CURRENT-DOLLAR NON-WAGE GOVERNMENT EXPENDITURES

     GEXPNW  =  GNWT * P * UGPBSS


EQ32LGD         LG: EMPLOYMENT IN THE GOVERNMENT SECTOR

     LG  =  LSS * LGST


EQ33IGI         GEXPW: CURRENT-DOLLAR GOVERNMENT WAGE EXPENDITURES

     GEXPW  =  W * WREL * LG


EQ34IGI        TAXIP: TARIFFS AND INDIRECT TAXES ON IMPORTS

     TAXIP  =  ( RINDT*( 1 + RTAR ) + RTAR) * PMNEID * MNEID


EQ35IGI        TAXIC: INDIRECT TAXES ON DOMESTIC NON-ENERGY GOODS

     TAXIC  =  RINDT * ( P/( 1 + RINDT ) ) * UGPC


EQ36IGI         RMTAX: MARGINAL TAX RATE ON PERSONAL INCOME

     RMTAX  =  1  -  AG40 * ( 1 - RATAXN )
```

```
EQ37IGL         ROYH: ENERGY INDIRECT TAXES CALCULATED TO EQUATE RETURNS TO CAPITAL IN ENERGY AND NON-ENERGY SECTORS

    ROYH  =  PEN * ( ENC + J1D(INVENT) )  +  PENW * PFX * ( XEN - MEN )  -  W * LEN  -  ( P*(RRK-(DELK*PI/P)*
             (J2A(KCT)/(J2A(KCT)+INVCD)-J2A(KENT)/(J2A(KENT)+(PEN/P)*J2A(INVENT))))/(1+RINDT)) * ( J2A(KENT)
             + (PEN/P)*J2A(INVENT) )


EQ38IGR         ROY: ROYALTIES AND REMITTANCES FROM CROWN CORPORATIONS

    ROY  =  ( ROYH + ROYEX) * ( QTIME.LT.3. )  +  ROYH * RTR * ( QTIME.GE.3. )


EQ39IGR         RATAX: AVERAGE PERSONAL INCOME TAX RATE

    RATAX  =  RTAXB  +  RRGTIN


EQ40IGI         TAXP: DIRECT TAX REVENUE FROM PERSONS

    TAXP  =  RATAX * YPERS


EQ41IGI         GFR: GOVERNMENT FINANCING REQUIREMENT

    GFR  =  GEXPW  +  GEXPNW  +  GTIN  +  RNU * LS * UIB  +  TRANSP  -  TAXIP  -  TAXIC  -  TAXP  -  TCC  -  ROY  +  TRANSF


EQ42AGS         LGT: TOTAL DOMESTIC-CURRENCY GOVERNMENT DEBT, END OF YEAR

    J1D(LGT)  =  GFR  -  J1D(HT)  +  PFX * J1D(FGBT+FGRT)
```

```
APPENDIX B   MODEL EQUATIONS
----------   ---------------


EQ43AGS        LGDT: STOCK OF GOVERNMENT DEBT HELD BY DOMESTIC RESIDENTS, END OF YEAR
    LGDT  =  LGT  -  LGFT


EQ44IGR        RRGTIN: PORTION OF PERSONAL TAX RATE ASSOCIATED WITH PAYMENT OF REAL INTEREST ON GOVERNMENT DEBT
    RRGTIN  =  (  GTIN - (J2A(LGT)-PFX*J2A(FGBT)) * .01*J1P(PC)  ) / YPERS


EQ45KFD        KCT: ACTUAL STOCK OF NON-ENERGY CAPITAL, END OF YEAR
    LOG(KCT)  =  ( ( 2. - (AF12+AF15*(STIME.GT.4.5)))*J1L(LOG(KCT)) + 2.*( DNUCSS + (AF12+AF15*(STIME.GT.4.5))*
                 LOG(KCD) ) + ( 1 + AF16*(1+AF12+AF15*(STIME.GT.4.5)))*( AF34*LOG(SALN/UGPCSS) + AF20*LOG(PS/PC)
                 + AF19*LOG((RRK*(J2A(KCT)+INVCD))/(RRKSS*(KCD+INVCD))) + AF56*LOG(TOBQ/TOBQSS) ) + ( 1 - (AF16*(3.
                 -(AF12+AF15*(STIME.GT.4.5)))))*J1L(AF34*LOG(SALN/UGPCSS)+AF20*LOG(PS/PC)+AF19*LOG((RRK*(J2A(KCT)
                 +INVCD)/(RRKSS*(KCD+INVCD)))+AF56*LOG(TOBQ/TOBQSS))) / ( 2.+AF12+AF15*(STIME.GT. 4.5) )


EQ46EFD        ENC: ANNUAL ENERGY USE BY FIRMS
    LOG(ENC)  =  ( ( 2. - (AF23+AF24*(STIME.GT.4.5)))*J1L(LOG(ENC)) + 2.*( DNUCSS + (AF23+AF24*(STIME.GT.4.5))*
                 J2A(LOG(ENCD)) ) + ( 1 + AF13*(1+AF23+AF24*(STIME.GT.4.5)))*( AF29*LOG(PENRSS/PENR) + AF33*(CAPU
                 -CAPUSS) + AF26*LOG(INVCD/J2A(INVCT)) ) + ( 1 - (AF13*(3.-(AF23+AF24*(STIME.GT.4.5)))))*J1L(AF29*
                 LOG(PENRSS)/PENR)+AF33*(CAPU-CAPUSS)+AF26*LOG(INVCD/J2A(INVCT)))) / ( 2.+AF23+AF24*(STIME.GT. 4.5) )
```

```
EQ47LFD         LC: EMPLOYMENT IN THE NON-ENERGY PRIVATE SECTOR, ANNUAL AVERAGE

     LOG(LC)  =  ( ( 2. - AF45)*J1L(LOG(LC)) + 2.*( DNPOP + AF45*J2A(LOG(LCD)) ) + ( 1 + AF51*(1+AF45) )*
                 ( AF48*LOG(INVCD/J2A(INVCT)) + AF50*LOG(WS/W) + AF52*(CAPU-CAPUSS) ) + ( 1-(AF51*(3.-AF45)) )*
                 J1L(AF48*LOG(INVCD/J2A(INVCT))+AF50*LOG(WS/W)+AF52*(CAPU-CAPUSS))) / ( 2. + AF45 )


EQ48UFL         UGPBSS: ZERO-EXCESS-PROFIT, FULL-EMPLOYMENT, REAL PRIVATE SECTOR OUTPUT (PERCEIVED POTENTIAL OUTPUT)

     UGPBSS  =  UGPCSS  +  ( PEN/P) * J1D(INVENT)  +  ( PENW/P) * PFX * ( XEN - MEN )


EQ49KFI         IC: ANNUAL GROSS INVESTMENT IN NON-ENERGY SECTOR

     IC  =  J1D(KCT)  +  DELK * J2A(KCT)


EQ50EFS         UGPEN: ENERGY OUTPUT

     UGPEN  =  ENC  +  J1D(INVENT)  +  XEN  -  MEN


EQ51UFS         CAPU: CAPACITY UTILIZATION RATE

     CAPU  =  CAPUSS  +  AF43 * LOG(SALES/UGPCSS)  +  AF14 * LOG(RRK*(J2A(KCT)+INVCD)/(RRKSS*(KCD+INVCD)))
              + AF38 * LOG(INVCD/J2A(INVCT))


EQ52UFS         UGPC: REAL NON-ENERGY OUTPUT

     UGPC  =  AF10 * ( ( AF08*AF05*((AF04*((J2A(KCT)*CAPU)**AF06)+(1-AF04)*(ENC**AF06))**(AF07/AF06))
              + (1-AF05)*((LC * PRODL)**AF07))**( 1/AF07 ) )
```

APPENDIX B   MODEL EQUATIONS

```
EQ53UFI        UGPB: TOTAL REAL PRIVATE SECTOR OUTPUT IN UGPC UNITS
     UGPB  =  UGPC  +  ( PEN/P) * J1D(INVENT)  +  ( PENW/P) * PFX * ( XEN - MEN )


EQ54UAD        XNEID: EXPORTS OF GOODS AND SERVICES EXCLUDING ENERGY AND INVESTMENT-INCOME RECEIPTS
     LOG(XNEID)  =  AT11  +  AT12 * J3A(LOG(PXNEID/(PFX*PW)))  +  AT13 * DUMCAR  +  AT16 * LOG(YW)
                    +  AT14 * (CAPU - CAPUSS )


EQ55UHD        MNEID: IMPORTS OF GOODS AND SERVICES EXCLUDING ENERGY AND INVESTMENT-INCOME PAYMENTS
     LOG(MNEID)  =  AT01  +  AT02 * J3A(LOG(P/(PMNEID*(1+RTAR))))  +  AT04 * LOG(UGPBSS)
                    +  AT03 * ( CAPU - CAPUSS )  +  AT05 * DUMCAR


EQ56UAI        XBAL: CURRENT ACCOUNT BALANCE (ADJUSTED FOR UNREPATRIATED PROFITS)
     XBAL  =  PXNEID * XNEID  -  PMNEID*MNEID  +  PENW * PFX * ( XEN - MEN )  -  RAC * J2A(LGFT)
             +  PFX * ( RACUS*J2A(FHT) + RACUSG*J2A(FGBT) )  -  FOPRO  -  TRANSF


EQ57AAS        FHT: NET FOREIGN ASSETS OF HOUSEHOLDS (LIABILITIES IF NEGATIVE)
     PFX * J1D(FHT)  =  XBAL  +  PI * J1D(KWT)  -  PFX * ( J1D(FGRT) + J1D(FGBT) )  +  J1D(LGFT)


EQ58IGR        TRANSP: CURRENT-DOLLAR GOVERNMENT TRANSFERS TO PERSONS
     TRANSP  =  TAXIP  +  TST * PC * UGPBSS
```

```
EQ59AHD        PEQ: EQUITY PRICE INDEX (SUM OF 3 ASSET-DEMAND FUNCTIONS; BONDS, FOREIGN ASSETS, EQUITIES)
    ( PBG*J1L(LGDT) + .5*J1D(LGDT) + PFX*( PBFH*J1L(FHT) + .5*J1D(FHT) ) + PEQ*J2A(QEQT)) / V  =  AS30 + AS40 + AS50
                 +  ( AS41 - AS51 ) * ( 1 - RATAX ) * ( RGE - AE88 - RFE )  +  ( AS42 - AS31) * ( 1 - RATAX ) *
                 ( RGE - REQE + AE89 )  +  ( AS32 - AS52 ) * ( 1 - RATAX ) * ( REQE - RFE - AE88 - AE89 )  +  AS33 *
                 (1-RATAX) * REQE  + AS53 * (1 - RATAX) * RFE  +  AS43 * (1 - RATAX) * ( RGE - DNPCE )
                 + AS44*DNPCE +(AS36+AS46+AS56)*( QTIME*( QTIME.LT.10.5 ) + 10.*( QTIME.GT.10.5 ) )


EQ60AMP        PBG: AVERAGE PRICE OF OUTSTANDING GOVERNMENT BONDS (PAR=1.0)
    PBG  =  J1L(RAC)/RG  +  ( 1 - J1L(RAC)/RG ) * EXP(-RG*(1-RATAX)/AS06 )


EQ61IGR        TCC: PROFITS-TAX REVENUE FROM CORPORATIONS
    TCC  =  RBTAX * AG34 * YB


EQ62AMI        PFX: CANADIAN DOLLAR PRICE OF U.S. DOLLAR
    LOG(PFX)  =  LOG(PFXE)  +  RUS  -  RFE


EQ63AFS        QEQT: STOCK OF DOMESTICALLY HELD CLAIMS TO CAPITAL, END OF YEAR
    PEQ * J1D(QEQT)  =  PI * ( J1D(KENT+KCT) - J1D(KWT) )  +  P * J1D(INVCT)  +  PEN * J1D(INVENT)
```

```
APPENDIX B   MODEL EQUATIONS
----------   ---------------

EQ64AHD        RGE: EXPECTED HOLDING-PERIOD YIELD ON GOVERNMENT BONDS (INVERTED DEMAND FUNCTION)
     RGE  =  REQE  -  AE89  +  ( 1/( AS42*(1-RATAX) ) ) * ( ( PFX*(PBFH*J1L(FHT)+.5*J1D(FHT)) + PBG*J1L(LGDT)
             + .5*J1D(LGDT))/V - ( AS40 + AS50 + (AS46+AS56)*(QTIME*(QTIME.LT.10.5)+10.*(QTIME.GT.10.5)) + (AS41-AS51)*
             (1-RATAX)*(RGE-RFE-AE88) + AS43*(1-RATAX)*(RGE-DNPCE) + AS44*DNPCE + AS52*(1-RATAX)*(RFE-REQE+AE88+AE89)
             + AS53 * (1-RATAX)*RFE ) )


EQ65AHD        RFE: EXPECTED DOMESTIC HOLDING-PERIOD YIELD ON FOREIGN ASSETS (INVERTED DEMAND FUNCTION)
     RFE  =  ( 1/( AS51+AS52+AS53 ) ) * ( AS51*( RGE - AE88 ) + AS52*( REQE - AE88 - AE89 ) + ( (PFX*(PBFH*J1L(FHT)
             + .5*J1D(FHT)))/V - AS50 - (AS56*(QTIME*(QTIME.LT.10.5)+10.*(QTIME.GT.10.5))))/( 1 - RATAX ) )


EQ66AMI        TOBQ: RATIO OF MARKET VALUE OF CAPITAL TO ITS REPLACEMENT COST
     TOBQ  =  PEQ * J2A(QEQT) / (  PI*J2A(KENT+KCT-KWT) + P*J2A(INVCT) + PEN*J2A(INVENT)  )


EQ67AMI        V: FINANCIAL WEALTH, ANNUAL AVERAGE MARKET VALUE
     V  =  J2A(HT)  +  PBG * J1L(LGDT)  +  .5 * J1D(LGDT)  +  PEQ * J2A(QEQT)  +  PFX * ( PBFH*J1L(FHT) + .5*J1D(FHT) )


EQ68AML        TOBQSS: LONG-RUN RATIO OF THE MARKET VALUE OF CAPITAL TO ITS REPLACEMENT COST
     TOBQSS  =  AS91 * J1L(TOBQ)  +  ( 1 - AS91) * J1L(TOBQSS)


EQ69UMI        PRELC: LONG-RUN RELATIVE PRICE, FIRM'S AVERAGE REVENUE/CONSUMPTION PRICE
     PRELC  =  J1L(PRELC)  +  AP20 * J1L(P/PC-PRELC)
```

```
EQ70AME         RG: BEFORE-TAX YIELD TO MATURITY ON GOVERNMENT BONDS

     RG  =  RGE  -  AE66 * LOG(AE50+(1-AE50)/PBG)


EQ71AMI          RRFINE: EXPECTED REAL RATE OF RETURN, AFTER TAX, ON HOUSEHOLD'S PORTFOLIO

     RRFINE  =  ( RGE*( PBG*J1L(LGDT) + .5*J1D(LGDT) ) + REQE*PEQ*J2A(QEQT) + RFE*PFX*( PBFH*J1L(FHT)
                + .5*J1D(FHT) ) ) / V  -  DNPCE


EQ72IHI         VHPVW: HUMAN WEALTH - WAGE AND NORMAL UNEMPLOYMENT COMPONENT

     VHPVW  =  AH00 * NPOP * ( ( PRODL*WRESS*(1-RATAXN)*PRELC)/( 1 + RINDT ) ) * ( ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24
               -(AH20*RDRH+(1-AH20)*RRFINE))) - 1)/( AH25*GWAGE + (1-AH25)*AE24 - (AH20*RDRH+(1-AH20)*RRFINE) ) + ( EXP
               (AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE)))*(EXP((AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24
               -(AH21*RDRH+(1-AH21)*RRFINE)))-1))/( AH26*GWAGE + (1-AH26)*AE24 - (AH21*RDRH+(1-AH21)*RRFINE) ) + ( EXP
               (AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE))+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-(AH21
               *RDRH+(1-AH21)*RRFINE)))*(EXP((AH37-AH36)*(AH27*GWAGE+(1-AH27)*AE24-RDRH))-1))/( AH27*GWAGE + (1-AH27
               )*AE24 - RDRH ) + ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE))+(AH36-AH35)*(AH26
               *GWAGE+(1-AH26)*AE24-(AH21*RDRH+(1-AH21)*RRFINE))+(AH37-AH36)*(AH27*GWAGE+(1-AH27)*AE24-RDRH))*(EXP(
               (AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24-RDRH))-1))/( AH28*GWAGE + (1-AH28)*AE24 - RDRH ) - ( EXP(AH35*
               (AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE))+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-(AH21*RDRH
               +(1-AH21)*RRFINE))+(AH37-AH36)*(AH27*GWAGE+(1-AH27)*AE24-RDRH)+(AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24
               -RDRH))/(AE24-RDRH) ) ) * ( 1 - RNAT + RNAT*( UIB/(WS*(1-(QTXRFM*RATAXN))) ) )
```

APPENDIX B **MODEL EQUATIONS**

```
EQ73IHI         VHPVU: HUMAN WEALTH - ADJUSTMENT FOR CYCLICAL UNEMPLOYMENT
     VHPVU  =  AHOO * NPOP * ( ( PRODL*WRESS*(1-RATAXN)*PRELC)/( 1 + RINDT ) ) * ( ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24
               -(AH20*RDRH+(1-AH20)*RRFINE+.6))) - 1)/( AH25*GWAGE + (1-AH25)*AE24 - (AH20*RDRH+(1-AH20)*RRFINE+.6)
               ) + ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE+.6)))*(EXP((AH36-AH35)*(AH26*GWAGE
               +(1-AH26)*AE24-(AH21*RDRH+(1-AH21)*RRFINE+.6)))-1))/( AH26*GWAGE + (1-AH26)*AE24 - (AH21*RDRH+(1-AH21
               )*RRFINE+.6) ) + ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE+.6))+(AH36-AH35)*(AH26
               *GWAGE+(1-AH26)*AE24-(AH21*RDRH+(1-AH21)*RRFINE+.6)))*(EXP((AH37-AH36)*(AH27*GWAGE+(1-AH27)*AE24-RDRH
               +.6))-1))/( AH27*GWAGE + (1-AH27)*AE24 - RDRH + .6 ) + ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH
               +(1-AH20)*RRFINE+.6))+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-(AH21*RDRH+(1-AH21)*RRFINE+.6))+(AH37-AH36
               )*(AH27*GWAGE+(1-AH27)*AE24-RDRH+.6))*(EXP((AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24-RDRH+.6))-1))/( AH28
               *GWAGE + (1-AH28)*AE24 - RDRH + .6 ) - ( EXP(AH35*(AH25*GWAGE+(1-AH25)*AE24-(AH20*RDRH+(1-AH20)*RRFINE
               +.6))+(AH36-AH35)*(AH26*GWAGE+(1-AH26)*AE24-(AH21*RDRH+(1-AH21)*RRFINE+.6))+(AH37-AH36)*(AH27*GWAGE+
               (1-AH27)*AE24-RDRH+.6)+(AH38-AH37)*(AH28*GWAGE+(1-AH28)*AE24-RDRH+.6))/(AE24-RDRH+.6) ) ) * ( RNAT - RNU
               + ( UIB/(WS*(1-(QTXRFM*RATAXN))))*( RNU - RNAT ) )


EQ74IHI         VHPVN: HUMAN WEALTH VALUED AT LONG-RUN EXPECTED REAL WAGE AND EXPECTED UNEMPLOYMENT
     VHPVN  =  VHPVT  +  VHPVW  +  VHPVU


EQ75IHI         VHPV: HUMAN WEALTH VALUED AT AVERAGE EXPECTED REAL WAGE AND EXPECTED UNEMPLOYMENT
     VHPV   =  VHPVT  +  ( 1 - AH97 + AH97*W*PS/( PC*WS ) ) * VHPVW  +  VHPVU
```

```
EQ76LHS         LS: LABOUR FORCE (PARTICIPATION RATE)
     LS / NPOP  =  AH00  -  ( ( AH01*AH02*((1+NK)**AH03)*((V/PC+VHPVN)/NPOP))/( (1-RMTAX+AH02*(1-RATAXN)*((1+NK)
                   **AH03) )*((WS/PS)*(1-RNU)+(UIB/PS)*RNU*((1-RMTAX)**(QTXRFM-1))) ) )  +  AH06 * LOG(PS/PC) *
                   ( V/PC + VNSS )/NPOP  +  AH62*( VHPV - VHPVN ) / NPOP


EQ77UHD         CON: REAL HOUSEHOLD EXPENDITURES ON DOMESTIC GOODS AND IMPORTS
     CON / NPOP  =  ( AH01*( (V/PC+VHPVN)/NPOP ) ) / ( 1 + ( AH02*(1-RATAXN)*((1+NK)**AH03))/( 1 - RMTAX ) )
                    +  AH05 *  LOG(PS/PC) * ( V/PC + VNSS )/NPOP  +  AH61*( VHPV - VHPVN ) / NPOP
                    +  AH04 * ( QTIME.LE. - 2. )


EQ78UMD         SALES: FINAL SALES WITH NORMAL INVENTORY GROWTH
     SALES  =  CON  +  IC  +  IEN  +  GEXPNW / PG  +  XNEID  -  MNEID  +  DNUCSS * INVCD


EQ79LMI          RNU: RATE OF UNEMPLOYMENT
     RNU  =  ( LS - LC - LEN - LG) / LS


EQ80UMI          INVCT: STOCK OF INVENTORIES, NON-ENERGY SECTOR, END OF YEAR
     J1D(INVCT)  =  UGPC  -  ( CON + IC + IEN + GEXPNW/PG + XNEID - MNEID )


EQ81EGR         PENR: REAL ENERGY PRICE AT FACTOR COST
     PENR  =  ( 1 - AG81) * J1L(PENR)  +  AG81 * J2A(PENRSS)
```

APPENDIX B   MODEL EQUATIONS

```
EQ82UMP        PD: PRICE OF DOMESTIC OUTPUT IN DOMESTIC MARKET

     .01 * J1P(PD)  =  (STIME.LT. .5) * ( AP54 + AP55* (QTIME.GT. -2.5) )  +  AP56 * (QTIME.GT.4.5)*(QTIME.LT.7.5)
                       +  ( QTIME.GT. -2.5) * DNPDE  +  AP50 * J2A(LOG(PS/PC))  +  AP51 * J2A(LOG(SALES/UGPCSS))
                       +  AP52 * J2A(LOG(INVCD/J2A(INVCT)))  +  AP53 * J1D(LOG(1+RINDT))


EQ83LMP         W: PRIVATE SECTOR ANNUAL WAGE

     .01 * J1P(W)  =  AP60 * J2A(LOG(WS/W))  +  AP61 * J2A(RNAT-RNU)  +  DNPRL  +  (AP65 + AP66*QTIME) * (QTIME.LT.8.5)
                      +  AP62 * ( QTIME.GT.4.5) * ( QTIME.LT.7.5 )  +  AP63 * ( AP64*DNPE + ( 1 - AP64)*DNPCE )
                      +  ( 1 - AP63) * J1L(AP64*.01*J1P(P)+(1-AP64)*.01*J1P(PC))


EQ84IHI         YPERS: PERSONAL INCOME

     YPERS  =  W * ( LC + LEN + WREL*LG )  +  TRANSP  -  TAXIP  +  RNU * LS * UIB  +  RAC * J2A(LGDT)
             +  RACUS * PFX * J2A(FHT)  +  YBD  -  FOPRO


EQ85UMI         PC: PRICE INDEX FOR CONSUMPTION

     LOG(PC)  =  AP02 * LOG(PD)  +  ( 1 - AP02) * LOG(PMNEID*(1+RTAR)*(1+RINDT)/AP03)


EQ86UMI         PI: INVESTMENT PRICE INDEX

     PI  =  PRELIP * P / ( 1 + RINDT )
```

```
EQ87UMI         PG: GOVERNMENT EXPENDITURE PRICE INDEX
    PG  =  PRELG * PD


EQ88EMI         PEN: ENERGY PRICE INDEX
    PEN  =  PENR * P / ( 1 + RINDT )


EQ89UMI         PMNEID: NON-ENERGY IMPORT PRICE INDEX
    PMNEID  =  PRELM * PFX * PW


EQ90UMP         PXNEID: NON-ENERGY EXPORT PRICE INDEX
    J1P(PXNEID)  =  AP76 * J1P(PD)  +  ( 1 - AP76) * J1P(PFX*PW)  +  AP77 * J1P(PCW2/PW)


EQ91IGI         CCAB: CAPITAL CONSUMPTION ALLOWANCES, PRIVATE SECTOR
    CCAB  =  DELK * J2A(KENT+KCT) * PI


EQ92IFI         YB: BUSINESS PROFITS (MODEL CONCEPT)
    YB  =  P * UGPB  -  W * ( LC + LEN )  -  CCAB  -  ROY  -  TAXIC


EQ93IFI         YBD: BUSINESS PROFITS AFTER PROFITS TAX
    YBD  =  YB  -  TCC
```

```
APPENDIX B   MODEL EQUATIONS
----------   ---------------

EQ94UMI         P: PRICE INDEX FOR NON-ENERGY BUSINESS OUTPUT (AVERAGE REVENUE INCLUSIVE OF INDIRECT TAXES)
    P  =  ( PC*CON + PI*( IC + IEN ) + GEXPNW + PXNEID*XNEID - PMNEID*MNEID ) / ( UGPC - J1D(INVCT) )


EQ95IAI         FOPRO: PROFITS ACCRUING TO FOREIGNERS
    FOPRO  =  YBD * J2A(KWT) / ( J2A(KCT+KENT+INVCT) + ( PEN/P)*J2A(INVENT) )


EQ96AMP         REQE: EXPECTED RATE OF RETURN ON CORPORATE LIABILITIES

    REQE  =  ( YBD - FOPRO) / ( PEQ*J2A(QEQT) )  +  AE40 * ( RRKSS - RRK )  +  J3A(.01*J1P(PI))


EQ97KMI         RRK: GROSS PROFIT RATE (RESIDUAL FACTOR RATE OF RETURN)
    RRK  * ( INVCD + J2A(KCT) )   =  UGPC  -  W/(P/(1+RINDT)) * LC  -  PEN/(P/(1+RINDT)) * ENC
                                          -  ( UGPCSS - WRESS*PRODL*LCD - PENRSS*ENCD - RRKSS*(KCD+INVCD) )


EQ98KML         RRKSS: MINIMUM-COST, FULL-EMPLOYMENT, ZERO-EXCESS-PROFIT, RETURN TO CAPITAL (NORMAL GROSS PROFIT RATE)
    RRKSS  =  ( KCD/( KCD + INVCD ) ) * CCRESS * CAPUSS  +  ( INVCD/( KCD + INVCD ) ) * CCRISS


EQ99UHP         RDRH: HOUSEHOLD REAL DISCOUNT RATE
    RDRH  =  J1L(RDRH)  +  AH99 * LOG(SALES/UGPCSS)
```

```
EQX01          GNE$: CURRENT DOLLAR GROSS NATIONAL EXPENDITURE (NATIONAL ACCOUNTS DEFINITION)

    GNE$  =  P * UGPB  +  GEXPW  +  PFX * ( RACUS*J2A(FHT) + RACUSG*J2A(FGBT) )  -  RAC*J2A(LGFT)  -  FOPRO  +  ZX01

EQX07          GNE: REAL (CONSTANT DOLLAR) GROSS NATIONAL EXPENDITURE (NATIONAL ACCOUNTS DEFINITION)

    GNE  =  UGPB  +  ( GEXPW + PFX*( RACUS*J2A(FHT) + RACUSG*J2A(FGBT) ) - RAC*J2A(LGFT) - FOPRO + ZX01) / PZ

EQX08          PGNE: GROSS NATIONAL EXPENDITURE DEFLATOR

    PGNE  =  GNE$ / GNE
```

APPENDIX C - Cross References

APPENDIX C-1	Endogenous Variables
APPENDIX C-2	Exogenous Variables
APPENDIX C-3	Parameters

APPENDIX C-1 CROSS REFERENCE FOR THE ENDOGENOUS VARIABLES

VARIABLE	FROM NORMALIZATION OF	ALSO APPEARS IN EQUATION(S)
BASETR	EQ12IHI	EQ13IHI
CAPU	EQ51UFS	EQ46EFD EQ47LFD EQ52UFS EQ54UAD EQ55UHD
CCAB	EQ91IGI	EQ92IFI
CCRDIF	EQ05KMP	EQ05KMP EQ11KML EQ14KML
CCRESS	EQ14KML	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ98KML
CCRISS	EQ11KML	EQ15LML EQ25UFL EQ98KML
CON	EQ77UHD	EQ78UMD EQ80UMI EQ94UMI
DNPCE	EQ03UHE	EQ59AHD EQ64AHD EQ71AMI EQ83LMP
DNPDE	EQ01UFE	EQ82UMP
DNPE	EQ02UFE	EQ83LMP
ENC	EQ46EFD	EQ37IGL EQ46EFD EQ52UFS EQ97KMI EQ50EFS
ENCD	EQ23EFL	EQ46EFD EQ97KMI
FHT	EQ57AAS	EQ27AMI EQ56UAI EQ57AAS EQ59AHD EQ64AHD EQ65AHD EQ67AMI EQ71AMI EQ84IHI EQX01 EQX07
FOPRO	EQ95IAI	EQ56UAI EQ84IHI EQ96AMP EQX01 EQX07
GEXPNW	EQ31UGD	EQ41IGI EQ78UMD EQ80UMI EQ94UMI
GEXPW	EQ33IGI	EQ41IGI EQX01 EQX07
GFR	EQ41IGI	EQ42AGS
GNE	EQX07	EQX08
GNE$	EQX01	EQX08

APPENDIX C-1 CROSS REFERENCE FOR THE ENDOGENOUS VARIABLES

VARIABLE	FROM NORMALIZATION OF	ALSO APPEARS IN EQUATION(S)
GTIN	EQ29IGI	EQ41IGI EQ44IGR
IC	EQ49KFI	EQ78UMD EQ80UMI EQ94UMI
INVCD	EQ25UFL	EQ37IGL EQ45KFD EQ46EFD EQ47LFD EQ51UFS EQ78UMD EQ82UMP EQ97KMI EQ98KML
INVCT	EQ80UMI	EQ46EFD EQ47LFD EQ51UFS EQ63AFS EQ66AMI EQ80UMI EQ82UMP EQ94UMI EQ95IAI
KCD	EQ22KFL	EQ45KFD EQ51UFS EQ97KMI EQ98KML
KCT	EQ45KFD	EQ37IGL EQ45KFD EQ49KFI EQ51UFS EQ52UFS EQ63AFS EQ66AMI EQ91IGI EQ95IAI EQ97KMI
KENT	EQ04KFI	EQ04KFI EQ37IGL EQ63AFS EQ66AMI EQ91IGI EQ95IAI
KWT	EQ06KAD	EQ06KAD EQ57AAS EQ63AFS EQ66AMI EQ95IAI
LC	EQ47LFD	EQ47LFD EQ52UFS EQ79LMI EQ84IHI EQ92IFI EQ97KMI
LCD	EQ20LFL	EQ21UFL EQ47LFD EQ97KMI
LG	EQ32LGD	EQ33IGI EQ79LMI EQ84IHI
LGDT	EQ43AGS	EQ59AHD EQ64AHD EQ67AMI EQ71AMI EQ84IHI
LGT	EQ42AGS	EQ26AMI EQ29IGI EQ42AGS EQ43AGS EQ44IGR
LS	EQ76LHS	EQ41IGI EQ79LMI EQ84IHI
LSS	EQ19LHL	EQ20LFL EQ32LGD
MNEID	EQ55UHD	EQ34IGI EQ56UAI EQ78UMD EQ80UMI EQ94UMI

APPENDIX C-1 CROSS REFERENCE FOR THE ENDOGENOUS VARIABLES

VARIABLE	FROM NORMALIZATION OF	ALSO APPEARS IN EQUATION(S)
P	EQ94UMI	EQ09EGR EQ69UMI EQ02UFE EQ08AHE EQ31UGD EQ35IGI EQ37IGL EQ48UFL EQ53UFI EQ55UHD EQ63AFS EQ66AMI EQ83LMP EQ86UMI EQ88EMI EQ92IFI EQ95IAI EQ97KMI EQX01
PBFH	EQ00AMP	EQ59AHD EQ64AHD EQ65AHD EQ67AMI EQ71AMI
PBG	EQ60AMP	EQ59AHD EQ64AHD EQ67AMI EQ70AME EQ71AMI
PC	EQ85UMI	EQ69UMI EQ03UHE EQ08AHE EQ12IHI EQ17UML EQ44IGR EQ45KFD EQ58IGR EQ75IHI EQ76LHS EQ77UHD EQ82UMP EQ83LMP EQ94UMI
PD	EQ82UMP	EQ01UFE EQ82UMP EQ85UMI EQ87UMI EQ90UMP
PEN	EQ88EMI	EQ37IGL EQ48UFL EQ53UFI EQ63AFS EQ66AMI EQ95IAI EQ97KMI
PENR	EQ81EGR	EQ81EGR EQ46EFD EQ88EMI
PENRSS	EQ10EMP	EQ15LML EQ81EGR EQ21UFL EQ22KFL EQ23EFL EQ46EFD EQ97KMI
PEQ	EQ59AHD	EQ63AFS EQ66AMI EQ67AMI EQ71AMI EQ96AMP
PFX	EQ62AMI	EQ09EGR EQ08AHE EQ29IGI EQ37IGL EQ42AAS EQ44IGR EQ48UFL EQ53UFI EQ54UAD EQ56UAI EQ57AAS EQ59AHD EQ64AHD EQ65AHD EQ67AMI EQ71AMI EQ84IHI EQ89UMI EQ90UMP EQX01 EQX07
PFXE	EQ08AHE	EQ08AHE EQ62AMI
PG	EQ87UMI	EQ78UMD EQ80UMI
PGNE	EQX08	NOT USED

APPENDIX C-1 CROSS REFERENCE FOR THE ENDOGENOUS VARIABLES

VARIABLE	FROM NORMALIZATION OF	ALSO APPEARS IN EQUATION(S)
PI	EQ86UMI	EQ37IGL EQ57AAS EQ63AFS EQ66AMI EQ91IGI EQ94UMI EQ96AMP
PMNEID	EQ89UMI	EQ34IGI EQ55UHD EQ56UAI EQ85UMI EQ94UMI
PRELC	EQ69UMI	EQ69UMI EQ16IHL EQ18LML EQ72IHI EQ73IHI
PS	EQ17UML	EQ01UFE EQ02UFE EQ03UHE EQ18LML EQ19LHL EQ45KFD EQ75IHI EQ76LHS EQ77UHD EQ82UMP
PXNEID	EQ90UMP	EQ54UAD EQ56UAI EQ90UMP EQ94UMI
QEQT	EQ63AFS	EQ59AHD EQ63AFS EQ66AMI EQ67AMI EQ71AMI EQ96AMP
RAC	EQ26AMI	EQ26AMI EQ29IGI EQ56UAI EQ60AMP EQ84IHI EQX01 EQX07
RACUS	EQ27AMI	EQ00AMP EQ27AMI EQ56UAI EQ84IHI EQX01 EQX07
RACUSG	EQ28AMI	EQ28AMI EQ29IGI EQ56UAI EQX01 EQX07
RATAX	EQ39IGR	EQ07IHE EQ40IGI EQ59AHD EQ60AMP EQ64AHD EQ65AHD
RATAXN	EQ07IHE	EQ07IHE EQ36IGI EQ16IHL EQ19LHL EQ72IHI EQ73IHI EQ76LHS EQ77UHD
RDRH	EQ99UHP	EQ05KMP EQ13IHI EQ16IHL EQ17UML EQ72IHI EQ73IHI EQ99UHP
REQE	EQ96AMP	EQ59AHD EQ64AHD EQ65AHD EQ71AMI
RFE	EQ65AHD	EQ59AHD EQ62AMI EQ64AHD EQ71AMI
RG	EQ70AME	EQ26AMI EQ60AMP
RGE	EQ64AHD	EQ59AHD EQ64AHD EQ65AHD EQ70AME EQ71AMI

APPENDIX C-1 CROSS REFERENCE FOR THE ENDOGENOUS VARIABLES

VARIABLE	FROM NORMALIZATION OF	ALSO APPEARS IN EQUATION(S)
RMTAX	EQ36IGI	EQ19LHL EQ76LHS EQ77UHD
RNU	EQ79LMI	EQ41IGI EQ73IHI EQ76LHS EQ83LMP EQ84IHI
ROY	EQ38IGR	EQ41IGI EQ92IFI
ROYH	EQ37IGL	EQ38IGR
RRFINE	EQ71AMI	EQ72IHI EQ73IHI
RRGTIN	EQ44IGR	EQ39IGR
RRK	EQ97KMI	EQ06KAD EQ37IGL EQ45KFD EQ51UFS EQ96AMP
RRKSS	EQ98KML	EQ45KFD EQ51UFS EQ96AMP EQ97KMI
SALES	EQ78UMD	EQ08AHE EQ24UFE EQ51UFS EQ82UMP EQ99UHP
SALN	EQ24UFE	EQ25UFL EQ45KFD
TAXIC	EQ35IGI	EQ41IGI EQ92IFI
TAXIP	EQ34IGI	EQ12IHI EQ41IGI EQ58IGR EQ84IHI
TAXP	EQ40IGI	EQ41IGI
TCC	EQ61IGR	EQ41IGI EQ93IFI
TOBQ	EQ66AMI	EQ68AML EQ45KFD
TOBQSS	EQ68AML	EQ68AML EQ45KFD
TRANSP	EQ58IGR	EQ12IHI EQ41IGI EQ84IHI
TTEN	EQ09EGR	EQ09EGR EQ10EMP
UGPB	EQ53UFI	EQ92IFI EQX01 EQX07
UGPBSS	EQ48UFL	EQ12IHI EQ31UGD EQ55UHD EQ58IGR
UGPC	EQ52UFS	EQ35IGI EQ53UFI EQ80UMI EQ94UMI EQ97KMI

APPENDIX C-1 CROSS REFERENCE FOR THE ENDOGENOUS VARIABLES

VARIABLE	FROM NORMALIZATION OF	ALSO APPEARS IN EQUATION(S)
UGPCSS	EQ21UFL	EQ08AHE EQ22KFL EQ23EFL EQ24UFE EQ45KFD EQ48UFL EQ51UFS EQ82UMP EQ97KMI EQ99UHP
UGPEN	EQ50EFS	NOT USED
UIB	EQ30IGR	EQ41IGI EQ72IHI EQ73IHI EQ76LHS EQ84IHI
V	EQ67AMI	EQ17UML EQ19LHL EQ59AHD EQ64AHD EQ65AHD EQ71AMI EQ76LHS EQ77UHD
VHPV	EQ75IHI	EQ76LHS EQ77UHD
VHPVN	EQ74IHI	EQ76LHS EQ77UHD
VHPVT	EQ13IHI	EQ16IHL EQ74IHI EQ75IHI
VHPVU	EQ73IHI	EQ74IHI EQ75IHI
VHPVW	EQ72IHI	EQ74IHI EQ75IHI
VNSS	EQ16IHL	EQ17UML EQ19LHL EQ76LHS EQ77UHD
W	EQ83LMP	EQ33IGI EQ37IGL EQ47LFD EQ75IHI EQ83LMP EQ84IHI EQ92IFI EQ97KMI
WRESS	EQ15LML	EQ16IHL EQ18LML EQ21UFL EQ22KFL EQ23EFL EQ72IHI EQ73IHI EQ97KMI
WS	EQ18LML	EQ19LHL EQ30IGR EQ47LFD EQ72IHI EQ73IHI EQ75IHI EQ76LHS EQ83LMP
XBAL	EQ56UAI	EQ57AAS
XNEID	EQ54UAD	EQ56UAI EQ78UMD EQ80UMI EQ94UMI
YB	EQ92IFI	EQ61IGR EQ93IFI
YBD	EQ93IFI	EQ84IHI EQ95IAI EQ96AMP
YPERS	EQ84IHI	EQ40IGI EQ44IGR

APPENDIX C-2 CROSS REFERENCE FOR THE EXOGENOUS VARIABLES

VARIABLE	IS USED IN EQUATION(S)
CAPUSS	EQ14KML EQ22KFL EQ46EFD EQ47LFD EQ14KML EQ22KFL EQ55UHD EQ98KML
CCRW	EQ05KMP EQ11KML EQ14KML
CCTAX1	EQ14KML
CCTAX2	EQ11KML
DELK	EQ05KMP EQ11KML EQ04KFI EQ37IGL EQ49KFI EQ91IGI
DNHE	EQ17UML
DNPOP	EQ47LFD
DNPRL	EQ83LMP
DNUBSS	EQ06KAD EQ17UML
DNUCSS	EQ45KFD EQ46EFD EQ78UMD
DUMCAR	EQ54UAD EQ55UHD
FGBT	EQ28AMI EQ29IGI EQ42AGS EQ44IGR EQ56UAI EQ57AAS EQX01 EQX07
FGRT	EQ42AGS EQ57AAS
GNWT	EQ31UGD
GTRAN	EQ13IHI
GWAGE	EQ16IHL EQ72IHI EQ73IHI
HT	EQ17UML EQ42AGS EQ67AMI
IEN	EQ04KFI EQ78UMD EQ80UMI EQ94UMI
INVENT	EQ37IGL EQ48UFL EQ53UFI EQ63AFS EQ66AMI EQ95IAI EQ50EFS

APPENDIX C-2 CROSS REFERENCE FOR THE EXOGENOUS VARIABLES

VARIABLE	IS USED IN EQUATION(S)
LEN	EQ20LFL EQ37IGL EQ79LMI EQ84IHI EQ92IFI
LGFT	EQ43AGS EQ56UAI EQ57AAS EQX01 EQX07
LGST	EQ20LFL EQ32LGD
MEN	EQ37IGL EQ48UFL EQ53UFI EQ56UAI EQ50EFS
NK	EQ19LHL EQ76LHS EQ77UHD
NPOP	EQ12IHI EQ13IHI EQ16IHL EQ19LHL EQ72IHI EQ73IHI EQ76LHS EQ77UHD
PCW2	EQ90UMP
PENW	EQ10EMP EQ37IGL EQ48UFL EQ53UFI EQ56UAI
PRELEN	EQ09EGR
PRELG	EQ87UMI
PRELIP	EQ14KML EQ86UMI
PRELM	EQ89UMI
PRODL	EQ16IHL EQ18LML EQ21UFL EQ52UFS EQ72IHI EQ73IHI EQ97KMI
PUS	EQ08AHE
PW	EQ09EGR EQ10EMP EQ54UAD EQ89UMI EQ90UMP
PZ	EQX07
QTIME	EQ15LML EQ25UFL EQ38IGR EQ59AHD EQ64AHD EQ65AHD EQ77UHD EQ82UMP EQ83LMP
QTXRFM	EQ16IHL EQ19LHL EQ72IHI EQ73IHI EQ76LHS
RATAXUS	EQ00AMP

APPENDIX C-2 CROSS REFERENCE FOR THE EXOGENOUS VARIABLES

VARIABLE	IS USED IN EQUATION(S)
RBTAX	EQ61IGR
REN	EQ09EGR
RINDT	EQ09EGR EQ16IHL EQ18LML EQ34IGI EQ35IGI EQ37IGL EQ72IHI EQ73IHI EQ82UMP EQ85UMI EQ86UMI EQ88EMI EQ97KMI
RNAT	EQ16IHL EQ19LHL EQ20LFL EQ72IHI EQ73IHI EQ83LMP
ROYEX	EQ38IGR
RRKUS	EQ06KAD
RTAR	EQ34IGI EQ55UHD EQ85UMI
RTAXB	EQ39IGR
RTR	EQ38IGR
RUIB	EQ16IHL EQ19LHL EQ30IGR
RUS	EQ00AMP EQ28AMI EQ27AMI EQ62AMI
STIME	EQ06KAD EQ08AHE EQ45KFD EQ46EFD EQ82UMP
TRANSF	EQ41IGI EQ56UAI
TST	EQ12IHI EQ58IGR
WREADJ	EQ15LML
WREL	EQ33IGI EQ84IHI
XEN	EQ37IGL EQ48UFL EQ53UFI EQ56UAI EQ50EFS
YW	EQ54UAD
ZX01	EQX01 EQX07

APPENDIX C-3 CROSS REFERENCE FOR THE PARAMETERS

PARAMETER	VALUE	T-RATIO	APPEARS IN EQUATION(S)
AA04	0.50000	IMPOSED	EQ09EGR
AE10	0.25000	IMPOSED	EQ03UHE
AE11	0.25000	IMPOSED	EQ02UFE
AE12	0.25000	IMPOSED	EQ01UFE
AE24	0.01300	IMPOSED	EQ13IHI EQ16IHL EQ72IHI EQ73IHI
AE40	0.05000	IMPOSED	EQ96AMP
AE50	0.30000	IMPOSED	EQ70AME
AE61	0.00000	IMPOSED	EQ08AHE
AE62	0.21000	IMPOSED	EQ08AHE
AE63	0.00000	IMPOSED	EQ08AHE
AE64	0.20000	IMPOSED	EQ08AHE
AE66	0.12000	IMPOSED	EQ70AME
AE67	1.00000	IMPOSED	EQ08AHE
AE88	0.00922	MEAN	EQ59AHD EQ64AHD EQ65AHD
AE89	0.00859	MEAN	EQ59AHD EQ64AHD EQ65AHD
AF01	0.26773	28.6	EQ15LML EQ25UFL
AF02	0.51194	5.4	EQ15LML EQ25UFL
AF03	0.01857	5.0	EQ15LML EQ25UFL
AF04	0.94091	247.1	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ52UFS
AF05	0.59226	20.7	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ52UFS
AF06	-0.33044	59.3	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ52UFS
AF07	-0.21985	7.2	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ52UFS
AF08	3.15450	4.5	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ52UFS
AF10	1541.8	4.5	EQ15LML EQ21UFL EQ22KFL EQ23EFL EQ52UFS
AF12	0.12907	2.9	EQ45KFD
AF13	0.30153	3.0	EQ46EFD
AF14	0.22590	2.6	EQ51UFS
AF15	0.10000	IMPOSED	EQ45KFD
AF16	0.09358	0.8	EQ45KFD
AF19	0.00000	IMPOSED	EQ45KFD
AF20	0.10635	2.4	EQ45KFD
AF23	0.05312	2.0	EQ46EFD
AF24	0.18000	IMPOSED	EQ46EFD
AF26	0.40000	3.6	EQ46EFD
AF29	0.11378	0.8	EQ46EFD
AF33	0.29294	2.3	EQ46EFD
AF34	0.22053	4.4	EQ45KFD
AF38	0.77700	1.6	EQ51UFS
AF40	0.60000	IMPOSED	EQ24UFE
AF43	0.58566	1.6	EQ51UFS
AF45	0.25894	1.7	EQ47LFD
AF48	0.23478	1.2	EQ47LFD
AF50	0.15900	2.2	EQ47LFD
AF51	0.41010	3.3	EQ47LFD

APPENDIX C-3 CROSS REFERENCE FOR THE PARAMETERS

PARAMETER	VALUE	T-RATIO	APPEARS IN EQUATION(S)
AF52	0.40378	5.1	EQ47LFD
AF56	0.02098	1.8	EQ45KFD
AG34	0.39121	40.1	EQ61IGR
AG40	0.86340	38.5	EQ36IGI
AG50	0.25000	IMPOSED	EQ07IHE
AG81	0.82690	6.3	EQ81EGR
AHOQ	0.79455	15.1	EQ16IHL EQ19LHL EQ72IHI EQ73IHI EQ76LHS
AH01	0.01483	197.5	EQ19LHL EQ76LHS EQ77UHD
AH02	0.05459	17.6	EQ19LHL EQ76LHS EQ77UHD
AH03	3.91370	25.9	EQ19LHL EQ76LHS EQ77UHD
AH04	149.81	5.7	EQ77UHD
AH05	0.00323	3.3	EQ77UHD
AH06	-0.108 E-6	1.2	EQ76LHS
AH11	0.30000	IMPOSED	EQ12IHI
AH20	0.95000	IMPOSED	EQ72IHI EQ73IHI
AH21	1.00000	IMPOSED	EQ72IHI EQ73IHI
AH25	1.00000	IMPOSED	EQ16IHL EQ72IHI EQ73IHI
AH26	0.75000	IMPOSED	EQ16IHL EQ72IHI EQ73IHI
AH27	0.50000	IMPOSED	EQ16IHL EQ72IHI EQ73IHI
AH28	0.25000	IMPOSED	EQ16IHL EQ72IHI EQ73IHI
AH30	0.90000	IMPOSED	EQ13IHI
AH31	0.68000	IMPOSED	EQ13IHI
AH32	0.51000	IMPOSED	EQ13IHI
AH33	0.20000	IMPOSED	EQ13IHI
AH34	0.00000	IMPOSED	EQ13IHI
AH35	1.00000	IMPOSED	EQ13IHI EQ16IHL EQ72IHI EQ73IHI
AH36	3.00000	IMPOSED	EQ13IHI EQ16IHL EQ72IHI EQ73IHI
AH37	6.00000	IMPOSED	EQ13IHI EQ16IHL EQ72IHI EQ73IHI
AH38	12.0000	IMPOSED	EQ13IHI EQ16IHL EQ72IHI EQ73IHI
AH61	0.00760	4.8	EQ77UHD
AH62	0.560 E-7	0.4	EQ76LHS
AH96	0.03243	MEAN	EQ05KMP
AH97	0.40000	IMPOSED	EQ75IHI
AH98	0.80000	IMPOSED	EQ05KMP
AH99	0.00300	IMPOSED	EQ99UHP
AP02	0.67860	10.8	EQ85UMI
AP03	1.18830	MEAN	EQ85UMI
AP20	0.70000	IMPOSED	EQ69UMI
AP50	0.19300	2.1	EQ82UMP
AP51	0.26180	2.5	EQ82UMP
AP52	0.35310	1.4	EQ82UMP
AP53	1.00000	IMPOSED	EQ82UMP
AP54	0.02170	4.4	EQ82UMP
AP55	-0.00660	0.1	EQ82UMP
AP56	-0.05540	5.3	EQ82UMP

APPENDIX C-3 CROSS REFERENCE FOR THE PARAMETERS

PARAMETER	VALUE	T-RATIO	APPEARS IN EQUATION(S)
AP60	0.10040	2.7	EQ83LMP
AP61	2.18780	7.6	EQ83LMP
AP62	-0.00400	0.6	EQ83LMP
AP63	1.00000	IMPOSED	EQ83LMP
AP64	0.48030	1.2	EQ83LMP
AP65	0.01220	5.6	EQ83LMP
AP66	-0.00160	4.9	EQ83LMP
AP76	0.59540	3.4	EQ90UMP
AP77	0.12620	2.5	EQ90UMP
AS00	-6.64060	241.7	EQ17UML
AS01	0.56030	1.4	EQ17UML
AS06	0.20371	4.8	EQ26AMI EQ60AMP
AS07	0.20371	IMPOSED	EQ00AMP EQ28AMI EQ27AMI
AS08	2.50000	IMPOSED	EQ28AMI EQ27AMI
AS09	30.0000	IMPOSED	EQ28AMI EQ27AMI
AS21	0.45569	2.1	EQ06KAD
AS24	0.01264	5.0	EQ06KAD
AS29	-0.09876	MEAN	EQ06KAD
AS30	0.87210	443.8	EQ59AHD
AS31	10.0000	0.8	EQ59AHD
AS32	0.29000	2.3	EQ59AHD
AS33	0.00000	IMPOSED	EQ59AHD
AS36	0.00230	6.3	EQ59AHD
AS40	0.09230	53.2	EQ59AHD EQ64AHD
AS41	1.24220	3.6	EQ59AHD EQ64AHD
AS42	10.0000	0.8	EQ59AHD EQ64AHD
AS43	0.19000	IMPOSED	EQ59AHD EQ64AHD
AS44	0.00000	IMPOSED	EQ59AHD EQ64AHD
AS46	-0.00150	4.5	EQ59AHD EQ64AHD
AS50	0.01170	6.4	EQ59AHD EQ64AHD EQ65AHD
AS51	1.24220	3.6	EQ59AHD EQ64AHD EQ65AHD
AS52	0.29000	2.3	EQ59AHD EQ64AHD EQ65AHD
AS53	0.00000	IMPOSED	EQ59AHD EQ64AHD EQ65AHD
AS56	-0.00090	2.5	EQ59AHD EQ64AHD EQ65AHD
AS91	0.10000	IMPOSED	EQ68AML
AT01	-1.63890	56.0	EQ55UHD
AT02	0.38358	2.3	EQ55UHD
AT03	0.93426	6.2	EQ55UHD
AT04	1.00000	IMPOSED	EQ55UHD
AT05	0.06281	12.4	EQ55UHD
AT11	1.91130	15.3	EQ54UAD
AT12	-0.78083	3.9	EQ54UAD
AT13	0.07016	10.1	EQ54UAD
AT14	-0.63071	3.5	EQ54UAD
AT16	1.00000	IMPOSED	EQ54UAD

REFERENCES

Abel, A. and O. Blanchard. "An Intertemporal Model of Saving and Investment." *Econometrica*, volume 51, no. 3, May 1983, pp. 675-692.

Armstrong, J. "CES Factor Demands." Research Department, Bank of Canada, RM-83-38, 1984 (unpublished).

Armstrong, J. (1985a). "Maximum Likelihood Estimation with Covariance Restrictions." Research Department, Bank of Canada, RM-83-56, 1985 (unpublished).

Armstrong, J. (1985b). "Discrete Representation of Continuous Relationship Nonlinear in Variables." Research Department, Bank of Canada, RM-83-75, 1985 (unpublished).

Armstrong, J. (1985c). "Maximum Likelihood Estimation with Qualitative Restrictions." Research Department, Bank of Canada, RM-83-84, 1985 (unpublished).

Armstrong, J. and D. Rose. "Discrete Time Representations of Continuous Processes." Research Department, Bank of Canada, RM-83-41, 1983 (unpublished).

Bergstrom, A.R., (ed.), *Statistical Inference in Continuous Time Economic Models*, Amsterdam: North Holland, 1976.

Boadway, R. "Corporate Taxation and Investment: A Synthesis of the Neo-classical Theory." *Canadian Journal of Economics*, vol. XIII, no. 2, May 1980, pp. 250-267.

Boothe, P.; K. Clinton; A. Côté; and D. Longworth. *International Asset Substitutability: Theory and Evidence for Canada*, Ottawa: Bank of Canada, 1985.

Brainard, W. and J. Tobin. "Pitfalls in Financial Model Building." *American Economic Review, Proceedings*, vol. 58, no. 2, 1968, pp. 99-122.

Feldstein, M. "Inflation, Tax Rules and Investment: Some Econometric Evidence." *Econometrica*, vol. 50, no. 4, July 1982, pp. 825-862.

Francis J. "Some Further Results of Estimation of a System of Supply and Demand Functions for Canadian Exports." Research Department, Bank of Canada, RM-79-84, 1979 (unpublished).

Helliwell, J.F.; M.E. MacGregor; and T. Padmore. "Economic Growth and Productivity in Canada, 1955-1990." Department of Economics, University of British Columbia, Discussion Paper No. 84-30, November 1984.

Helliwell, J.F.; R.N. McRae; P. Boothe; A. Hansson; M. Margolick; T. Padmore; A. Plourde; and R. Plummer. "Energy and the National Economy: An Overview of the Mace Model." Paper prepared for the 16th Annual Meeting of the Canadian Economics Association, University of Ottawa, June 1982.

Helliwell, J.F.; L.H. Officer; H.T. Shapiro; and I.A. Stewart. The Structure of RDX1. Bank of Canada Staff Research Studies, No. 3, Ottawa: Bank of Canada, 1969.

Helliwell, J.F.; H.T. Shapiro; G.S. Sparks; I.A. Stewart; F.W. Gorbet; and D.R. Stephenson, The Structure of RDX2. Bank of Canada Staff Research Studies, No. 7, (2 vols.), Ottawa: Bank of Canada, 1971.

Jonson, P.D. et al., (ed.), Conference in Applied Economic Research, Reserve Bank of Australia, 1977.

Martin, J. and P.R. Masson. "Exchange Rates and Portfolio Balance." NBER Working Paper No. 377, August 1979.

Masson, P.R. and R.B. Montador. "Consumption and Labour Supply in SAM." Research Department, Bank of Canada, RM-82-29, 1982 (unpublished).

Masson, P.R.; R.B. Montador; and J. Selody. "Household Consumption and Labour Supply: A Provisional Consensus." Research Department, Bank of Canada, RM-81-8, 1981 (unpublished).

Masson, P.R.; D. Rose; and J. Selody. Building a Small Macro-Model for Simulation: Some Issues. Technical Report No. 22, Ottawa: Bank of Canada, 1980.

Montador, R.B. Inventory Behaviour in Canada: Its Importance and Some Evidence of its Determinants. Technical Report, Ottawa: Bank of Canada, (forthcoming).

Murray, J.D. "The Tax Sensitivity of U.S. Direct Investment in Canadian Manufacturing." Journal of International Money and Finance, vol, 1, 1982, pp. 117-140.

Robertson, H. and M. McDougall (1982a). The Equations of RDXF, September 1980 Version. Technical Report No. 25, Ottawa: Bank of Canada, 1982.

Robertson, H. and M. McDougall (1982b). The Structure and Dynamics of RDXF, September 1980 Version. Technical Report No. 26, Ottawa: Bank of Canada, 1982.

Rose, D.; D. Lecavalier; and R.B. Montador. "Consumption and Labour Supply in SAM: Some Modifications to the Integral Approximations and the Introduction of a Variable Discount Rate." Research Department, Bank of Canada, RM-83-52, 1983 (unpublished).

Rose, D. and J. Selody. "The Government Sector of SAM." Research Department, Bank of Canada, RM-83-94, 1983 (unpublished).

Rose, D.; J. Selody; and P.R. Masson. "Simulated Macroeconomic Consequences of Alternative Paths of Adjustment of Canadian Energy Prices to World Levels." *Proceedings of the Fifth Pacific Basin Central Bank Economists' Conference: Supply-Side Shocks, the Balance of Payments and Monetary Policy*, (K. Clinton (ed.)) Ottawa: Bank of Canada, 1982.

Rose, D.; J. Selody; and J. St-Gelais. "The Influence of Some Adjustment Processes on the Response to Monetary Shocks in SAM." Working Paper No. W14 for the study *Price Flexibility and Business Cycle Fluctuation in Canada - A Survey* submitted to the Royal Commission on the Economic Union and Development Prospects for Canada. Bank of Canada 1985.

Selody, J. and K. Lynch. *Modelling Government Fiscal Behaviour in Canada*. Technical Report No. 34, Ottawa: Bank of Canada, 1983.

Stuber, G. *The Slowdown in Productivity Growth in the 1975-83 Period - A Survey of Possible Explanations*. Technical Report, Ottawa: Bank of Canada (forthcoming).